Consensus of multi-agent systems with nonlinear dynamics and sampled-data information: a delayed-input approach

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SUMMARY

This paper is concerned with the problem of consensus in directed networks of multiple agents with intrinsic nonlinear dynamics and sampled-data information. A new protocol is induced from a class of continuous-time linear consensus protocols by implementing data-sampling technique and a zero-order hold circuit. On the basis of a delayed-input approach, the sampled-data multi-agent system is converted to an equivalent nonlinear system with a time-varying delay. Theoretical analysis on this time-delayed system shows that consensus with asymptotic time-varying velocities in a strongly connected network can be achieved over some suitable sampled-data intervals. A multi-step procedure is further presented to estimate the upper bound of the maximal allowable sampling intervals. The results are then extended to a network topology with a directed spanning tree. For the case of the topology without a directed spanning tree, it is shown that the new protocol can still guarantee the system to achieve consensus by appropriately informing a fraction of agents. Finally, some numerical simulations are presented to demonstrate the effectiveness of the theoretical results and the dependence of the upper bound of maximal allowable sampling interval on the coupling strength. Copyright © 2012 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Many real systems in nature and human society can be modeled by multi-agent systems, where typical examples include animal groups, unmanned air vehicles, sensor networks, clusters of satellites, to name just a few [1–4]. One critical issue arising from multi-agent systems is to develop distributed control strategies based only on local relative information that can guarantee the whole system to evolve into a coordinated behavior. Particularly, consensus in multi-agent systems, which means all the agents will reach an agreement on certain concern or interest, has been extensively studied in the past few years, with many profound results established [2–6].

The consensus problem has a long-standing record in the field of computer science, especially in distributed computation and automata theory. In the context of multi-agent systems, in recent years, we have witnessed a dramatic advance of various distributed strategies that can achieve consensus. In [7], Vicsek \textit{et al.} proposed a simple model for phase transition of a group of self-driven particles and numerically depicted the evolution of the model. By using tools from matrix theory and graph theory, some theoretical analysis of the consensus problem on the linearized Vicsek’s model

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was provided in [8]. Then, a general framework of the consensus problem for networks of dynamic agents with a fixed or switching topology and with communication time-delays was suggested and studied by Olfati-Saber et al. [9]. The consensus conditions derived in [9] were further relaxed in [10], showing that consensus in a network with dynamically changing topology can be reached if and only if the time-varying network topology contains a spanning tree frequently enough as the network evolves in time [10]. Consensus tracking control of multi-agent systems with an active leader was investigated in [11, 12]. In fact, extensive research on the consensus problem is still ongoing today, such as consensus-seeking subjected to stochastic disturbances [13, 14], consensus with diverse time delays [15], and consensus with higher-order dynamics [16, 17].

The research work along this line can be divided into consensus in discrete-time and continuous-time systems, respectively, according to the dynamical nature of agents. Generally, the analytical methods for these two kinds of systems are different. In many applications, a continuous system or process may use the data sampled at discrete time instants because of the implementation of digital sensors, filters, and controllers. Motivated by this observation, formation control of multi-agent systems with intermittent communication was investigated in [18, 19]. With the use of energy-based analysis, some stability conditions of sampled-data systems were obtained and verified by simulations [18, 19]. In [20], the average consensus problem for a class of first-order multi-agent systems with noise disturbances and sampled-data communications was considered. It was shown that an arbitrarily small static mean-square consensus error can be achieved by appropriately choosing the sampling time interval [20]. In [21], two sampled-data-based coordination algorithms were proposed and analyzed for multi-vehicle systems with second-order linear dynamics under dynamic directed interactions. Some sufficient conditions were derived on the interaction graph, the damping gain, and the sampling period to guarantee coordination by using some properties of infinite products of stochastic matrices [21]. Then, by using tools from polynomial theory, some necessary and sufficient conditions for consensus in first-order and second-order multi-agent systems with a fixed directed topology and periodic sampling were obtained in [22] and [23], respectively. In [24], some sufficient conditions were derived to guarantee the states of first-order multi-agent systems with delayed sampling and undirected switching topologies to converge to consensus. By Lyapunov stability theory, some sufficient conditions for consensus of second-order multi-agent systems with a fixed topology and asynchronous sampling were given in [25].

