Abstract

Curvelet is often used in image multi-scale analysis for its description ability to the image edges such as curve and line characteristics. On the basis of analyzing several common algorithms of image fusion, a new multi-focus image fusion method based on Curvelet transform is proposed according to the question of image fusion. The principle of Curvelet transform is described and the fusion rule of coefficients is analyzed. Firstly, two different focus images were decomposed using Curvelet transform separately, and then in the Curvelet domain of the two transformed images, the new Curvelet coefficients were acquired by adopting an efficient fusion rule, in which the low-frequency coefficients were integrated using the weighted average method, and high-frequency coefficients were integrated using regional energy analysis method. Finally, the fused coefficients are reconstructed to obtain fusion results. Experiments are conducted on multi-focus images by using different methods and the performances were evaluated with the indicators such as mean error, variance and information entropy. Experimental results show that the method is more suitable for multi-focus image fusion than some other ways, and the fusion image will have more information.

Keywords: Depth of field, Multi-focus Image, Image Fusion, Curvelet Transform

1. Introduction

The use of image fusion techniques has gained significant popularity over the past decade. It is improved with the development of digital image processing and image analysis technology, and gradually demonstrates its wide range of applications in the automatic target recognition, remote sensing, medical image processing and other fields. In the image fusion research, multi-focus image fusion is one of the major categories with representative, which generally refers to the conditions that due to the limited depth-of-focus of optical lenses in CCD devices, it is often not possible to get an image that contains all relevant objects in focus. Consequently, the image obtained will not be in focus everywhere, i.e., if one object in the scene is in focus, another one will be out of focus [1]. In order to resolve that problem, many fusion techniques have been developed. The simplest image fusion method just takes the pixel-by-pixel average of the source images. The algorithms are rather easy to implement and time efficient [2]. This, however, often leads to undesirable side effects such as reduced contrast. In recent years, many researchers recognized that multi-scale transforms are very useful for analyzing the information content of images for the purpose of fusion. So, various alternatives based on multi-scale transforms have been proposed [3,4], such as Laplacian pyramid-based, gradient pyramid-based, ratio pyramid-based, et al.. The basic idea of multi-scale transform is to perform a multi-resolution decomposition on each source image, then integrate all these decompositions to produce a composite representation [5,6]. The fused image is finally reconstructed by performing an inverse multi-resolution transform. The pyramid-based method is simple and has good performance generally. However, in the pyramid reconstruction, it is sometimes unstable,

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especially when there are multiple significant differences in the source image, the fused image will appear plaques. Wavelet transform is a multi-resolution analysis method, too. It can decompose an image into an approximation and the lowest level of detail in different directions at different scales, and it is one of the most commonly used image fusion methods [9-11]. Wavelets are very effective in representing objects with isolated point singularities, while wavelet bases are not the most significant in representing objects with singularities along lines. As a consequence, the method based on the WT can not excavate the edge quality and detail information [12].

To above deficiencies of wavelet transform, Donoho et al. proposed the theory of Curvelet transform [13], which takes edge as the basic description element and is well suitable for the characteristics of image. The research results show that Curvelet transform theory can be better used in image denoising [14], feature extraction [15], image fusion [16], etc., and good results were obtained. But they used the first generation of Curvelet transform. In 2004, Candès, who proposed the theory of second generation of Curvelet transform [17,18], and in 2005 the fast implementation of second generation Curvelet transform algorithm is given. Compared with first generation Curvelet transform, the second generation can overcome many of the shortcomings, not only has the simple structure, but also greatly reduces the amount of data redundancy, and its fast algorithm is easier to be understood. Therefore, combining with the characteristics of multi-focus images, we discussed the principles of the second generation Curvelet transform and explored a new multi-focus image fusion method based on Curvelet transform in this paper. It is realized by adopting different strategies on low-frequency coefficients and high frequency coefficients to extract the features of the original images. Experimental results show that the algorithm is with satisfactory results.

The rest of this paper is organized as follows. The principle of Curvelet transform is described in section 2. In section 3, a new image fusion approach for multi-focus images based on Curvelet transform is presented; it describes different methods for merging the coefficients obtained during the Curvelet transform process and the main steps of this fusion method. After that, evaluation rules of fused image is proposed, which is followed by the discussion of experiments in section 5. Finally, in section 6, the conclusions are presented.

