

Renormalization effects on neutrino masses and mixing in a string-inspired $SU(4) \times SU(2)^2 \times U(1)_X$ model

A Psallidas¹

Theoretical Physics Division, Ioannina University, GR-45110 Ioannina, Greece

Abstract. In this talk, we discuss the implications of the renormalization group equations for the neutrino masses and mixing angles in a supersymmetric string-inspired $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_X$ model with matter in fundamental and antisymmetric tensor representations only. The quark, charged lepton and neutrino Yukawa matrices are distinguished by different Clebsch-Gordan coefficients due to contracting over $SU(4)$ and $SU(2)_R$ indices. In order to permit for a more realistic, hierarchical light neutrino mass spectrum with bi-large mixing a second $U(1)_X$ breaking singlet with fractional charge is introduced. By numerical investigation we find a region in the model parameter space where the neutrino mass-squared differences and mixing angles at low energy are consistent with experimental data.

1. Introduction

The success of the minimal supersymmetric extension of the standard model theory (MSSM) in explaining the low energy parameters is undisputed. The measured values of the strong coupling constant α_3 , α_{em} and the weak mixing angle $\sin^2 \theta_W$ are correctly predicted when the unification scale is taken to be of the order 10^{16} GeV. Furthermore, neutrino oscillation data [1] suggest that the search for physics beyond the successful Standard Model (SM) and MSSM is a natural step.

In particular, experimental facts imply intriguing relations between the large mixing angles in the neutrino sector and the smaller ones in the quark sector. For instance, the mixing angle $\theta_{\nu_{12}}$ and the Cabibbo mixing θ_C could satisfy the so called Quark-Lepton Complementarity (QLC) relation $\theta_{\nu_{12}} + \theta_C \approx \frac{\pi}{4}$ [2]. It has been shown [3] that this relation can be reproduced if some symmetry exists among quarks and leptons, namely if in a unified framework they form part of the same multiplet. Therefore, moving beyond the MSSM attempts have been made [4] in order to realize QLC in the context of models unifying quarks and leptons such as the Pati-Salam [5]. Apart from the QLC relations, a different possibility is offered from the symmetry $L_e - L_\mu - L_\tau$, which implies an inverted neutrino mass hierarchy and bimaximal mixing $\theta_{\nu_{12}} = \theta_{\nu_{23}} = \frac{\pi}{4}$, with $\theta_{\nu_{13}} = 0$ [6]. This symmetry alone does not give a consistent description of current experimental data, but additional corrections and renormalization effects have still to be taken into account. It has been shown [7] in the context of MSSM extended by a spontaneously broken $U(1)_X$ factor, that the neutrino sector respects an $L_e - L_\mu - L_\tau$ symmetry. Small corrections from other singlet

¹ Talk presented at the ‘‘Corfu Summer Institute’’, Corfu-Greece, September 4-14, 2005. Work in collaboration with T Dent, G K Leontaris, and J Rizos.

vevs, which are usually present in a string spectrum, can lead to a soft breaking of this symmetry and describe accurately the experimental neutrino data.

Also, neutrino oscillation data imply that neutrino mass squared differences are tiny, with $\Delta m_{sol}^2 \approx 10^{-5} \text{ eV}^2$ and $\Delta m_{atm}^2 \approx 10^{-3} \text{ eV}^2$. The observed smallness of the neutrino masses as compared to those of quarks and charged leptons finds an attractive explanation in the seesaw mechanism. In order to realize this mechanism a theory beyond the MSSM is needed that will incorporate the right-handed neutrinos and suppress adequately the neutrino masses. Examples of higher symmetries including the SM gauge group and incorporating the right-handed neutrino in the fermion spectrum, are the partially unified Pati–Salam model, based on $SU(4) \times SU(2)_L \times SU(2)_R$, and the fully unified $SO(10)$. When embedded into perturbative string or D-brane models, these may be extended by additional abelian or discrete fermion family symmetries. Thus fermion masses and mixing angles can be compared to the predictions of various types of models with full or partial gauge unification and flavor symmetries.

An issue of utmost importance is the evolution of the neutrino parameters from the high energy scale where the neutrino mass matrices are formed, down to their low energy measured values, according to the renormalization group equations (RGEs). Of course, one attempt could be to determine the neutrino mass matrices from experimental data directly at the weak scale. Nevertheless, the Yukawa couplings and other relevant parameters are not known at the unification scale. A knowledge of these quantities at the unification mass could provide a clue for the structure of the unified or partially unified theory and the exact (family) symmetries determining the neutrino mass matrices at the GUT scale. In the literature there have been made attempts [8, 9, 10] to determine the neutrino mass parameters in a top-bottom or bottom-up approach.

In this talk, we explore further the neutrino mass spectrum of a model with gauge symmetry $SU(4) \times SU(2) \times SU(2) \times U(1)_X$ based on the 4-4-2 models [13, 14, 15], whose implications for quark and lepton masses were recently investigated in [11]. Various attractive features characterize these models. Large Higgs representations, which are problematic to obtain in string models, are not required. Also, third generation fermion Yukawa couplings are unified [16] up to small corrections. Unification of gauge couplings is allowed and, if one assumes the model embedded in supersymmetric string, might be predicted [17]. Moreover, the doublet-triplet splitting problem is absent.

String derived models usually predict in the spectrum of the effective field theory model a large number of neutral singlet fields charged differently under the extra $U(1)_X$ symmetry. Some of them are required to obtain non-zero vevs of the order of the $U(1)_X$ breaking scale due to the D- and F-flatness conditions. In the present model, in order to describe accurately the low energy neutrino data we introduce two such singlets charged under $U(1)_X$ [19]. The previous model [11] with one such singlet could easily give rise to a spectrum of light neutrinos with normal hierarchy and bi-large mixing. However, after study of the renormalization group (RG) evolution and unification it was found that the scale of light neutrino masses too large to be compatible with observation. If we impose the correct scale of light neutrino masses, then some heavy right-handed neutrinos (RHN) would have masses above the unification scale, which is incompatible with our effective field theory approach.

Therefore, from the superpotential three *a priori* independent expansion parameters appear, one coming from the Higgses which acquire v.e.v.'s at the $SU(4) \times SU(2)_R$ breaking scale and two from the singlets. Also, in the superpotential there may appear nonrenormalizable operators involving products of $SU(4) \times SU(2)_R$ breaking Higgses and, consequently, different contractions of gauge group indices are possible leading to different contributions depending on the Clebsch factors. We use a minimal set of nontrivial Clebsch factors to construct the Dirac mass matrices. Right-handed ($SU(2)_L$ singlet) neutrinos obtain Majorana masses through nonrenormalizable couplings to the $U(1)_X$ -charged singlets and to Higgses, while light neutrinos will acquire masses

via the see-saw mechanism.

In this talk, we discuss the numerical solution of the renormalization group equations for the neutrino masses and mixing angles above, between and below the see-saw scales, for several sets of order 1 parameters which specify the heavy RHN matrix. In each case the results at low energy are consistent with current experimental data, and provide further predictions for the 1-3 neutrino mixing angle and for neutrinoless double beta decay.

