The Algorithm for Blind Multi-user Detector Based on Subspace Tracking

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Abstract: Multi-user detection (MUD) is an efficient technique for interference suppression that reduces the Multiple access interference (MAI) and improves the performance and increases the capacity of the system. Nowadays most of the research to MUD focuses on the blind multi-user detector because it does not require training sequences and can save the spectrum resource. By applying an improved subspace tracking algorithm to a modified subspace-based linear MMSE multi-user detector, a blind multi-user detector is presented. For the improved subspace tracking algorithm can reduce considerably computational complexity while keeping satisfactory convergence speed and stability and the modified MMSE multi-user detector doesn’t require the estimation of eigenvalue matrix, there can be significant elevation in the detection performance. Simulation results demonstrate preliminarily the conclusions above. Copyright © 2014 IFSA Publishing, S. L.

Keywords: Blind multi-user detection (MUD), Multi-user access interference (MAI), Affine projection algorithm (APA), Subspace tracking.

1. Introduction

In recent years, feature subspace method has been widely used in the field of signal processing. In 1996, Stephen Bensley E proposed the blind subspace estimation method for channel parameters in CDMA systems [1]. Xiaodong Wang proposes a subspace blind multi-user detection algorithm based on the method above in 1998. Based on subspace approach to blind multi-user detection, the number of users is to decide the signal subspace tracking and system in determining the number of users, actually is to judge the signal subspace dimension. Its role is to distinguish between signal and noise subspace. The estimation of signal subspace only need to know the timing information, the expected user, the system user and spread spectrum waveform. Algorithm complexity can be reduced to O (NK) (where N is the spreading gain, K is the number of users). Therefore, multi-user detection based on subspace method has become a hot research direction.

2. The Subspace Approach and Theory

The model of the base band signals can be considered as:

\[ r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t), \quad t \in [0, T], \]  

(1)

For direct sequence spread spectrum multiple access modes, the feature wave of user k is:

\[ s_k(t) = \sum_{j=0}^{N-1} \beta_j^k \varphi(t - jT_c), \quad t \in [0, T], \]  

(2)
where $N$ stands for the processing gain, $(\beta_k^r, \ldots, \beta_{N-1}^r)$ presents the feature series which has been given to user $k$, and $\beta_j^t \in [-1, +1]$. $\varphi$ is the code of the normalized waveform with $T_e$ to be the time interval and $NT_e = T$.

At the receiving end, to code matched filtering, and then use code rate sampling, in a symbol interval can get N code matched filter output of sample vector $r$. Hence, synchronization model as mentioned in Eq. (1) can be available written in vector form as:

$$r = \sum_{k=1}^{K} A_k b_k s_k + \sigma n,$$  \hspace{1cm} (3)

And the parameter $s_k$ is the normalized features of user $k$ wave vector:

$$s_k = \frac{1}{\sqrt{N}}[\beta_k^r, \beta_1^r, \ldots, \beta_{N-1}^r]^T,$$  \hspace{1cm} (4)

where $\sigma$ is the Gaussian white noise vector with mean to 0, and the covariance matrix to $I_N$ (unit matrix).

For general condition, assuming that the characteristic waveforms of the $k$ users are linear independent, then they can be record as $S = [s_1, \ldots, s_K]$ and $A = \text{diag}(A_1^2, \ldots, A_K^2)$, So, the auto-correlation matrix of the received signal vector $r$ is

$$R = E\{rr^T\} = \sum_{k=1}^{K} A_k^2 s_k s_k^T + \sigma^2 I_N,$$  \hspace{1cm} (5)

Do eigenvalue decomposition for matrix $R$, we can get:

$$R = U \Lambda U^T = [U_1, U_n][\Lambda_1 0 0 \Lambda_n][U_1^T 0 U_n^T],$$  \hspace{1cm} (6)