2. Curvelet transform

2.1. Continuous Curvelet transform

Suppose in a 2-dimensions space $\mathbb{R}^2$, with spatial variable $x$, with $\omega$ a frequency domain variable, and with $r$ and $\theta$ polar coordinates in the frequency-domain. There are two windows $W(r)$ and $V(t)$, which we will call the “radial window” and “angular window”, respectively. These are both smooth, nonnegative and real-valued, with $W$ taking positive real arguments and supported on $r \in (1/2,2)$ and $V$ taking real arguments and supported on $t \in (-1,1)$. These windows will always obey the admissibility conditions:

$$a \sum_{j=\infty}^{\infty} W^2 (2^j r) = 1 \quad r \in (3/4,3/2) c$$

$$\sum_{l=\infty}^{\infty} V^2 (t-l) = 1 \quad t \in (-1/2,1/2)$$

Now, for each scale $j \geq j_0$, we introduce the frequency window $U_j$ defined in the Fourier domain by
where \( \lfloor j/2 \rfloor \) is the integer part of \( j/2 \). Thus, \( U_j \) is a “wedge” window in polar coordinates which applied with scale-dependent window widths in each direction. Define the waveform \( \phi_j(x) \) by means of its Fourier transform \( \phi_j(\omega) = U_j(\omega) \), then, all Curvelets at scale \( 2^{-j} \) are obtained by rotations and translations of \( \Phi_j \). Introduce the equispaced sequence of rotation angles \( \theta_l = 2\pi \times 2^{-\lfloor j/2 \rfloor} \times l \) and the sequence of translation parameters \( k = (k_1, k_2) \in \mathbb{Z}^2 \). With these notations, we define Curvelets at scale \( 2^{-j} \), orientation angle \( \theta_l \) and position \( x_k^{(j)} = R_{\theta_l}^{-1}(k_1 \times 2^{-j} \times k_2 \times 2^{-j/2}) \) by

\[
\phi_{j,l,k}(x) = \phi_j[R_{\theta_l}(x - x_k^{(j-1)})]
\]

where \( R_\theta \) is the rotation by \( \theta \) and \( R_\theta^{-1} \) its inverse

\[
R_\theta = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad R_\theta^{-1} = R_\theta^T = R_{-\theta}
\]

Then the definition of Curvelet transform is:

\[
c(j,l,k) = \langle f, \phi_{j,l,k} \rangle = \int_{\mathbb{R}^2} f(x) \overline{\phi_{j,l,k}(x)} dx
\]

In the frequency domain, it is more convenient to realize Curvelet transform algorithm, the frequency domain Curvelet transform:

\[
c(j,l,k) = \frac{1}{(2\pi)^2} \int \hat{f}(\omega) \overline{\phi_{j,l,k}(\omega)} d\omega = \frac{1}{(2\pi)^2} \int \hat{f}(\omega) U_j(\omega) e^{i\phi_{j,l,k}(\omega)} d\omega
\]

As in wavelet theory, we also have coarse scale elements. We introduce the low-pass window \( W_0 \) obeying

\[
\left| W_0(r) \right|^2 + \sum_{j=0}^{\infty} \left| W(2^{-j} r) \right|^2 = 1
\]

and for \( k_1, k_2 \in \mathbb{Z} \), define coarse scale Curvelets as:

\[
\begin{align*}
\phi_{j_{0}, k}(x) &= \phi_{j_{0}}(x - 2^{-j_{0}} k) \\
\phi_{j_{0}}(\omega) &= 2^{-j_{0}} W_0(2^{-j_{0}} |\omega|)
\end{align*}
\]

Hence, coarse scale Curvelets are non-directional. The “full” Curvelet transform consists of the fine-scale directional elements and of the coarse-scale isotropic father wavelets. Figure 1 summarizes the key components of the construction.
2.2. Discrete Curvelet transform

In the continuous-time definition, the window $U_j$ divided the frequency domain into the rings with different angles smoothly. But this division does not fit the image of the two-dimensional Cartesian coordinate system. Therefore, it is instead of concentric squares $U_j$, which was expressed as

$$U_j(\omega) = \tilde{W}_j(\omega)V_j(\omega)$$

where, $\tilde{W}_j(\omega)$ is band-pass radius window with the expression:

$$\tilde{W}_j(\omega) = \sqrt{\Phi_j^2(\omega) - \Phi_j^2(\omega)} , j \geq 0$$

where $\Phi$ is defined as the product of low-pass one dimensional windows:

$$\Phi_j(\omega_1, \omega_2) = \phi(2^{-j} \omega_1)\phi(2^{-j} \omega_2)$$

Then, angular window is

$$V_j(\omega) = V(2^{1/2} \omega_2 / \omega_1) .$$

Introduce now the set of equispaced slopes $\tan \theta_l = l \cdot 2^{[l/2]}, l = -2^{[l/2]}, \ldots, 2^{[l/2]} - 1$, and define

$$U_{j,\theta}(\omega) = \tilde{W}_j(\omega)V_j(S_{\theta_l} \omega)$$

where $S_{\theta_l}$ is the shear matrix,

$$S_{\theta_l} = \begin{bmatrix} 1 & 0 \\ -\tan \theta_l & 1 \end{bmatrix}$$

The discrete Curvelet form is:
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\[ \tilde{\phi}_{j,k}(x) = 2^{3/4} \phi_{j,k} \left[ S_{b_j}^T (x - S_{b_j}^T b) \right] \] (15)

Where \( b \) is belong to the discrete values \( (k, 2^{-j}, k, 2^{-j/2}) \)

The discrete Curvelet transform is defined as:
\[ c(j, l, k) = \int \hat{f}(\omega) \tilde{U}_j(\omega) e^{i c b, S_{b_j}^T \omega} d\omega = \int \hat{f}(S_{b_j} \omega) \tilde{U}_j(\omega) e^{i c b, \omega} d\omega \] (16)

A Curvelet in the space domain and frequency domain expressions is shown in Figure 2.

3. Image fusion based on curvelet transform

Curvelet inherited the excellent features from wavelet analysis in special and frequency domains, and it has the better ability to express the edge of the image, the transformed energy is more concentrated. Therefore, the introduction of Curvelet transform image fusion, can be used to better extract Curvelet of the characteristics of the original image, the fused images will provide more information.

3.1. Fusion strategies

The key step in image fusion based on Curvelet transform is coefficient combination, namely, the process of merge the coefficients in an appropriate way in order to obtain the best quality in the fused image. To the multi-focus images of the same scene but different focal planes, their low-frequency layer coefficients have little differences, but the high-frequency coefficients have an apparent gap. High-frequency coefficients usually fluctuated around zero, the larger the absolute value of coefficients the more dramatic the gray-scale changes, that is to say, it contain important image information, such as image edges, lines and regional borders. Therefore, based on the characteristics of Curvelet transform and the imaging mechanism of multi-focus image, this paper put forward a multi-focus image fusion strategy that the low-frequency coefficients were integrated using the weighted average, and high frequency coefficients were integrated using regional-energy-based method.

To conduct Curvelet transform on images \( A \) and \( B \), then, the corresponding low-frequency coefficients \( C_{i,A}^{L}(i, j) \) and \( C_{i,B}^{L}(i, j) \) with high-frequency coefficients \( C_{i,A}^{H}(i, j) \) and \( C_{i,B}^{H}(i, j) \).
\( C_{i,B}^{H}(i,j) \) are obtained, respectively. To mark the fusion image \( F \), then, the specific fusion rule is as follows:

1. The fusion of low frequency coefficients is based on the weighted average method. Its formula can be expressed as:

\[
C_{i,F}^{L}(i,j) = \alpha \times C_{i,A}^{L}(i,j) + (1-\alpha) \times C_{i,B}^{L}(i,j)
\]

(17)

The weighted average method can effectively suppress image noise; but, it can reduce the contrast of the image to some extent.

2. The fusion of high-frequency coefficients is using regional energy analysis method. The local energy is first used to calculate the local matching of two image's corresponding spatial information. If the local matching is less than a given matching threshold, it indicating that the two images in the local area "energy" is vary greatly. Then, select the center pixel of "energy" more large area as the location of fused image pixel. When the matching is greater than a given threshold, it adopts the weighted integration method; the center pixel with larger "local energy" represents the distinct characteristics of the original image. The algorithm is as follows:

Step 1: Calculate the “local energy” \( E_{i,A} \) and \( E_{i,B} \) of two images, respectively.

\[
E_{i}(i,j) = \sum_{m \in J, n \in K} w_{l}(m,n)[R_{l}(i+m, j+n)]^{2}
\]

(18)

Where \( E_{i}(i,j) \) indicates the local energy of the region which takes \( (i,j) \) as the center on layer \( l \); \( R_{l} \) is the distinct details, \( 0 \leq l < N \); \( w_{l}(m,n) \) is the weight coefficient corresponding to \( R_{l} \), and \( J,K \) indicate the sizes of local region where \( m,n \) varying inside.