2. Description of the model

In this section the noteworthy characteristics of the string inspired Pati-Salam model extended by a $U(1)_X$ family-symmetry are presented. The total gauge group of the model is $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_X$. The field content includes three copies of $(4, 2, 1) + (\bar{4}, 1, 2)$ representations to accommodate the three fermion generations $F_i + \bar{F}_i$ ($i = 1, 2, 3$),

$$F_i = \begin{pmatrix} u_i & u_i & u_i & \nu_i \\ d_i & d_i & d_i & e_i \end{pmatrix}_{\alpha_i}, \quad \bar{F}_i = \begin{pmatrix} u_i^c & u_i^c & u_i^c & \nu_i^c \\ d_i^c & d_i^c & d_i^c & e_i^c \end{pmatrix}_{\bar{\alpha}_i} \quad (1)$$

where the subscripts $\alpha_i, \bar{\alpha}_i$ indicate the $U(1)_X$ charge. So, F_i includes all left-handed SM fermions (quarks and leptons), whilst \bar{F}_i contains their right-handed partners, including the right-handed neutrinos. Note that $F + \bar{F}$ makes up the **16**-spinorial representation of $SO(10)$. In order to break the Pati-Salam symmetry down to SM gauge group, Higgs fields $H = (4, 1, 2)$ and $\bar{H} = (\bar{4}, 1, 2)$ are introduced

$$H = \begin{pmatrix} u_H & u_H & u_H & \nu_H \\ d_H & d_H & d_H & e_H \end{pmatrix}_x, \quad \bar{H} = \begin{pmatrix} u_{\bar{H}}^c & u_{\bar{H}}^c & u_{\bar{H}}^c & \nu_{\bar{H}}^c \\ d_{\bar{H}}^c & d_{\bar{H}}^c & d_{\bar{H}}^c & e_{\bar{H}}^c \end{pmatrix}_{\bar{x}}$$

which acquire vevs of the order M_G along their neutral components

$$\langle H \rangle = \langle \tilde{\nu}_H \rangle = M_G, \quad \langle \bar{H} \rangle = \langle \tilde{\nu}_{\bar{H}}^c \rangle = M_G \quad (2)$$

breaking the symmetry at M_G :

$$SU(4) \times SU(2)_L \times SU(2)_R \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y. \quad (3)$$

The Higgs sector also includes the $h = (1, \bar{2}, 2)$ field which after the breaking of the PS symmetry is decomposed to the two Higgs superfields of the MSSM. Further, two $D = (6, 1, 1)$ scalar fields are introduced to give mass to color triplet components of H and \bar{H} via the terms HHD and $\bar{H}\bar{H}D$ [13]. The scalar fields D and h make up the **10** representation of $SO(10)$. Also, in the string version of the model one encounters the following kinds of fractionally charged states:

$$H'(4, 1, 1), \quad \bar{H}'(\bar{4}, 2, 1), \quad h_L(1, 2, 1), \quad h_R(1, 1, 2).$$

Such states usually create problems in the low energy effective theory. Due to the fact that the lightest fractionally charged particle is expected to be stable, if its mass is around the TeV scale, then the estimation of its relic abundances [20] contradicts the upper experimental bounds by several orders of magnitude. However, the problem doesn't exist either if these states become massive at a high scale of the order of the heterotic string scale or if they become integrally charged modifying the hypercharge generator. Also, the problem can be solved constructing models containing a hidden gauge group [21] forcing the fractional charged states to form bound states [22].

Under the symmetry breaking (3) the following decompositions take place :

$$F_L(4, \mathbf{2}, \mathbf{1}) \rightarrow Q(\mathbf{3}, \mathbf{2}, -\frac{1}{6}) + \ell(\mathbf{1}, \mathbf{2}, \frac{1}{2})$$

$$\begin{aligned}
\bar{F}_R(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) &\rightarrow u^c(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3}) + d^c(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}) + \nu^c(\mathbf{1}, \mathbf{1}, 0) + e^c(\mathbf{1}, \mathbf{1}, -1) \\
\bar{H}(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) &\rightarrow u_H^c(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3}) + d_H^c(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}) + \nu_H^c(\mathbf{1}, \mathbf{1}, 0) + e_H^c(\mathbf{1}, \mathbf{1}, -1) \\
H(\mathbf{4}, \mathbf{1}, \mathbf{2}) &\rightarrow u_H(\mathbf{3}, \mathbf{1}, -\frac{2}{3}) + d_H(\mathbf{3}, \mathbf{1}, \frac{1}{3}) + \nu_H(\mathbf{1}, \mathbf{1}, 0) + e_H(\mathbf{1}, \mathbf{1}, 1) \\
D(\mathbf{6}, \mathbf{1}, 1) &\rightarrow D_3(\mathbf{3}, \mathbf{1}, \frac{1}{3}) + \bar{D}_3(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}) \\
h(\mathbf{1}, \mathbf{2}, \mathbf{2}) &\rightarrow h^d(\mathbf{1}, \mathbf{2}, \frac{1}{2}) + h^u(\mathbf{1}, \mathbf{2}, -\frac{1}{2})
\end{aligned} \tag{4}$$

where the fields on the left appear with their quantum numbers under the Pati-Salam gauge symmetry, while the fields on the right are shown with their quantum numbers under the SM symmetry.

Finally, two scalar singlet fields ϕ , χ are introduced, charged under $U(1)_X$ whose vevs will play an essential role in the fermion mass matrices through non-renormalizable terms of the superpotential. In the stable SUSY vacuum the two singlets obtain vevs to satisfy the D-flatness condition including the anomalous Fayet-Iliopoulos term [28]. The anomalous D-flatness conditions allow solutions where the vevs of the conjugate fields $\bar{\phi}$ and $\bar{\chi}$ are zero and we will restrict our analysis to this case. Note that in general a string model may have more than two singlets and more than one set of Higgses H_i , \bar{H}_i , with different $U(1)_X$ charge. All such fields may in principle also obtain vevs, however we find that two of them are sufficient to give a set of mass matrices in accordance with all experimental data. Hence we consider any additional singlet vevs to be significantly smaller.

A matter that should be dealt with caution is the breaking of the Pati-Salam symmetry. The Higgses H_i , \bar{H}_i may obtain masses through $H\bar{H}\phi$, $H\bar{H}\chi$ and $H\bar{H}\phi\chi$ couplings. However, in order to break the Pati-Salam group while preserving SUSY we require that one H - \bar{H} pair be massless at this level. This ‘‘symmetry-breaking’’ Higgs pair could be a linear combination of fields with different $U(1)_X$ charges, which would in general complicate the expressions for fermion masses. The chiral spectrum is summarized in Table 1. We choose the charge of the Higgs field h to be $-\alpha_3 - \bar{\alpha}_3$ so that that the 3rd. generation coupling $F_3\bar{F}_3h$ is allowed at tree-level.

Table 1. Field content and $U(1)_X$ charges

	SU(4)	SU(2) _L	SU(2) _R	U(1) _X
F_i	4	2	1	α_i
\bar{F}_i	$\bar{4}$	1	$\bar{2}$	$\bar{\alpha}_i$
H	4	1	2	x
\bar{H}	$\bar{4}$	1	$\bar{2}$	\bar{x}
ϕ	1	1	1	z
χ	1	1	1	z'
h	1	$\bar{2}$	2	$-\alpha_3 - \bar{\alpha}_3$
D_1	6	1	1	$-2x$
D_2	6	1	1	$-2\bar{x}$