And $U = [U_1, U_n]$, $\Lambda = \text{diag}(\Lambda_1, \Lambda_n)$ contains $k$ maximum eigenvalues of matrix $R$. $U_s = [u_1, \ldots, u_K]$ includes the corresponding orthogonal eigenvectors. $\Lambda_s = \sigma^2 I_{N-K}$ and $U_n = [u_{K+1}, \ldots, u_N]$ respectively contain $N-K$ minimum eigenvalues $\sigma^2$ and corresponding eigenvectors. It is easy to see that the domain $S$ and the range of values of the characteristic matrix of the matrix $U_s$ are equal, which means $\text{range}(S) = \text{range}(U_s)$. Signal characteristic matrix $U_s$ of the domain space is known as the signal subspace. The orthogonal complement is called noise subspace, which is formed by the noise characteristic matrix $U_n$.

A weight vector of linear multi-user detectors is assumed to be $c_k (k = 1, 2, \ldots, K)$. Users 1 is expected user, then the expected user information $b_1$ can be represented by symbols $\hat{b}_1$.

$$\hat{b}_1 = \text{sgn}(c_1^T r).$$  \hspace{1cm} (7)

Based on subspace theory, multi-user detection subspace method use linear multi-user detector weight vector subspace parameter, and selects the subspace tracking algorithm, and the subspace tracking parameter evaluation, in order to eventually find out weight vector linear multi-user detector. General linear multi-user detector need to order $N \times N$ received signal autocorrelation matrix inversion, and the computation is large. The introduction of subspace method makes the calculation into approximation to subspace tracking algorithm. The subspace tracking algorithm only needs to do signal subspace tracking, and signal feature vector is $N \times K$ order. So the calculation is reduced greatly.

The introduction of subspace method, can only through the relevant user receives waveform and timing information and make the de-correlation and the weight vectors of linear minimum mean square error detector is blind. With the previous minimum output energy (MOE) algorithm, compared with the detector algorithm with lower complexity and higher SNR performance. And using the subspace method can joint was carried out on the fading channel estimation and multi-user detection, when the signal in the multi-path channel is damaged, almost does not cause performance degradation [2].

The de-correlation detector and the weight vectors of the linear MMSE detector can be available replace by signal subspace parameter $(U_s, A)$ and $\sigma$.

Subspace representation of de-correlation multi-user detector is

$$w_i = \frac{1}{s_i^T U_i (A - \sigma^2 I_N)^{-1} U_i^T s_i} U_i (A - \sigma^2 I_N)^{-1} U_i^T s_i,$$  \hspace{1cm} (8)

The subspace of linear MMSE detector is

$$c_i = \frac{1}{s_i^T U_i A_i^{-1} U_i^T s_i} U_i A_i^{-1} U_i^T s_i,$$  \hspace{1cm} (9)

We can see that the de-correlation detector and linear MMSE detector and you need to use the signal subspace parameter, $U_s$ and $A$. The subspace method makes the detector a key to tracking the core.
algorithm of signal subspace, and it needs to be able to real-time tracking of the received signal covariance matrix eigenvalue \( R = E \{ r r^T \} \) and eigenvector. In this way, the signal subspace parameter is affirmative, and the linear multi-user detection is obtained. In this paper, the main research minimum mean square error (MMSE) multi-user detector subspace method.

3. The Basic Concept of Subspace Tracing

The classic subspace tracking methods are Eigenvalue De-composition (EVD) and Singular Value Decomposition (SVD). Although its performance is good, but the complexity \( O(N^3) \) is high, which is hard for engineering implementation. So the more extensive we considered is the fast subspace tracking algorithm. They can reduce the computing complexity to \( O(NK) \) or even to \( O(N^2) \). The following are some approaches on the field of complex subspace tracking.

3.1. Projection Approximation Subspace Tracking (PAST)

The PAST algorithm proposed by Yang to the signal subspace tracking problem is converted into the constrained minimization problem. Through a projection approximation to simplify the minimization problem for index weighted least squares problem, use the least squares (RLS) minimization problem for index weighted least mean square problem, use the least squares (RLS) projection approximation to simplify the constrained minimization problem. Through a projection approximation, the signal subspace tracking problem is converted into a quadratic function of \( W(n) \) at time \( n \) on the column vector \( W(n) \).