It has been observed that only static consensus can be achieved in most of the previously mentioned multi-agent systems using sampled-data information feedback. There is little work reported in the literature on consensus with time-varying final velocities for multi-agent systems. Very recently, a new class of consensus algorithms for coupled harmonic oscillators with a directed topology have been introduced and discussed in [26], where time-varying asymptotic oscillatory velocities were generated. For multi-agent systems with intrinsic nonlinear dynamics, it is the agent’s intrinsic dynamics that determine the final consensus states as the coupling term gradually disappears when consensus is achieved asymptotically. Thus, a consensus trajectory must be a solution of an isolated system, and the consensus trajectory may be an isolated equilibrium point, a periodic orbit, or even a chaotic orbit in some applications. Pertinent works along this line include [27–29], where consensus problem of continuous-time coupled systems are investigated. Prior to the protocols proposed in this paper, the coupling laws used in the papers mentioned earlier are based on a common assumption that each agent can communicate with its neighbors all the time, which is clearly impractical in many cases. In this paper, therefore, we focus on solving the sampled-data consensus problem for multi-agent systems with intrinsic nonlinear dynamics, where each agent’s dynamics are driven by both a nonlinear term depending on its own state and a navigational feedback term based on the sampled relative states between its own and the neighbors’ ones. Note that the sampled-data-based feedback requires much less information, computational power and bandwidth in sensing, and data communications in a network of multi-agent systems. On the basis of a delayed-input approach, the sampled-data multi-agent system is first converted to a nonlinear system with delay in the feedback. Theoretical analysis is then performed to show that consensus with time-varying velocities in a strongly connected network can be achieved by choosing appropriate sampling time intervals. Furthermore, several feasible linear matrix inequalities are established for estimating the maximal allowable upper bound of sampling intervals. The results are consequently extended to the case...
where the communication topology contains only a directed spanning tree. Analysis is also provided for the case where the topology does not contain any spanning tree to show that the protocol can still guarantee the whole group of agents to achieve consensus by appropriately informing a fraction of agents. Finally, some numerical simulations are presented to demonstrate the effectiveness of the theoretical results.

The rest of the paper is organized as follows. In Section 2, some preliminaries and the model description are given. The main results are discussed in Section 3. In Section 4, some simulation examples are shown to illustrate the theoretical results. Conclusions are finally drawn in Section 5.

Throughout the paper, let \( \mathbb{R} \) and \( \mathbb{N} \) be the sets of real and natural numbers, and \( \mathbb{R}^{N \times N} \) be the \( N \times N \) real matrix space. \( I_N \) (\( O_N \)) represents the \( N \)-dimensional identity (zero) matrix, and \( 1_N \) (\( 0_N \)) indicates the \( N \)-dimensional column vector with each entry being 1 (0). Moreover, matrices are assumed to have compatible dimensions if not explicitly stated. A column vector \( x \in \mathbb{R}^N \) is said to be positive if and only if all entries \( x_i > 0 \), \( i = 1, 2, \ldots, N \). \( \| \cdot \| \) and \( \otimes \) represent the Euclidean norm and the Kronecker product, respectively.

2. PRELIMINARIES AND MODEL DESCRIPTION

2.1. Preliminaries

A directed graph \( G(\mathcal{V}, \mathcal{E}, \mathcal{A}) \) consists of a set of vertices \( \mathcal{V} = \{v_1, v_2, \ldots, v_N\} \), a set of directed edges \( \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \), and a weighted adjacency matrix \( \mathcal{A} = [a_{ij}]_{N \times N} \) having nonnegative entries. An edge \( e_{ij} \) in graph \( G \) is denoted by the ordered pair of vertices \( (v_j, v_i) \), where \( v_j \) and \( v_i \) are called the parent and child vertices, respectively, and \( e_{ij} \in \mathcal{E} \) if and only if \( a_{ij} > 0 \). For simplicity, denote \( G(\mathcal{V}, \mathcal{E}, \mathcal{A}) \) by \( G(\mathcal{A}) \) if no confusion will arise. A path from node \( v_k \) to \( v_j \) is a sequence of edges, \( (v_k, v_{k_1}), (v_{k_1}, v_{k_2}), \ldots, (v_{k_l}, v_j) \), with distinct vertices \( v_{k_m}, m = 1, 2, \ldots, l \). A directed graph is called strongly connected if and only if there is a directed path between any pair of distinct vertices. Furthermore, a directed graph contains a directed spanning tree if there exists a vertex called root such that there exists a directed path from this root to every other vertex. The Laplacian matrix \( L = [l_{ij}]_{N \times N} \) of \( G(\mathcal{A}) \) is defined as \( l_{ii} = \sum_{j \neq i} a_{ij}, l_{ij} = -a_{ij} \) for \( i \neq j \). Clearly, matrix \( L \) is symmetric if the graph is undirected. For a directed graph, the Laplacian matrix \( L \) has the following properties.