Now we proceed to terms in the superpotential which can give rise to fermion masses. Charged fermions obtain only Dirac type mass terms, whilst neutral ones may obtain Dirac and Majorana

masses. Dirac type mass terms arise after electroweak symmetry-breaking from couplings of the form

$$\begin{aligned} \mathcal{W}_D = & y_0^{33} F_3 \bar{F}_3 h + F_i \bar{F}_j h \left(\sum_{m>0} y_m^{ij} \left(\frac{\phi}{M_U} \right)^m + \sum_{m'>0} y_{m'}^{ij} \left(\frac{\chi}{M_U} \right)^{m'} + \sum_{n>0} y_n^{ij} \left(\frac{H\bar{H}}{M_U^2} \right)^n + \right. \\ & \left. \sum_{k,\ell>0} y_{k,\ell}^{ij} \left(\frac{\phi}{M_U} \right)^k \left(\frac{\chi}{M_U} \right)^\ell + \sum_{p,q>0} y_{p,q}^{ij} \left(\frac{H\bar{H}}{M_U^2} \right)^p \left(\frac{\phi}{M_U} \right)^q + \sum_{r,s>0} y_{r,s}^{ij} \left(\frac{H\bar{H}}{M_U^2} \right)^r \left(\frac{\chi}{M_U} \right)^s + \dots \right) \end{aligned} \quad (5)$$

where $m, m', n, k, l, p, q, r, s$ are appropriate integers. Apart from the heaviest generation, all masses arise at non-renormalizable level, suppressed by powers of the fundamental scale or unification scale M_U . The couplings $y_{m,m'}^{ij}, y_n^{ij}$ etc. are non-vanishing and generically of order 1 whenever the $U(1)_X$ charge of the corresponding operator vanishes, thus:

$$\alpha_i - \alpha_3 + \bar{\alpha}_j - \bar{\alpha}_3 = \{-mz, -m'z', -n(x + \bar{x}), -kz - \ell z', -p(x + \bar{x}) - qz, -r(x + \bar{x}) - sz'\}. \quad (6)$$

Other higher-dimension operators may arise by multiplying any term by factors such as $(H\bar{H})^\ell \phi^s / M_U^{2\ell+s}$ where $\ell(x + \bar{x}) + sz = 0$. Such terms are negligible unless the leading term vanishes.

Neutrinos may in addition receive also Majorana type masses. These arise from the operators

$$\begin{aligned} \mathcal{W}_M = & \frac{\bar{F}_i \bar{F}_j H H}{M_U} \left(\mu_0^{ij} + \sum_{t>0} \mu_t^{ij} \left(\frac{\phi}{M_U} \right)^t + \sum_{t'>0} \mu_{t'}^{ij} \left(\frac{\chi}{M_U} \right)^{t'} + \right. \\ & \sum_{w>0} \mu_w^{ij} \left(\frac{H\bar{H}}{M_U^2} \right)^w + \sum_{k',\ell'>0} \mu_{k',\ell'}^{ij} \left(\frac{\phi}{M_U} \right)^{k'} \left(\frac{\chi}{M_U} \right)^{\ell'} + \sum_{p',q'>0} \mu_{p',q'}^{ij} \left(\frac{H\bar{H}}{M_U^2} \right)^{p'} \left(\frac{\phi}{M_U} \right)^{q'} + \\ & \left. \sum_{r,s>0} \mu_{r',s'}^{ij} \left(\frac{H\bar{H}}{M_U^2} \right)^{r'} \left(\frac{\chi}{M_U} \right)^{s'} + \dots \right). \end{aligned} \quad (7)$$

where $t, t', w, k', \ell', p', r', s'$ are appropriate integers. Couplings of this type are non-vanishing whenever the following conditions are satisfied:

$$\bar{\alpha}_i + \bar{\alpha}_j + 2x = \{-tz, -t'z', -w(x + \bar{x}), -k'z - \ell'z' - p'(x + \bar{x}) - q'z, -r'(x + \bar{x}) - s'z'\}. \quad (8)$$

3. Fermion mass matrices

3.1. General structure

As can be seen from the superpotential Yukawa couplings (2) and (2), three different expansion parameters appear in the construction of the fermion mass matrices. These are

$$\epsilon \equiv \frac{\langle \phi \rangle}{M_U}, \quad \epsilon' \equiv \frac{M_G^2}{M_U^2}, \quad \epsilon'' \equiv \frac{\langle \chi \rangle}{M_U} \quad (9)$$

given $\langle H\bar{H} \rangle = M_G^2$. Note that, for non-renormalizable Dirac mass terms involving several products of $H\bar{H}/M_U^2$, the gauge group indices may be contracted in different ways [15]. This can lead to different contributions to the up, down quarks and charged leptons, depending on the Clebsch factors $C_{n(u,d,e,\nu)}^{ij}$ multiplying the effective Yukawa couplings. Also, although the

Clebsch coefficient for a particular operator O_n may vanish at order n , the coefficient for the operator $O_{(n+p);q}$ containing p additional factors ($H\bar{H}$) and q factors of ϕ and/or χ is generically nonzero.

In our analysis we wish to estimate the effects of the second singlet (χ) contributions on the neutrino sector as compared to the analysis presented in [11] without affecting essentially the results in the quark sector. In order to obtain a set of fermion mass matrices with the minimum number of new operators, we assume fractional $U(1)_X$ charges for H, \bar{H} and χ fields, while the combination $H\bar{H}$ and the singlet ϕ are assumed to have integer charges. Thus $\alpha_i, \bar{\alpha}_i, x + \bar{x}$ and z are integers, while z', x and \bar{x} are fractional. As a result, the Dirac mass terms involving vevs of χ are expected to be subleading compared to other terms. Suppressing higher-order terms involving products of ϵ, ϵ' and ϵ'' , the Dirac mass terms at the unification scale are

$$m_{ij} \approx \delta_{i3}\delta_{j3}m_3 + \left(\epsilon^m + (\epsilon'')^{m'} + C_{ij}(\epsilon')^n \right) v_{u,d} \quad (10)$$

where $m_3 \equiv v_{u,d}y_0^{33}$, with v_u and v_d being the up-type and down-type Higgs vevs respectively, and we omit the order-one Yukawa coefficients y_m^{ij} etc. for simplicity.

The Majorana mass terms are proportional to the combination HH (see Eq. (2)) which has fractional $U(1)_X$ charge. Thus, terms proportional to χ/M_U become now important for the structure of the mass matrix. The general form of the Majorana mass matrix is then

$$M_N \approx M_R \left(\mu_t^{ij} \epsilon^t + \mu_{l'}^{ij} (\epsilon'')^{l'} + \mu_w^{ij} (\epsilon')^w + \mu_{k',l'}^{ij} \epsilon^{k'} (\epsilon'')^{l'} + \mu_{p',q'}^{ij} (\epsilon')^{p'} \epsilon^{q'} + \mu_{r',s'}^{ij} (\epsilon')^{r'} (\epsilon'')^{s'} \right) \quad (11)$$

where we define $M_R \equiv M_G^2/M_U \equiv \epsilon' M_U$.

3.2. Specific choice of $U(1)_X$ charges

Before we proceed to a specific, viable set of mass matrices, we first make use of the observation [11] that the form of the fermion mass terms above is invariant under the shifts

$$\alpha_i \rightarrow \alpha_i + \zeta, \quad \bar{\alpha}_i \rightarrow \bar{\alpha}_i + \bar{\zeta}, \quad x \rightarrow x - \bar{\zeta}, \quad \bar{x} \rightarrow \bar{x} + \bar{\zeta} \quad (12)$$

so that we are free to assign $\alpha_3 = \bar{\alpha}_3 = 0$. We further fix $x + \bar{x} = 1$ and $z = -1$. The resulting $U(1)_X$ charges are presented in Table 2. We will choose the values of x and z' to be fractional

Table 2. Specific choice of $U(1)_X$ charges.

field	F_1	F_2	F_3	\bar{F}_1	\bar{F}_2	\bar{F}_3	h	H	\bar{H}	ϕ	χ
$U(1)_X$	-4	-3	0	-2	1	0	0	x	\bar{x}	-1	z'

such that the v.e.v. of χ only affects the overall scale of neutrino masses, as explained below.