For stationary signal, the difference between \( W_H^H(i-1)r(i) \) and \( W_H^H(n)r(i) \) is very small, so from Eq. (11) we can get the fixed cost function:

\[
J(W(n)) = \sum_{i=1}^{n} \lambda^{n-i} \| r(i) - W(n)y(i) \|^2 ,
\]

At this point, \( J(W(n)) \) is a quadratic function of \( W(n) \). In this way, through the projection approximation method, the error performance of curved surface \( J(W(n)) \) is improved. The PAST method of the main advantage is that introducing Eq. (13) as shown in the index weighted least mean square criterion. If \( W(n) = C_{3y}(n)C_{3y}^{-1}(n) \), then \( J(W(n)) \) is minimized, and there are:

\[
J(W(n)) = \sum_{i=1}^{n} \lambda^{n-i} \| r(i) - W(n)y(i) \|^2 ,
\]

\[
C_{3y}(n) = \sum_{i=1}^{n} \lambda^{n-i} y(i)y^H(n) = \lambda C_{3y}(n-1) + r(i)y^H(n) ,
\]

\[
C_{3y}(n) = \sum_{i=1}^{n} \lambda^{n-i} y(i)y^H(n) = \lambda C_{3y}(n-1) + y(i)y^H(n) .
\]

The one iterative computational complexity of \( N \times K \) dimension matrix \( C_{3y}(n) \) and the \( K \times K \) dimension matrix \( C_{3y}^{-1}(n) \) are respectively \( O(NK) \) and \( O(K^2) \). \( C_{3y}^{-1}(n) \) can be obtained...
through the matrix inversion lemma or Cholesky factor recursive calculation of QR decomposition.

3.2. Projection Approximation Subspace Tracking with Deflation (PASTd)

PASTd algorithm [5-6] is on the basis of use of compression mapping technology with the PAST to realize the signal eigenvalue and eigenvector of tracking. It is first proposed by the B. Yang in 1995. The computational complexity can be reduced from $O(NK + O(K^2))$ of the PAST algorithm to $4NK + O(K)$ with good performance. MMSE detector based on PASTd algorithm is proposed by Xiaodong Wang in 1998. The PASTd algorithm adaptive update process is as follows:

\[
\tau(n) = \frac{1}{\|\bar{q}(n)\|^2} \left( \frac{1}{1 + \|\bar{p}(n)\|\|\bar{q}(n)\|^2} - 1 \right)
\]

\[
\bar{f}(n) = \tau(n)W(n-1)\bar{q}(n) + \left( 1 + \tau(n)\|\bar{q}(n)\|^2 \right)\bar{p}(n)
\]

\[
Z(n) = \frac{1}{\beta}Z(n-1) - \gamma(n)\bar{q}(n)\bar{q}^H(n)
\]

$\beta (0 < \beta < 1)$ is the forgetting factor. The algorithm complexity is $4NK + O(K^2)$ and only slightly higher than the PASTd algorithm, but it ensures the orthogonality for each iteration of the signal subspace tracking. Reference [7] proved that if the first K eigenvalues of covariance matrix $C$ strict larger than the following N- K eigenvalues, were OPAST algorithm is global convergence.