Lemma 1 ([30])

Suppose that a directed graph \( G(\mathcal{A}) \) is strongly connected. Then, its Laplacian matrix \( L \) is irreducible and satisfies \( L1_N = 0 \). Furthermore, there exists a positive vector \( \xi = (\xi_1, \xi_2, \ldots, \xi_N)^T \) such that \( \xi^T L = 0 \) and \( \xi^T 1_N = 1 \).

Lemma 2 (Frobenius normal form [31])

There exist a permutation matrix \( W \) with order \( N \) and an integer \( m \geq 1 \), such that

\[
W^T L W = \begin{bmatrix}
\mathcal{T}_{11} & O & \cdots & O \\
\mathcal{T}_{21} & \mathcal{T}_{22} & \cdots & O \\
\vdots & \vdots & \ddots & \vdots \\
\mathcal{T}_{m1} & \mathcal{T}_{m2} & \cdots & \mathcal{T}_{mm}
\end{bmatrix}
\]

(1)

where \( \mathcal{T}_{11} \in \mathbb{R}^{q_1 \times q_1}, \mathcal{T}_{22} \in \mathbb{R}^{q_2 \times q_2}, \ldots, \mathcal{T}_{mm} \in \mathbb{R}^{q_m \times q_m} \) are irreducible square matrices, \( \sum_{k=1}^{m} q_k = N \), which are uniquely determined within a simultaneous permutation of their lines, but their ordering is not necessarily unique.

Remark 1

The process of permuting the Laplacian matrix \( L \) of \( G(\mathcal{A}) \) into its Frobenius normal form is a standard problem in graph algorithms, which can be solved by the depth-first search algorithm in linear time [32].
Suppose that $\mathcal{G}(\mathcal{A})$ is a directed network and its Laplacian matrix $L$ is in the Frobenius normal form and $G_1, G_2, \cdots, G_m$ are the strongly connected components of $\mathcal{G}(\mathcal{A})$ with the adjacency matrices $A_1 = \text{diag}(L_{11}) - L_{11}, A_2 = \text{diag}(L_{22}) - L_{22}, \cdots, A_m = \text{diag}(L_{mm}) - L_{mm}$. Then, $\mathcal{G}^*(\mathcal{A}^*)$ is called a condensation network of $\mathcal{G}(\mathcal{A})$ if there is a connection from a node in $\mathcal{V}(\mathcal{G}_i)$ to a node in $\mathcal{V}(\mathcal{G}_j)$, $i \neq j$, then the weights $A_i^* > 0$; otherwise, $A_i^* = 0$ for $i, j = 1, 2, \cdots, m$; $\mathcal{A}_i^* = 0$ for $i = 1, 2, \cdots, m$. Furthermore, for each $i = 2, 3, \cdots, m$, there is an integer $j < i$ such that $\mathcal{A}_i^* > 0$ if and only if the directed network $\mathcal{G}(\mathcal{A})$ contains a directed spanning tree [31].

Before we proceed, Finsler’s lemma is introduced, which will be used in deriving the theoretical results later.

**Lemma 3** ([33, 34])
Suppose that $x \in \mathbb{R}^n$, $P = P^T \in \mathbb{R}^{n \times n}$, and $H \in \mathbb{R}^{m \times n}$ such that $\text{Rank}(H) = l < n$. Then, the following statements are equivalent:

1. $x^TPx < 0, \forall x \in \{x : Hx = 0, x \neq 0\}$,
2. $P - \sigma H^TH < 0$, for some scalar $\sigma \in \mathbb{R}$,
3. $\exists X \in \mathbb{R}^{n \times m}$ such that $P + XH + H^TX^T < 0$,
4. $H^TH < 0$, where $H^\perp$ is the kernel of $H$, that is, $HH^\perp = 0$.

**Remark 2**
It has been shown that various problems in control theory can be solved by combining Lyapunov control approach with Finsler’s lemma. In most applications, Finsler’s lemma is referred to as Elimination Lemma in which the redundant variables in matrix inequalities can be eliminated [33, 34].