The charge entries of the common Dirac mass matrix for quarks, charged leptons and neutrinos are then

$$Q_X[M_D] = \begin{pmatrix} -6 & -3 & -4 \\ -5 & -2 & -3 \\ -2 & 1 & 0 \end{pmatrix}, \quad (13)$$

and the charge matrix for heavy neutrino Majorana masses is

$$Q_X[M_N] = 2x + \begin{pmatrix} -4 & -1 & -2 \\ -1 & 2 & 1 \\ -2 & 1 & 0 \end{pmatrix} \quad (14)$$

Now, we relate $\epsilon, \epsilon', \epsilon''$ with a single expansion parameter η , assuming the relations

$$\epsilon = b_1 \sqrt{\eta}, \quad \epsilon'' \equiv b_2 \eta, \quad \epsilon' = \sqrt{\eta} \quad (15)$$

where b_1, b_2 are numerical coefficients of order one. Then the effective Yukawa couplings for quarks and leptons may include terms

$$Y_f^{ij} = b_1^m \eta^{m/2} + b_1^{m+1} \eta^{1+m/2} + C_f^{ij} \eta^{n/2} + b_1 \eta^{1+n/2} + \dots \quad (16)$$

with $f = u, d, e, \nu$, up to order 1 coefficients y_f^{ij} . Which of these terms survives, depends on the sign of the charge of the corresponding operator. For a negative charge entry, the first two terms are not allowed and only the third and fourth contribute. Further, if a particular C_f^{ij} coefficient is zero, then we consider only the fourth term.

Therefore, we need to specify the Clebsch-Gordan coefficients C_f^{ij} for the terms involving powers of $\langle H\bar{H} \rangle / M_U^2$. These coefficients could be found if the fundamental theory was completely specified at the unification or string scale. In the absence of a specific string model, here we present a minimal number of operators which lead to a simple and viable set of mass matrices. Up to possible complex phases, we choose $C_d^{12} = C_d^{22} = \frac{1}{3}$, $C_u^{23} = 3$ and $C_u^{11} = C_u^{12} = C_u^{21} = C_u^{22} = C_u^{31} = C_\nu^{22} = C_\nu^{31} = 0$ with all others being equal to unity. The effective Yukawa matrices at the GUT scale obtained under the above assumptions are

$$Y_u = \begin{pmatrix} b_1 \eta^4 & b_1 \eta^{5/2} & \eta^2 \\ b_1 \eta^{7/2} & b_1 \eta^2 & 3\eta^{3/2} \\ b_1 \eta^2 & b_1 \eta^{1/2} & 1 \end{pmatrix}, \quad Y_d = \begin{pmatrix} \eta^3 & \frac{\eta^{3/2}}{3} & \eta^2 \\ \eta^{5/2} & \frac{\eta}{3} & \eta^{3/2} \\ \eta & b_1 \eta^{1/2} & 1 \end{pmatrix},$$

$$Y_e = \begin{pmatrix} \eta^3 & \eta^{3/2} & \eta^2 \\ \eta^{5/2} & \eta & \eta^{3/2} \\ \eta & b_1 \eta^{1/2} & 1 \end{pmatrix}, \quad Y_\nu = \begin{pmatrix} \eta^3 & \eta^{3/2} & \eta^2 \\ \eta^{5/2} & b_1 \eta^2 & \eta^{3/2} \\ b_1 \eta^2 & b_1 \eta^{1/2} & 1 \end{pmatrix}. \quad (17)$$

where we suppress order one coefficients. The quark sector as well as the neutrino sector were studied in [11]. However, full renormalization group effects were not calculated for the neutrino sector and as it turns out one singlet is inadequate to accommodate the low energy data. Consequently, we introduced the second singlet χ , with fractional charge, whose v.e.v. affects only the overall scale of neutrino masses.

The desired matrix for the right handed Majorana neutrinos may result from more than one choice of charge for the H field and the χ singlet field. These can be seen in Table 3.

Table 3. Possible choices for the $U(1)_X$ charges of the H and χ fields, namely x and x' respectively.

$Q_X[H]$	$-\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{5}{3}$	$-\frac{6}{5}$	$-\frac{7}{8}$	$-\frac{1}{4}$	$-\frac{3}{4}$
$Q_X[\chi]$	$-\frac{5}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{3}{5}$	$-\frac{5}{4}$	$-\frac{5}{2}$	$-\frac{3}{2}$

Table 4. Operators producing the Majorana right handed neutrino matrix M_N .

M_N entry	Operator	vev
M_N^{11}	$\left(\frac{H\bar{H}}{M_U^2}\right)^7 \frac{\chi}{M_U}$	$b_2\eta^{9/2}$
M_N^{12}	$\left(\frac{H\bar{H}}{M_U^2}\right)^4 \frac{\chi}{M_U}$	$b_2\eta^3$
M_N^{13}	$\left(\frac{H\bar{H}}{M_U^2}\right)^5 \frac{\chi}{M_U}$	$b_2\eta^{7/2}$
M_N^{22}	$\left(\frac{H\bar{H}}{M_U^2}\right) \frac{\chi}{M_U}$	$b_2\eta^{3/2}$
M_N^{23}	$\left(\frac{H\bar{H}}{M_U^2}\right)^2 \frac{\chi}{M_U}$	$b_2\eta^2$
M_N^{33}	$\left(\frac{H\bar{H}}{M_U^2}\right)^3 \frac{\chi}{M_U}$	$b_2\eta^{5/2}$

We choose the H charge to be $x = -\frac{6}{5}$ so that $2x$ is non-integer, and set the χ singlet charge to $-\frac{3}{5}$. The analysis for the quarks and charged leptons remains the as in [11] since operators with nonzero powers χ^r do not exist for powers $r < 5$ and are negligible compared to the leading terms.

With these assignments, the charge entries of the heavy Majorana matrix Eq. (14) are:

$$Q_X[M_N] = \begin{pmatrix} -\frac{32}{5} & -\frac{17}{5} & -\frac{22}{5} \\ -\frac{17}{5} & -\frac{7}{5} & -\frac{7}{5} \\ -\frac{22}{5} & -\frac{7}{5} & -\frac{12}{5} \end{pmatrix}. \quad (18)$$

Due to the fractional $U(1)_X$ charges contributions from ϕ or $H\bar{H}$ alone vanish. However, we also have the singlets $H\bar{H}\chi/M_U^3$ with vev $b_2\eta^{3/2}$ and $\phi\chi/M_U^2$ with a vev $b_1b_2\eta^2$, while for some entries one may have to consider higher order terms since the leading order will be vanishing. In Table 4 we explicitly write the operator for every entry of M_N . The Majorana right-handed neutrino mass matrix is then

$$M_N = \begin{pmatrix} \mu_{11}\eta^{9/2} & \mu_{12}\eta^3 & \mu_{13}\eta^{7/2} \\ \mu_{12}\eta^3 & \mu_{22}\eta^{3/2} & \mu_{23}\eta^2 \\ \mu_{13}\eta^{7/2} & \mu_{23}\eta^2 & \eta^{5/2} \end{pmatrix} b_2 M_R \quad (19)$$

with $M_R = \epsilon' M_U = \sqrt{\eta} M_U$.

Having defined the Dirac and heavy Majorana mass matrices for the neutrinos, it is straightforward to obtain the light Majorana mass matrix from the see-saw formula

$$m_\nu = -m_{D\nu} M_N^{-1} m_{D\nu}^T \quad (20)$$

at the GUT scale.