3.3. Orthogonal predict approximation subspace tracking algorithm (OPAST)

OPAST algorithm is proposed by k. Abed - Meraim on the basis of PAST algorithm. It is against the disadvantages that the orthogonality of $U(n)$ is weak in PAST algorithm. It introduces orthogonalization formula

\[
U(n) = U(n)\left[ U^H(n)U(n) \right]^{-1/2}, \quad [\bullet]^{-1/2}
\]

is the inverse square root of matrix. Signal subspace orthogonal forecast into iterative subspace tracking algorithm is as follows:

for $n = 1, 2, ...$
\[
U(n) = U(n) + \bar{f}(n)\bar{q}^H(n)
\]
\[
\bar{q}(n) = \frac{1}{\beta}Z(n-1)\bar{y}(n)
\]
\[
\bar{y}(n) = W^H(n-1)r(n)
\]
\[
\gamma(n) = \frac{1}{1 + \bar{y}^H(n)\bar{q}(n)}
\]
\[
\bar{p}(n) = \gamma(n)\left( r(n) - W(n-1)\bar{y}(n) \right)
\]

4. The Improved Subspace Blind Multi-user Detector

PASTd algorithm and OPAST algorithm was presented on the basis of PAST algorithm. PASTd algorithm uses compression technology to simplify OPAST algorithm. It makes the complexity fall from $O(NK + O(K^2))$ to $4NK + O(K)$, but the compression technology also caused the orthogonality of algorithm decrease. OPAST algorithm improved the PAST algorithm by introducing orthogonalization formula, which greatly enhances the orthogonality of the algorithm and makes the algorithm convergence speed quickly. But the algorithm stability is bad, and it is more sensitive to the cumulative error. The algorithm with the increase of the number of iterations divergence problem may also come up. An accelerated subspace tracking algorithm is proposed in reference [8] on the basis of PASTd algorithm. Though the computational complexity of this algorithm is a little more than the PASTd algorithm, its convergence speed is slightly increased. It also has good steady-state performance and global convergence. In this paper, each iteration of the algorithm in the process of the signal subspace parameter $U_i$ is orthogonalization again in order to enhance its orthogonality and to accelerate the convergence speed. The improved algorithm called newPASTd algorithm.

For conventional linear MMSE detector based on subspace, the section 2 shows that linear MMSE MUD demodulation with the signal subspace parameter vector $U_i$ and the signal characteristic value matrix $A_i$. By using of $A_i$, the approximation error is introduced in the estimate. A kind of modified MMSE multi-user detector is presented in reference [9]. In this kind of modified MMSE detector, if you choose a kind of orthogonality of subspace tracking algorithm, it only need the parameters of $U_i$ without the need of eigenvalue matrix $A_i$ to effectively demodulate the interested users. This paper argues that the newPASTd
algorithm used for the correction of the MMSE detector will get a good detector, and it can also make the complexity, convergence and steady-state performance benefit.

4.1. The Modified MMSE Multi-user Detector

From section 2, we know that, based on subspace theory, the linear MMSE multi-user detector weight vector is expressed as:

\[
W = \frac{1}{s^T U_s A_s^{-1} U_s^T s_1} U_s A_s^{-1} U_s^T s_1, \tag{16}
\]

where \( W \) is the linear multi-user detector weight vector as expected 1 user. The expected user information can be expressed as:

\[
h_i = \text{sgn}(W^T r), \tag{17}
\]

As Eq.(17) shows, Solving this kind of classic linear MMSE MUD demodulating vector need to use signal subspace parameter \( U_s \) and the signal characteristic value matrix \( \Lambda_s \). By using of \( \Lambda_s \) the approximation error is introduced in the estimate. If the value of each estimated column vector of signal subspace by subspace tracking algorithm of \( U_s \) is orthogonal, then \( \text{rang}(U_s) = \text{rang}(S) = \text{rang}(U_s) \). The parameter \( S \) is the same as defined in section 2. The received signal \( r \) is projected into the subspace \( U_s \), a \( K \) dimensions vector is gotten as \( \tilde{U}_s^H r [10] \). Its autocorrelation matrix is:

\[
Y = E[(\tilde{U}_s^H r)(\tilde{U}_s^H r)^H] = \tilde{U}_s^H E(r r^H) \tilde{U}_s = \tilde{U}_s^H \Sigma \tilde{U}_s; \tag{18}
\]

Form Eq.(18), the linear multi-user detector of user 1 is

\[
w_{1\text{new}} = [s_1^T \tilde{U}_s Y^{-1} \tilde{U}_s^H s_1]^{-1} \tilde{U}_s Y^{-1} \tilde{U}_s^H s_1, \tag{19}
\]

\( C, Y \) can be gotten form the following computation:

\[
C(M) = \sum_{n=1}^{M} \beta^{M-n} r(n) r(n)^H, \tag{20}
\]

\[
Y(M) = \tilde{U}_s^H C(M) \tilde{U}_s, \tag{21}
\]

\( M \) is the current number of data transmission for each user and \( \beta (0 \leq \beta \leq 1) \) is called forgetting factor.