Step 2: Calculate the local matching factor \( M_{i,AB} \) of spatial information on corresponding layers.

\[
M_{i,AB} = \frac{2 \times \sum_{m \in J, n \in K} w_{l}(m,n)R_{i,A}(i+m, j+n)R_{i,B}(i+m, j+n)}{E_{i,A}(i,j)+E_{i,B}(i,j)}
\]

(19)

\( M_{i,AB} \) reflects the approximate extent of the regional energy in two regions. When \( A \) and \( B \) are closer to the time, \( M_{i,AB} \rightarrow 1 \). And when \( A \) and \( B \) have large gap, \( M_{i,AB} \rightarrow 0 \).

Step 3: Determine the matching coefficient:

Set the matching threshold \( T \) and weight factor \( \alpha \), then

\[
C_{i,F}^{H}(i,j) = \alpha C_{i,A}^{H}(i,j) + (1-\alpha)C_{i,B}^{H}(i,j)
\]

(20)

When \( M_{i,AB}(i,j) \geq T \), it indicates that the two image are related. Then, the component parts of the details were extracted as a component of the fused image. And when \( M_{i,AB}(i,j) < T \), it indicates that the two images are unrelated.

3.2. Fusion Steps
The basic steps of image fusion based on Curvelet transform are as follows. Here, we only take the fusion of two source images as an example, though it can be extended to handle more than two images straightforwardly. Suppose $A$ and $B$ are original images of registration, $F$ is the fusion image.

1. Perform Curvelet transform on two original images respectively, and obtain the image Curvelet coefficients including coarse-scale coefficients and fine-scale coefficients.
2. Obtain the new Curvelet coefficient by fusing the high and the low frequency coefficients, the fusion rule is what has been discussed in last section.
3. Obtain the fusion coefficients.
4. Perform Curvelet inverse transform on the new fused Curvelet coefficients, the reconstructed image will be fused image.

In this approach, we can obtain an optimum fused image which has richer information in the spatial domain. Figure 3 gives an overview of algorithm organization.

![Figure 3. Image fusion flow based on Curvelet transform](image)

4. Evaluation of fusion image

In order to verify the efficiency of image fusion, it needs a method of evaluation [19-21]. The evaluation methods commonly can be divided into two broad categories: the subjective assessment method and the objective evaluation method.

4.1. Subjective evaluation.

Subjective assessment method is a man-made visual analysis for fused image, it is simple and intuitive. In addition to this, it has many advantages, such as it can be used to determine whether the image has shadow, whether the fusion image texture or color information is consistent, and whether the clarify has been reduced et al.. Therefore, the subjective assessment method is often used to compare the edges of fused images. It can get the differences of images in space decomposed force and clarity intuitively.

4.2. Objective evaluation

As the subjective assessment methods are not comprehensive and with certain one-sidedness. When the observation conditions change, the assessment results may be different. So, researchers made a number of methods named objective evaluation, those are quantitative analysis. For the metallographic image fusion evaluation, we should take consideration of the enhancement of spatial details and the maintenance of spectral information comprehensively. In this section we describe a number of different focus functions studied in this paper. Let $f(i, j)$ be the gray level intensity of pixel $(i, j)$.

Mean gray(MG). It is the average value of all image pixels, which reflects the average brightness for human eyes. The formula is:
Standard deviation (STD). It is an important index to weigh the information capability of images and it reflects the discrete level of gray-scale image’s mean value. The standard deviation is defined as

$$\text{STD} = \sqrt{\frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (f(i, j) - \hat{\mu})^2}{MN}}$$  \hspace{1cm} (23)$$

The greater the standard error, the more dispersed the distribution of gray-scale image, the better the quality of fused image. That is to say, it contains more information.

Information entropy(IE). Image entropy is an important indicator for evaluating the richness of image information; it represents the property of combination between images. The entropy of an image is

$$H = -\sum_{i=0}^{L-1} p_i \log p_i$$  \hspace{1cm} (24)$$

where $H$ is the entropy, $L$ is the overall gray-scales of image, $p_i$ is the probability of gray level $i$.

Average gradient(AG). It reflects the contrast between the details variation of pattern on the image, so it is often used to evaluate the clarity of the image.

$$\overline{G} = \frac{1}{(M-1)(N-1)} \sum_{i=1}^{M-1} \sum_{j=1}^{N-1} \sqrt{\left(\frac{\partial f(x_i, y_j)}{\partial x_i}\right)^2 + \left(\frac{\partial f(x_i, y_j)}{\partial y_i}\right)^2}/2$$  \hspace{1cm} (25)$$

Generally speaking, larger the value of $\overline{G}$ , clearer that image.