3.3. Setting the expansion parameters

Given the fermion mass textures in terms of the $U(1)_X$ charges and expansion parameters, we need now to determine the values of the latter in order to obtain consistency with the low energy experimentally known quantities (masses and mixing angles). Note that the coefficient b_2 defined in Eq. (15) will determine the overall neutrino mass scale through Eq. (19).

Consistency with the measured values of quark masses and mixings fixes the value of $\eta \approx 5 \times 10^{-2}$: for example the CKM mixing angle θ_{12} is given by $\sqrt{\eta} \approx 0.22$ up to small corrections [11]. Hence the ratio of the SU(4) breaking scale M_G to the fundamental scale M_U is also fixed through $\frac{M_G^2}{M_U^2} = \sqrt{\eta} \approx 0.22$: the Pati-Salam group is unbroken over only a small range of energy. We perform a renormalization group analysis in order to check the consistency of this prediction with the low-energy values of the gauge couplings α_s , α_{em} and the weak mixing angle $\sin^2 \theta_W$ [18]

$$\sin^2 \theta_W = 0.23120, \quad \alpha_3 = 0.118 \pm 0.003, \quad a_{em} = \frac{1}{127.906}. \quad (21)$$

At this point it should be noted that in our approach we assume that the underline theory is characterized by a single coupling constant. In D-brane constructions however, the gauge couplings do not necessarily satisfy the usual unification condition. The reason is that in this case, the volume of the internal space enters between gauge and string couplings, thus the actual values of the gauge couplings may differ at the unification scale. Nevertheless, it is possible that some internal volume relations allow for a partial unification [12]. If the underlying model at M_U has a single unified gauge coupling, then M_G is fixed to be just below the unification scale according to the analysis of gauge coupling unification in the MSSM. Because of this fact, the low energy measured range for α_3 affects the unification of the gauge couplings. Thus, we add the following extra states

$$h_L = (1, 2, 1), \quad h_R = (1, 1, 2) \quad (22)$$

which are usually present in a string spectrum [13]. It turns out that we need 4 of each of these extra states for $M_G = 9.32 \times 10^{15}$ GeV to be consistent with the value of e' deduced from quark mass matrices.

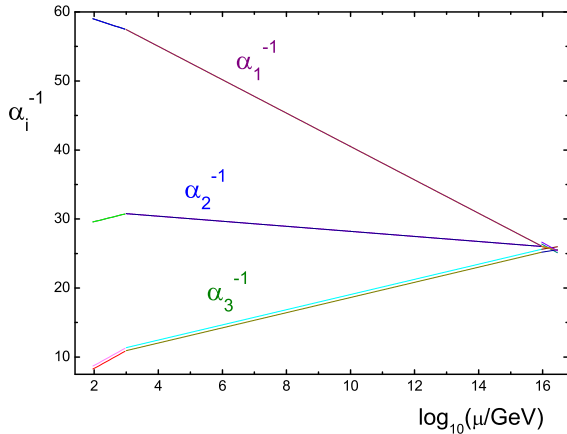


Figure 1. Evolution of the gauge couplings. The two lines for α_3 indicate the range of initial conditions at M_Z .

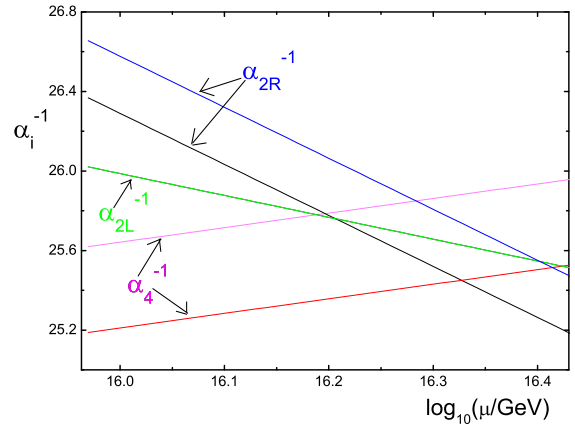


Figure 2. Close-up of the gauge couplings in the Pati-Salam energy region.

In Figure 1 we plot the evolution of the gauge couplings from M_Z to M_U . In Figure 2 we show in more detail the evolution of the gauge couplings in the Pati-Salam energy region. The two bands for the α_4 and α_{2R} couplings are due to strong coupling uncertainty at M_Z . For $\alpha_3(M_Z) = 0.1176$, as can be seen from Figure 3, we obtain $M_U = 1.96 \times 10^{16}$ GeV.

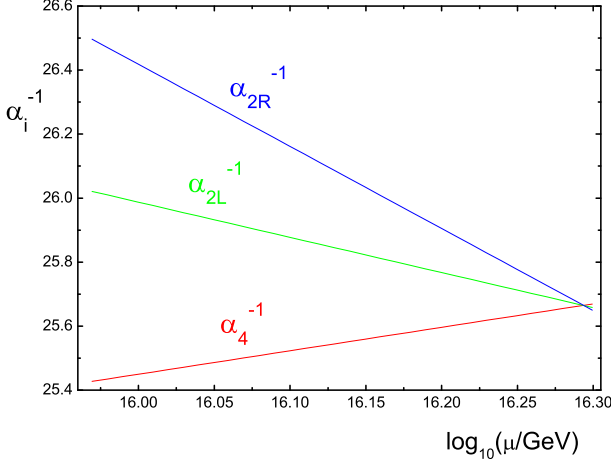


Figure 3. The unification point of Pati-Salam gauge couplings for $\alpha_3(M_Z) = 0.1176$.

The remaining parameters to be determined are b_1 , b_2 and μ_{ij} of the right handed neutrino mass matrix. We find that M_N is proportional to b_2 , $M_N \approx b_2 M_R M'_N$, thus b_2 is related to the scale of the light matrix m_ν . Also, the choice $b_1 \approx 1.1$ leads to agreement with the data, while implying $\epsilon = 1.1\sqrt{\eta} \approx 0.25$.

4. Running of neutrino masses and mixing angles

The comparison between the high energy theoretical prediction and the experimental observation is performed using the renormalization group equations. The mass matrix for the light neutrinos is an outcome of the see-saw mechanism and the effects induced by the RGEs are very important. Indeed, the low energy neutrino data could be considerably different from the results at the see-saw scale due to RG corrections above and below the see-saw thresholds. The running of neutrino masses and mixing angles has been extensively discussed for energies below the seesaw scales [23, 24, 25] as well as above [26, 8, 9]. The running of the effective neutrino mass matrix m_ν above and between the see-saw scales is split into two terms,

$$m_\nu = -\frac{v^2}{4} \left(\kappa + 2 Y_\nu M^{-1} Y_\nu^T \right). \quad (23)$$

where κ is related to the coefficient of the effective 5 dimensional operator $LLh_u h_u$, (n) labels the effective field theories with M_n right handed neutrino integrated out ($M_n \geq M_{n-1} \geq M_{n-2}, \dots$) and $Y_\nu^{(n)}$ are the neutrino couplings at an energy scale M between two RH neutrino masses $M_n \geq M \geq M_{n-1}$, while $Y_\nu = 0$ below the lightest RH neutrino mass. These effective parameters govern the evolution below the highest seesaw scale and obey the differential equation [23, 24, 25]

$$16\pi^2 \frac{dX^{(n)}}{dt} = (Y_e Y_e^\dagger + Y_\nu Y_\nu^\dagger)^T X^{(n)} + X^{(n)} (Y_e Y_e^\dagger + Y_\nu Y_\nu^\dagger)^T + (2\text{Tr}(Y_\nu Y_\nu^\dagger + 3Y_u Y_u^\dagger) - 6/5g_1^2 - 6g_2^2) X^{(n)} \quad (24)$$

where $X = \kappa, Y_\nu M^{-1} Y_\nu^T$. The RGEs have been solved both numerically and also analytically [25, 8, 9]. Numerically, below the lightest heavy RH neutrino mass large renormalization effects can be experienced only in the case of degenerate light neutrino masses for very large $\tan \beta$ [27, 25]. Above this mass things are more complicated due to the non-trivial dependence of the heavy

Table 5. Numerical values of parameters $\mu_{11}, \mu_{12}, \mu_{13}, \mu_{22}, \mu_{23}$ at M_G .