Replacing \( W \) in Eq. (16) as modified MMSE detector by \( w_{1\text{new}} \), we only need the signal subspace without characteristic value matrix. Because \( \text{rang}(\tilde{U}_s) = \text{rang}(U_s) \), so \( W \) is equivalent to \( w_{1\text{new}} \). The improved linear MMSE detector avoids the estimate of the signal characteristic value of matrix \( A_s \), it thus improves the performance of the detector by reducing introduced error of the subspace estimation.

4.2. The Improved Subspace Tracking Algorithm (new PASTd Algorithm)

The PASTd algorithm has been discussed in section 2, and its cost function can be rewritten as:

\[
J[U_s(n)] = E\left\{ ||r(n) - U_s(n)y(n)||^2 \right\}, \tag{22}
\]

\( r(n) \) is the receiving data vector, and \( y(n) = U_s^H (n - 1)r(n) \). The adaptive update process with PASTd algorithm is as following:

for \( i = 1, \ldots, K \)

\[
y_i(n) = u_i^T (n - 1)x_i(n) \]

\[
e_i(n) = u_i(n - 1)y_i(n) - x_i(n) \]

\[
z_i(n) = \beta z_i(n - 1) + |y_i(n)|^2 \]

\[
u_i(n) = u_i(n - 1) - e_i(n) y_i^* (n)/z_i(n) \]

\[
x_{i+1}(n) = x_i(n) - z_i(n) u_i(n) \]

\( 0 < \beta \leq 1 \) is the forgetting factor, and \( x_i(n) = r(n) \). \( z_i(n) \) is the \( i \) th eigenvalue, and \( A_s(n) = \text{diag}(z_1(n), \ldots, z_K(n)) \).

In conclusion, this paper designed the modified MMSE detection algorithm based on newPASTd is summarized as follows:

For \( n = 1, 2, \ldots, M \)

1. Receive data vector first.
2. Through newPASTd tracking algorithm to estimate the orthogonal signal subspace parameter \( U_s \).
3. Adaptive computing \( C, Y \) based on Eq.(20) and Eq.(21).
4. Get the adjusting vector by set \( U_s, Y, s_i \) to Eq.(19), and the final output will be decide by Eq.(17).

5. Simulation and Analyzing

5.1. The Simulation and Analysis of Subspace Tracking Algorithm

In this section, we do simulation and analysis with space tracking algorithms with PASTd and OPAST. Considering that the synchronous DS-CDMA system is under the condition of additive white Gaussian noise channel, we use Gold spread...
spectrum sequence with 31 long bits. Assuming that the signal-to-noise ratio of user $k$ is \( SNR = 10 \log \left( \frac{A_k^2}{\sigma^2} \right) \), the multiple access interference is \( 10 \log \left( \frac{A_k^2}{A_1^2} \right) \), system has six users, the signal-to-noise ratio of the expected user 1 is 20 dB, five other users from multiple access interference are 15 dB and the forgetting factor $\beta$ is 0.996. Then the signal-to-noise ratio can be gotten as:

\[
SIR = \frac{A_1^2 \left( \mathbf{e}^T(n) \mathbf{s}_1 \right)^2}{\sum_{i=2}^{K} A_i^2 \left( \mathbf{e}^T(n) \mathbf{s}_i \right)^2 + \sigma^2 \mathbf{e}^T(n) \mathbf{e}(n)}.
\]  