### 2.2. Model description

The commonly studied first-order continuous-time multi-agent system model is described by [9, 10]

$$
\dot{x}_i(t) = -\alpha \sum_{j=1}^{N} l_{ij} x_j(t), \quad i = 1, 2, \cdots, N,
$$

(2)

where $x_i \in \mathbb{R}^n$ is the position state of the $i$th agent, $\alpha$ represents the coupling strength, and $L = [l_{ij}]_{N \times N}$ is the Laplacian matrix of the communication topology $\mathcal{G}(\mathcal{A})$. When the multiple agents reach consensus, all the position states of agents converge to $\sum_{j=1}^{N} \xi_j x_j(0)$ that depends only on the initial positions of the agents, where $\xi = (\xi_1, \cdots, \xi_N)^T$ is the nonnegative left eigenvector of $L$ associated with the eigenvalue 0, satisfying $\xi^T1_N = 1$. However, in realistic applications of multi-agent formations, the velocity of each agent is generally not a constant but a time-varying variable. To cope with this, consider the following multi-agent dynamical system:

$$
\dot{x}_i(t) = f(x_i(t), t) + u_i(t), \quad i = 1, 2, \cdots, N,
$$

(3)

where $f(x_i(t), t) \in \mathbb{R}^n$ describes the intrinsic nonlinear dynamics of the $i$th agent, and $u_i(t) \in \mathbb{R}^n$ is a control input to be designed. It is assumed that the relative states between each pair of neighboring agents are measured at discrete sampling times $t_k, k \in \mathbb{N}$, and the control inputs are generated on the basis of a zero-order hold. It is also assumed that there exists a positive constant $h$ such that $t_{k+1} - t_k \leq h, k \in \mathbb{N}$. The objective here is to design a distributed control protocol $u_i(t)$ on the basis of the relative sampling-data information such that $\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0$, for all $i, j = 1, 2, \cdots, N$. An admissible protocol can be induced from (2) by employing sampling technique and a zero-order hold circuit, that is,

$$
u_i(t) = -\alpha \sum_{j=1}^{N} l_{ij} x_j(t_k), \quad t_k \leq t < t_{k+1}, \quad k \in \mathbb{N}, \quad i = 1, 2, \cdots, N.
$$

(4)
Substituting (4) into (3) gives
\[ \dot{x}_i(t) = f(x_i(t), t) - \alpha \sum_{j=1}^{N} l_{ij} x_j(t_{k}), \quad t_k \leq t < t_{k+1}, \quad k \in \mathbb{N}, \quad i = 1, 2, \cdots, N. \quad (5) \]

Clearly, if consensus can be achieved, it is natural to require a consensus state \( s(t) \in \mathbb{R}^n \) of the system (5) to be a possible trajectory of an isolated node satisfying
\[ \dot{s}(t) = f(s(t), t) \quad (6) \]

Here, \( s(t) \) may be an isolated equilibrium point, a periodic orbit, or even a chaotic orbit in some applications.

**Remark 3**
Note that synchronous consensus in multi-agent system (3) with sampled-data information is considered in this paper. The asynchronous consensus in multi-agent systems with intrinsic nonlinear dynamics and sampled-data information is still a challenge, although asynchronous consensus in linear multi-agent systems with time-invariant velocity has been addressed in [35–37].

### 3. MAIN RESULTS

In this section, the consensus problem for system (5) with a directed topology and sampled-data information is investigated.

**Assumption 1**
There exists a nonnegative constant \( \rho \) such that
\[ \| f(x_1, t) - f(x_2, t) \| \leq \rho \| x_1 - x_2 \|, \quad \forall \ x_1, x_2 \in \mathbb{R}^n; \quad \forall \ t \geq 0. \quad (7) \]

Note that Assumption 1 is a Lipschitz condition, satisfied by many well-known systems.

Let \( d_k(t) = t - t_k \), for \( t \in [t_k, t_{k+1}) \), \( k \in \mathbb{N} \). Then, one has \( t_k = t - d_k(t) \) with \( 0 \leq d_k(t) < h \), so system (5) can be written as
\[ \dot{x}_i(t) = f(x_i(t), t) - \alpha \sum_{j=1}^{N} l_{ij} x_j(t - d_k(t)), \quad t_k \leq t < t_{k+1}, \quad i = 1, 2, \cdots, N, \quad k \in \mathbb{N}. \quad (8) \]

**Remark 4**
By using a delayed-input approach, the sampled-data feedback system (5) is equivalently transformed into a continuous-time system with a time-varying delay in the feedback as shown in (8). In the following, theoretical analysis is performed on the basis of the time-varying delay system (8).