Solution	μ_{11}	μ_{12}	μ_{13}	μ_{22}	μ_{23}
1.	0.10535	0.10972	0.86012	0.10491	0.91014
2.	0.11939	0.10954	0.80912	0.10683	0.93832
3.	0.10392	0.11787	0.97796	0.10512	0.98749
4.	0.09143	0.10962	0.87616	0.10063	0.93798
5.	0.12697	0.12745	0.99860	0.11652	0.99980
6.	0.10920	0.09638	1.00975	0.10238	0.93561
7.	0.10124	0.11682	0.98568	0.10688	0.99156
8.	0.12358	0.09514	0.99580	0.10434	0.95646
9.	0.13006	0.11973	1.02235	0.10378	0.89460
10.	0.12665	0.12137	1.00029	0.10695	0.91578

neutrino mass couplings, unless $M_{\nu c}$ is diagonal. For the leptonic mixing angles, in the case of normal hierarchy relevant to our model, one expects negligible effects for the solar mixing angle while θ_{13} and θ_{23} are expected to run faster [10].

On the other hand, studying the analytical expressions obtained after approximation, exactly the opposite behavior is predicted and the solar mixing angle receives larger renormalization effects than θ_{13} or θ_{23} . However, possible cancellations may occur and enhancement or suppression factors may appear, thus the numerical solutions may differ considerably from these estimates.

In our string-inspired model the Dirac and heavy Majorana mass matrices at the unification scale are parametrized in terms of order-1 superpotential coefficients $\mu_{ij}(M_U)$ whose exact numerical values are not known. The flavour structure at the unification scale might also be different from that at the electroweak scale M_Z . Thus, even if the Yukawa parameters are determined at M_Z , to understand the structure of the theory at M_U , and consequently any possible family symmetry, we would certainly need the parameter values at M_U .

In this section we study the renormalization group flow of the neutrino mass matrices in a “top-down” approach from the Pati-Salam scale M_G to the weak scale. We choose sets of values of the undetermined order 1 coefficients at the high scale and run the renormalization group equations down to M_Z where we calculate $\Delta m_{\nu_{ij}}^2$ and $\theta_{\nu_{ij}}$ and compare them with the experimental values. Study of a bottom-up approach has been performed [10] and we will compare our results to this work. The renormalization group analyses of the neutrino parameters, successively integrating out the right handed neutrinos, is performed using the software packages REAP/MPT [8].

We generate appropriate numerical values for the coefficients $\mu_{11}, \mu_{12}, \mu_{13}, \mu_{22}, \mu_{23}$, so that after the evolution of m_ν to low energy we obtain values in agreement with the experimental data. The coefficient μ_{33} is set to unity (which can always be done by adjusting the value of b_2). Experimentally acceptable solutions can be seen in Table 5. In Table 6 we present the resulting values of θ_{ij} and Δm_{ij}^2 at the scale M_Z . The mass-squared differences lie in the ranges $\Delta m_{atm}^2 = [1.33 - 3.39] \times 10^{-3} \text{eV}^2$, $\Delta m_{sol}^2 = [7.24 - 8.85] \times 10^{-5} \text{eV}^2$. These are consistent with the experimental data $\Delta m_{atm,exp}^2 = [1.3 - 3.4] \times 10^{-3} \text{eV}^2$ and $\Delta m_{sol,exp}^2 = [7.1 - 8.9] \times 10^{-5} \text{eV}^2$. The mixing angles are also found in the allowed ranges $\theta_{12} = [29.4 - 37.6]$, $\theta_{23} = [36.9 - 51.0]$ and $\theta_{13} = [0.86 - 12.50]$.

In figure 4 we plot the running of the three light neutrino Majorana masses ($m_1 < m_2 < m_3$) in the energy range $M_G - M_Z$. The initial (GUT) neutrino eigenmasses are all larger than

Table 6. Values of the physical parameters Δm_{atm}^2 , Δm_{sol}^2 , θ_{12} , θ_{13} , θ_{23} at M_Z (mass units eV^2).

Solution	$\Delta m_{atm}^2(M_Z) \cdot 10^3$	$\Delta m_{sol}^2(M_Z) \cdot 10^5$	$\theta_{12}(M_Z)$	$\theta_{13}(M_Z)$	$\theta_{23}(M_Z)$
1.	2.7149	7.9621	29.442	3.9859	44.114
2.	2.3145	7.9514	34.289	12.507	51.047
3.	1.8978	8.6141	30.560	0.8656	46.230
4.	3.0062	8.3217	34.347	1.8512	44.333
5.	3.3905	7.2468	30.245	2.9355	36.900
6.	3.2459	7.5351	34.296	1.3701	46.947
7.	2.0171	7.9464	34.432	1.0086	50.279
8.	1.3321	7.9060	37.646	6.1490	43.067
9.	2.4867	8.8561	29.592	5.6007	42.970
10.	2.1652	7.8869	29.189	3.1512	37.220

their low energy values. Significant running is observed mainly for the heaviest eigenmass m_{ν_3} . For experimentally acceptable mass-squared differences $\Delta m_{\nu_{ij}}^2$ at M_Z , in all cases their corresponding values at the GUT scale lie out of the acceptable range. In this scenario with hierarchical light neutrino masses, we find that large renormalization effects occur above the heavy neutrino threshold since the Yukawa couplings Y_ν are large and the second term in (23) dominates. Also, since $m_{\nu_1} < \sqrt{\Delta m_{sol}^2}$, the solar angle turns out to be more stable compared to the running of the θ_{23} , as can be seen in Figure 5. These results are in agreement with the findings of [10].

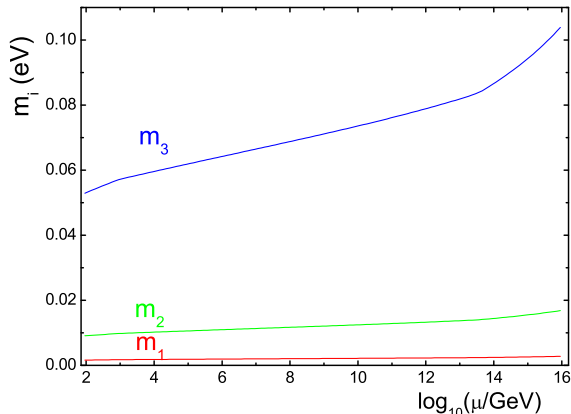


Figure 4. The running of the light neutrino masses.

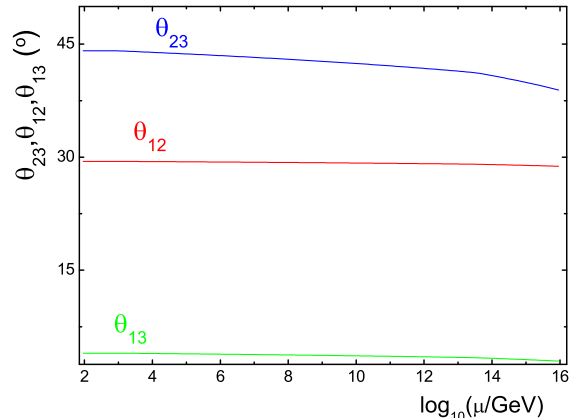


Figure 5. The evolution of the mixing angles.