(23)

Fig. 1 is SIR performance for the subspace tracking algorithm of PASTd. We can see from Fig. 1, the PASTd algorithm has a good steady-state value, which can reach to 15 dB. But it has very slow convergence, which needs about 1300 times to reach a stable value. If too much interference users, or if the initial value of convergence value is far, the algorithm may not be to convergence. So, the application in multi-user detection PASTd algorithm subspace tracking, general with classic first singular value decomposition (SVD) algorithm for 50 or 100 iterations, to speed up the convergence speed. The complexity of PASTd algorithm is low, only $O(NK)$. The parameter $N$ is spread spectrum gain, and $K$ stands for the number for system users.

Fig. 2 is SIR performance for the subspace tracking algorithm of OPAST. The eigenvalue estimation of the algorithm uses the inverse of the diagonal elements of $Z(n)$. As seen from Fig. 2, OPAST algorithm has high convergence speed, and after about 400 times of iteration convergence to the steady state value, but the steady state value is not high, only about 7 dB.

5.2. The Analysis of Blind Multi-user Detector Based on Improved Algorithm

In this section, we do MMSE MUD simulation and comparison with space tracking algorithms as PASTd, OPAST and newPASTd. Considering that the synchronous DS - CDMA system is under the condition of additive white Gaussian noise channel, we use Gold spread spectrum sequence with 31 long bits. Assuming that the signal-to-noise ratio of user $k$ is \( SNR = 10 \log \left( \frac{A_k^2}{\sigma^2} \right) \), the multiple access interference is \( 10 \log \left( \frac{A_k^2}{A_1^2} \right) \), system has six users, the signal-to-noise ratio of the expected user 1 is 20 dB, and five other users from multiple access interference are 15 dB.

Fig. 3 shows the performance of three kinds of algorithm in one user. We can see that the newPASTd MUD and OPAST MUD have fast convergence speed, and the subspace tracking effectively. OPAST MUD has fastest convergence speed, but the steady state value is poorer and its SNR is low, which means its multiple access interference ability is poor. PASTd MUD convergence speed is very slow, and it is not suitable for real-time processing system.

Fig. 4 compares the three algorithms in the expected user steady-state performance under different signal-to-noise ratio. For each SNR, the corresponding SI is the mean value for 2000 iterations by SIR. Seen from the diagram, multiple access interference among users dominant communication environment, newPASTd MUD steady-state has higher value; OPAST MUD steady-state value is extremely low. The PASTd MUD steady-state value is also poor. Fig. 4 shows newPASTd MUD has good steady-state performance and good performance of multiple access interference.
solved the problems that slow convergence speed of the PASTd MUD and that poor steady-state performance of the OPAST MUD.

6. Conclusions

The paper first introduced the synchronous DS-CDMA system de-correlation detector and MMSE multi-user detector in the subspace representation. Then it analyzes and compares several low complexity subspace-tracking algorithms. Finally by using strong orthogonality and low complexity newPASTd algorithm in MMSE detector it gets a blind multi-user detector. It suppresses the eigenvalue estimates that affect the performance of detection, solved the problems that slow convergence speed of the PASTd MUD and that poor steady-state performance of the OPAST MUD.

Fig. 3. The output SNR and samples.

Fig. 4. The output SNR and its SI.

Fig. 5 shows the SNR of three algorithms at 20 dB. We can see that due to accumulated subspace eigenvalue matrix and the estimated error, the OPAST MUD and PASTd MUD makes the detection performance poorer and bit error rate higher. The newPASTd MUD performance is superior to both.

6. Conclusions

The paper first introduced the synchronous DS-CDMA system de-correlation detector and MMSE multi-user detector in the subspace representation. Then it analyzes and compares several low complexity subspace-tracking algorithms. Finally by using strong orthogonality and low complexity newPASTd algorithm in MMSE detector it gets a blind multi-user detector. It suppresses the eigenvalue estimates that affect the performance of detection.

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