Therefore, in order to make the evaluation of image quality more effective and more comprehensive. In this paper, we adopt a comprehensive evaluation which makes the combination of the subjective visual evaluation and objective evaluation.

5. Experimental results analysis

In order to verify the performance of the proposed fusion algorithm, we have designed the experiments on two images using Matlab. Each set image has different focuses and is partly blurring, they have done registration strictly with the size 256 × 256 pixels. The proposed method compared with other fusion methods such as average method, PCA method, neighbor variance method, wavelet transform method and pyramid method.

Experimental results are shown in following figures. Fig.4(a) and Fig.4(b) show a pair of multi-focus test images which focus on bottom and top focal plane, respectively. Fig.4(c), Fig.4(d), Fig.4(e), Fig.4(f), Fig.4(g) and Fig.4(h) show the results of average method, PCA method, neighbor variance method, wavelet transform method and pyramid method, and proposed method, respectively. It can be seen from that the fused image produced by those methods are basically a combination of the good-focus parts in the source images to some extent.
From the visual point of view, various algorithms have achieved the integration of two source image information. Both the spatial resolutions and clarity have been enhanced in comparison to the original images to some extent. However, by the comparison, it can be seen that the fusion results of neighbor variance method, wavelet transform method and pyramid method, and proposed method are better than that of average method and PCA method, and the proposed method is the best, it has obvious advantages in the details of information. Comparing with Fig.4(c) and Fig.4(d), it is more clarify. Comparing with Fig.4(e), it doesn’t have the shadow, and comparing with Fig.4(f), it has the ability to perform spatial detail information. Therefore, from a subjective evaluation of view point, the overall effectiveness of the new method is better.

In addition to the visual analysis, we extended our investigation to a quantitative analysis. From the perspective of an objective assessment, Table 1 presents a comparison of the experimental results of image fusion using the average method, PCA method, neighbor variance method, wavelet transform method and pyramid-based method, and proposed method in terms of standard error, entropy, and the mean gradient. In Table 1, the entropy value and gradient value of proposed method fused image are greater than those of other methods; this is consistent with the results of subjective assessment. It means that the proposed method has the stronger ability to obtain the details from both original images and enhance the spatial information than other methods; its fusion strategy is feasible.

### Table 1. Quantitative result for various fusion methods.

<table>
<thead>
<tr>
<th>Fused images</th>
<th>MG</th>
<th>STD</th>
<th>IE</th>
<th>AG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image (c)</td>
<td>173.7</td>
<td>42.4</td>
<td>7.29</td>
<td>8.96</td>
</tr>
<tr>
<td>Image (d)</td>
<td>173.5</td>
<td>42.5</td>
<td>7.29</td>
<td>9.02</td>
</tr>
<tr>
<td>Image (e)</td>
<td>169.8</td>
<td>43.4</td>
<td>7.34</td>
<td>14.19</td>
</tr>
<tr>
<td>Image (f)</td>
<td>173.8</td>
<td>44.2</td>
<td>7.34</td>
<td>13.72</td>
</tr>
<tr>
<td>Image (g)</td>
<td>173.6</td>
<td>47.5</td>
<td>7.37</td>
<td>14.90</td>
</tr>
<tr>
<td>Image (h)</td>
<td>173.1</td>
<td>47.4</td>
<td>7.38</td>
<td>15.11</td>
</tr>
</tbody>
</table>
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From the subjective and objective analysis above, it can be seen that the amount of image information will have different levels of improvement after fusing with different methods, and finer features can be obtained in the fusion results. However, compared to the other traditional methods, the Curvelet-based algorithm proposed in this paper not only makes the detail image more clearly but also renders the image edge smoother. Its fusion performance is better.

6. Conclusions

This paper presented a Curvelet based approach for the fusion of multi-focus images. An experimental study was conducted by applying the proposed method, as well as other image fusion methods, to the fusion of multi-focus images. The experimental study shows that the application of the Curvelet transform in the fusion of multi-focus is superior to the application of the traditional methods, both visually and quantitatively. The obtained Curvelet fusion results have higher clarity values than the traditional fusion results. Also, curves of visual details are better in the Curvelet fusion results than in the traditional fusion results. Based on the experimental results, it can be seen that it is well suited for extracting detailed information from an image. However, in terms of complexity and real-time, there are still a lot of questions need to be further improved, and as a new multi-scale geometric analysis tool, it has a potential range of applications in the field of image fusion.

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8. References