The evolution of the atmospheric and solar mass differences is been depicted in figure 6. In figure 7 we plot the distribution Δm_{atm}^2 versus Δm_{sol}^2 at the two scales M_G (Table 7) and M_Z for the ten models of Table 5. We find that the hierarchy of the neutrino masses at the Pati-Salam breaking scale tends to be greater than that at low energies. Several models predict

Table 7. Values of the physical parameters Δm_{atm}^2 and Δm_{sol}^2 at M_G ; the effective neutrino mass $\langle m \rangle$ related to $\beta\beta_{0\nu}$ decay; and the parameter b_2 which determines the scale of the light matrix m_ν .

Solution	$\Delta m_{atm}^2(M_G) \cdot 10^3$	$\Delta m_{sol}^2(M_G) \cdot 10^5$	$\langle m \rangle$	b_2
1.	10.5294	27.6096	0.00361	3.41
2.	8.1987	30.1333	0.00633	2.88
3.	7.2533	31.6108	0.00607	1.50
4.	11.7749	30.4352	0.00753	1.28
5.	14.093	23.7416	0.00346	2.85
6.	12.347	28.2642	0.00822	1.26
7.	7.4256	30.855	0.00821	1.23
8.	5.2910	29.8775	0.00852	1.17
9.	9.74656	30.5418	0.00379	3.64
10.	8.99695	25.8838	0.00327	3.42

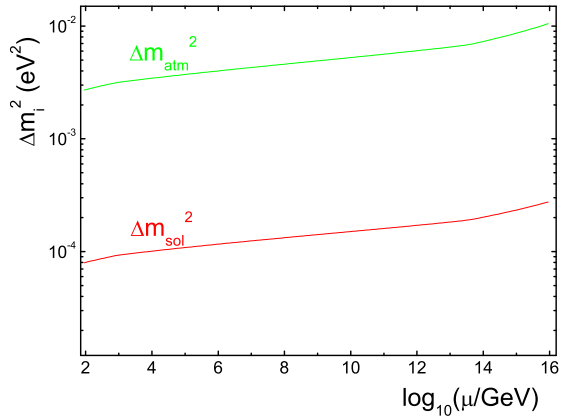


Figure 6. Running of Δm_{sol}^2 and Δm_{atm}^2 .

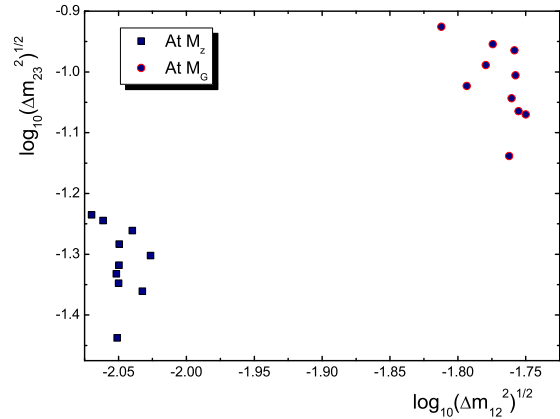


Figure 7. Δm_{12}^2 and Δm_{23}^2 at M_Z and at M_G .

$\Delta m_{23}^2/\Delta m_{21}^2$ out of the experimental range at M_G , although after the running at M_Z they are consistent with the data.

Finally, we check the predictions of our model for the effective neutrino mass parameter relevant for $\beta\beta_{0\nu}$ decay. This parameter can be written in terms of the observable quantities as

$$|\langle m \rangle| = \left| (m_1 \cos^2 \theta_\odot + e^{i\alpha} \sqrt{\Delta m_{sol}} \sin^2 \theta_\odot) \cos^2 \theta_{13} + \sqrt{\Delta m_{atm}} \sin^2 \theta_{13} e^{i\beta} \right|. \quad (25)$$

In the last column of Table (7) the $\beta\beta_{0\nu}$ -decay predictions are presented for solutions 1-10. Many current experiments attempt to measure this quantity [29]; the best current limit on the effective mass is given by the Heidelberg–Moscow collaboration [30]

$$\langle m \rangle \leq 0.35 z \text{ eV}, \quad (26)$$

where the parameter $z = \mathcal{O}(1)$ allows for uncertainty arising from nuclear matrix elements.

In a recent analysis of neutrinoless double beta decay [31] the allowable range of the effective mass parameter was given for specific scenarios. In the case of the normal hierarchy the bounds are

$$0 < \langle m \rangle < 0.007 \text{ eV} \quad (27)$$

thus our results are in the experimentally acceptable region.

5. Conclusions

In this talk, we studied the running of neutrino masses and mixing angles in a supersymmetric string-inspired $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_X$ model. An accurate description of the low energy neutrino data forced us to introduce two singlets charged under the $U(1)_X$, leading to two expansion parameters. The mass matrices are then constructed in terms of three expansion parameters

$$\epsilon \equiv \frac{\langle \phi \rangle}{M_U}, \quad \epsilon' \equiv \frac{\langle H \bar{H} \rangle}{M_U^2}, \quad \epsilon'' \equiv \frac{\langle \chi \rangle}{M_U}, \quad (28)$$

where ϕ and χ are singlets and H, \bar{H} the $SU(4) \times SU(2)_R$ -breaking Higgses. The model is simplified by the fractional $U(1)_X$ charges of H and χ , which ensure that the parameter ϵ'' only appears as a prefactor to the heavy Majorana neutrino masses.

The expansion parameter arising from the Higgs v.e.v.'s defines the ratio of the $SU(4)$ breaking scale M_G to the unification scale M_U : we performed a renormalization group analysis of gauge couplings under this constraint and found successful unification with the addition of extra states usually present in a string spectrum.

Assuming that only the third generation of quarks and charged leptons acquire masses at tree level and under a specific choice of $U(1)_X$ charges as well as Clebsch factors, we examined the implications for the light neutrino masses resulting from the see-saw formula. We found that the light neutrino mass spectrum is hierarchical and that the mass hierarchy tends to be larger at the GUT scale than at M_Z due the renormalization group running. The solar mixing angle θ_{12} is stable under RG evolution while larger renormalization effects are found for the atmospheric mixing angle θ_{23} and θ_{13} , always with their values at M_Z in agreement with experiment.

Acknowledgments

This research was funded by the program ‘HERAKLEITOS’ of the Operational Program for Education and Initial Vocational Training of the Hellenic Ministry of Education under the 3rd Community Support Framework and the European Social Fund.