#### 3.1. Consensus in strongly connected networks with sampled-data information

In this subsection, consensus in strongly connected networks of multiple agents with sampled-data information is studied.

Let \( e_i(t) = x_i(t) - x_0(t) \) represent the position vector of the \( i \)th agent relative to the weighted average position of all the agents in system (5), where \( x_0(t) = \sum_{j=1}^{N} \xi_i x_j(t) \), and \( \xi = (\xi_1, \xi_2, \cdots, \xi_N)^T \) is the positive left eigenvector of Laplacian matrix \( L \) associated with its zero eigenvalue, satisfying \( \xi^T 1_N = 1 \). Then, for \( t \in [t_k, t_{k+1}) \) and arbitrarily given \( k \in \mathbb{N} \), one has the following error dynamical system:
\[ \dot{e}_i(t) = f(x_i(t), t) - f(x_0(t), t) - \sum_{j=1}^{N} \xi_j [f(x_j(t), t) - f(x_0(t), t)] - \alpha \sum_{j=1}^{N} l_{ij} e_j(t - d_k(t)), \]
\[ i = 1, 2, \cdots, N. \quad (9) \]

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Let \( e(t) = [e_1^T(t), \ldots, e_N^T(t)]^T \) and \( e(t - d_k(t)) = [e_1^T(t - d_k(t)), \ldots, e_N^T(t - d_k(t))]^T \). Then, system (9) can be written as
\[
\dot{e}(t) = \left( (I - 1_N \bar{\xi}^T) \otimes I_n \right) F(x(t), t) - \alpha (L \otimes I_n) e(t - d_k(t)), \quad t \in [t_k, t_{k+1}),
\]
where \( F(x(t), t) = f(x(t), t) - 1_N \otimes f(x_0(t), t) \) and \( f(x(t), t) = \left[ f^T(x_1(t), t), \ldots, f^T(x_N(t), t) \right]^T \).

**Theorem 1**

Suppose that the network is strongly connected and Assumption 1 holds. Then, consensus in system (5) is achieved if there exist symmetric matrices \( P, Q \in \mathbb{R}^{N \times N} \) such that \( E^T P E > 0 \), \( E^T Q E > 0 \), and the following LMI holds:
\[
\begin{bmatrix}
    h^2 E^T Q E - E^T E \\
    -E^T P E \\
    -\alpha h^2 L^T Q E \\
    0
\end{bmatrix} < 0,
\]
\[
E = \begin{bmatrix}
    I_{N-1} \\
    -\frac{\bar{\xi}^T}{\bar{\xi}^T}
\end{bmatrix} \in \mathbb{R}^{N \times (N-1)}, \quad \bar{\xi} = [\xi_1, \ldots, \xi_{N-1}]^T \in \mathbb{R}^{N-1},
\]
and \( \xi = [\xi_1, \xi_2, \ldots, \xi_N]^T \) is the positive left eigenvector of Laplacian matrix \( L \) associated with its zero eigenvalue, satisfying \( \xi^T 1_N = 1 \).

**Proof**

Construct the following Lyapunov–Krasovskii functional:
\[
V(t) = e^T(t) (P \otimes I_n) e(t) + 2h \int_{t-h}^t \int_{t-h}^t e^T(s) (Q \otimes I_n) e(s) ds d\theta,
\]
where symmetric matrices \( P \) and \( Q \in \mathbb{R}^{N \times N} \) satisfy \( E^T P E > 0 \) and \( E^T Q E > 0 \), with
\[
E = \begin{bmatrix}
    I_{N-1} \\
    -\frac{\bar{\xi}^T}{\bar{\xi}^T}
\end{bmatrix} \in \mathbb{R}^{N \times (N-1)},
\]
where \( \bar{\xi} = [\xi_1, \ldots, \xi_{N-1}]^T \in \mathbb{R}^{N-1}, \xi = [\xi_1, \xi_2, \ldots, \xi_N]^T \in \mathbb{R}^N \) is the positive left eigenvector of Laplacian matrix \( L \) associated with its zero eigenvalue, satisfying \( \xi^T 1_N = 1 \). Since \( (\xi^T \otimes I_n) e(t) = 0 \), \( V(t) \) defined by (13) is a valid Lyapunov–Krasovskii functional for system (10).