References

- [1] Bahcall J N, Gonzalez-Garcia M C and Pena-Garay C 2004 *JHEP* **0408** 016 (*Preprint* hep-ph/0406294)
Altarelli G 2004 *Nucl. Phys. Proc. Suppl.* **137** 105 (*Preprint* hep-ph/0405182)
Maltoni M, Schwetz T, Tortola M A and Valle J W F 2004 *New J. Phys.* **6** 122 (*Preprint* hep-ph/0405172)
Gonzalez-Garcia M C and Pena-Garay C 2003 *Phys. Rev. D* **68** 093003 (*Preprint* hep-ph/0306001)
- [2] Petcov S T and Smirnov A Y 1994 *Phys. Lett. B* **322** 109 (*Preprint* hep-ph/9311204)
- [3] Minakata H and Smirnov A Y 2004 *Phys. Rev. D* **70** 073009 (*Preprint* hep-ph/0405088)
For a recent review see e.g. Minakata H 2005 *Preprint* hep-ph/0505262
- [4] Antusch S, King S F and Mohapatra R N 2005 *Phys. Lett. B* **618** 150 (*Preprint* hep-ph/0504007)
Frampton P H and Mohapatra R N 2005 *JHEP* **0501** 025 (*Preprint* hep-ph/0407139)
For a review see Mohapatra R N and Smirnov A Y 2006 *Preprint* hep-ph/0603118
- [5] Pati J C and Salam A 1974 *Phys. Rev. D* **10** 275
Mohapatra R N and Pati J C 1975 *Phys. Rev. D* **11** 2558
- [6] Petcov S T 1982 *Phys. Lett. B* **110** 245
Barbieri R, Hall L J, Smith D R, Strumia A and Weiner N 1998 *JHEP* **9812** 017 (*Preprint* hep-ph/9807235)

- Joshiyura A S and Rindani S D 2000 *Eur. Phys. J. C* **14** 85
 Mohapatra R N, Perez-Lorenzana A and Sousa Pires C A 2000 *Phys. Lett. B* **474** 355
 Shafi Q and Tavartkiladze Z 2000 *Phys. Lett. B* **482** 145
 Lavoura L 2000 *Phys. Rev. D* **62** 093011
 Grimus W and Lavoura L 2000 *Phys. Rev. D* **62** 093012
 Kitabayashi T and Yasue M 2001 *Phys. Rev. D* **63** 095002
 Babu K S and Mohapatra R N 2002 *Phys. Lett. B* **532** 77
 Mohapatra R N, Nasri S and Yu H B 2006 *Preprint* hep-ph/0603020
- [7] Leontaris G K, Rizos J and Psallidas A 2004 *Phys. Lett. B* **597** 182 (*Preprint* hep-ph/0404129)
 Leontaris G K, Psallidas A and Vlachos N D 2005 *Preprint* hep-ph/0511327
 See also Mconald K L and McKellar B H J 2006 *Phys. Rev. D* **73** 073004 (*Preprint* hep-ph/0603129)
- [8] Antusch S, Kersten J, Lindner M, Ratz M and Schmidt M A 2005 *JHEP* **0503** 024 (*Preprint* hep-ph/0501272)
- [9] Mei J W 2005 *Phys. Rev. D* **71** 073012 (*Preprint* hep-ph/0502015)
- [10] Ellis J, Hektor A, Kadastik M, Kannike K and Raidal M 2005 *Phys. Lett. B* **631** 32 (*Preprint* hep-ph/0506122)
- [11] T. Dent, G. Leontaris and J. Rizos 2005, *Phys. Lett. B* **605** 399 (*Preprint* hep-ph/0407151)
- [12] Leontaris G K and Rizos J 2001, *Phys. Lett. B* **510** 295 (*Preprint* hep-ph/0012255)
- [13] Antoniadis I and Leontaris G K 1989 *Phys. Lett. B* **216** 333
 Antoniadis I, Leontaris G K and Rizos J 1990 *Phys. Lett. B* **245** 161
 Leontaris G K and Rizos J 1999 *Nucl. Phys. B* **554** 3 (*Preprint* hep-th/9901098)
- [14] Allanach B C and King S F 1996 *Nucl. Phys. B* **459** 75 (*Preprint* hep-ph/9509205)
 Pati J C 2004 *Nucl. Phys. Proc. Suppl.* **137** 127 (*Preprint* hep-ph/0407220)
 Pati J C 2005 *Preprint* hep-ph/0507307
- [15] Allanach B C, King S F, Leontaris G K and Lola S 1997 *Phys. Rev. D* **56** 2632 (*Preprint* hep-ph/9610517)
- [16] Allanach B C and King S F 1995 *Phys. Lett. B* **353** 477 (*Preprint* hep-ph/9504406)
- [17] Leontaris G K and Tracas N D 1996 *Phys. Lett. B* **372** 219 (*Preprint* hep-ph/9511280)
- [18] Eidelman S *et al.* 2004 *Phys. Lett. B* **592** 1
- [19] Irges N, Lavignac S and Ramond P 1998 *Phys. Rev. D* **58** 035003 (*Preprint* hep-ph/9802334)
 Altarelli G and Feruglio F 1998 *Phys. Lett. B* **439** 112 (*Preprint* hep-ph/9807353)
 Ellis J R, Leontaris G K, Lola S and Nanopoulos D V 1999 *Eur. Phys. J. C* **9** 389 (*Preprint* hep-ph/9808251)
 Leontaris G K and Rizos J 2000 *Nucl. Phys. B* **567** 32 (*Preprint* hep-ph/9909206)
- [20] Athanasiu A, Atick J, Dine M and Fischler W 1988 *Phys. Lett. B* **214** 55 (*Preprint* hep-ph/9601337)
- [21] Leontaris G K 1996 *Phys. Lett. B* **372** 212 (*Preprint* hep-ph/9601337)
- [22] Ellis J, Lopez J L and Nanopoulos D V 1990 *Phys. Lett. B* **247** 257
 Kalara S, Lopez J L and Nanopoulos D V 1992 *Phys. Lett. B* **275** 304
- [23] Chankowski P H and Pluciennik Z 1993 *Phys. Lett. B* **316** 312 (*Preprint* hep-ph/9306333)
 Babu K S, Leung C N and Pantaleone J T 1993 *Phys. Lett. B* **319** 191 (*Preprint* hep-ph/9309223)
 Antusch S, Drees M, Kersten J, Lindner M and Ratz M 2001 *Phys. Lett. B* **519** 238 (*Preprint* hep-ph/0108005)
 Antusch S, Drees M, Kersten J, Lindner M and Ratz M 2002 *Phys. Lett. B* **525** 130 (*Preprint* hep-ph/0110366)
- [24] Antusch S, Kersten J, Lindner M and Ratz M 2002 *Phys. Lett. B* **538** 87-95 (*Preprint* hep-ph/0203233)
 Casas J A, Espinosa J R, Ibarra A and Navarro I 2000 *Nucl. Phys. B* **573** 652 (*Preprint* hep-ph/9910420)
 Chankowski P H, Krolkowski W and Pokorski S 2000 *Phys. Lett. B* **473** 109 (*Preprint* hep-ph/9910231)
- [25] Antusch S, Kersten J, Lindner M and Ratz M 2003 *Nucl. Phys. B* **674** 401 (*Preprint* hep-ph/0305273)
- [26] Grzadkowski B and Lindner M 1987 *Phys. Lett. B* **193** 71
 Grzadkowski B, Lindner M and Theisen S 1987 *Phys. Lett. B* **198** 64
 Casas J A, Espinosa J R, Ibarra A and Navarro I 1999 *Nucl. Phys. B* **556** 3-22 (*Preprint* hep-ph/9904395)
 Casas J A, Espinosa J R, Ibarra A and Navarro I 2000 *Nucl. Phys. B* **569** 82-106 (*Preprint* hep-ph/9905381)
- [27] Ellis J R and Lola S 1999 *Phys. Lett. B* **458** 310 (*Preprint* hep-ph/9904279)
 Barbieri R, Ross G G and Strumia A 1999 *JHEP* **9910** 020 (*Preprint* hep-ph/9906470)
- [28] Dine M, Seiberg N and Witten E 1987 *Nucl. Phys. B* **289** 589
- [29] For a review see e.g. Petcov S T 2005 *Phys. Scripta* **T121** 94 (*Preprint* hep-ph/0504166)
- [30] Klapdor-Kleingrothaus H V *et al.* 2001 *Eur. Phys. J. A* **12** 147 (*Preprint* hep-ph/0103062)
- [31] Choubey S and Rodejohann W 2005 *Phys. Rev. D* **72** 033016 (*Preprint* hep-ph/0506102)