For \( t \in [t_k, t_{k+1}) \) and arbitrarily given \( k \), taking the time derivative of \( V(t) \) along the trajectories of (10) gives
\[
\dot{V}(t) = 2e^T(t) (P \otimes I_n) \dot{e}(t) + 2h^2 \dot{e}^T(t) (Q \otimes I_n) \dot{e}(t) - 2h \int_{t-h}^t \dot{e}^T(s) (Q \otimes I_n) \dot{e}(s) ds
\]
\[
= 2e^T(t) (P \otimes I_n) \left\{ \left( (I - 1_N \bar{\xi}^T) \otimes I_n \right) F(x(t), t) - \alpha (L \otimes I_n) e(t - d_k(t)) \right\}
\]
\[
+ 2h^2 \dot{e}^T(t) (Q \otimes I_n) \dot{e}(t) - h \int_{t-h}^t \dot{e}^T(s) (Q \otimes I_n) \dot{e}(s) ds - h \int_{t-h}^t \dot{e}^T(s) (Q \otimes I_n) \dot{e}(s) ds
\]
\[
\leq 2e^T(t) (P \otimes I_n) \left\{ \left( (I - 1_N \bar{\xi}^T) \otimes I_n \right) F(x(t), t) - \alpha (L \otimes I_n) e(t - d_k(t)) \right\}
\]
\[
+ 2h^2 \dot{e}^T(t) (Q \otimes I_n) \dot{e}(t) - h \int_{t-h}^t \dot{e}^T(s) (Q \otimes I_n) \dot{e}(s) ds - (h - d_k(t))
\]
\[
\times \int_{t-h}^{t-d_k(t)} \dot{e}^T(s) (Q \otimes I_n) \dot{e}(s) ds - d_k(t) \int_{t-d_k(t)}^t \dot{e}^T(s) (Q \otimes I_n) \dot{e}(s) ds.
\]
It follows from Jensen’s inequality [38] that
\[
-\int_{t-h}^{t} \dot{e}^T(s) (Q \otimes I_n) \dot{e}(s) ds \leq -[e(t) - e(t - h)]^T (Q \otimes I_n) [e(t) - e(t - h)],
\]
(15)
\[
-(h - d_k(t)) \int_{t-h}^{t-d_k(t)} \dot{e}^T(s) (Q \otimes I_n) \dot{e}(s) ds \leq -[e(t - d_k(t)) - e(t - h)]^T
\times (Q \otimes I_n) [e(t - d_k(t)) - e(t - h)],
\]
(16)
\[
and -d_k(t) \int_{t-d_k(t)}^{t} \dot{e}^T(s) (Q \otimes I_n) \dot{e}(s) ds \leq -[e(t) - e(t - d_k(t))]^T
\times (Q \otimes I_n) [e(t) - e(t - d_k(t))].
\]
(17)

Let \(e(t) - e(t - h) = \mu(t), e(t - d_k(t)) - e(t - h) = v(t)\) and \(e(t) - e(t - d_k(t)) = \omega(t)\). Then, according to (15)–(17) and Assumption 1, one gets
\[
\dot{V}(t) \leq 2e^T(t) (P \otimes I_n) \left\{ \left[ (1 - I_N \xi^T) \otimes I_n \right] F(x(t), t) - \alpha (L \otimes I_n) e(t - d_k(t)) \right\}
\]
\[+ h^2 \dot{e}^T(t) (Q \otimes I_n) \dot{e}(t) - \mu^T(t) (Q \otimes I_n) \mu(t)
\]
\[\leq -v^T(t) (Q \otimes I_n) v(t) - \omega^T(t) (Q \otimes I_n) \omega(t)
\]
\[
\leq 2e^T(t) (P \otimes I_n) \left\{ \left[ (1 - I_N \xi^T) \otimes I_n \right] F(x(t), t) - \alpha (L \otimes I_n) e(t - d_k(t)) \right\}
\]
\[- F^T(x(t), t) F(x(t), t) + \rho e^T(t) e(t) + h^2 \dot{e}^T(t) (Q \otimes I_n) \dot{e}(t) - \mu^T(t) (Q \otimes I_n) \mu(t)
\]
\[-v^T(t) (Q \otimes I_n) v(t) - \omega^T(t) (Q \otimes I_n) \omega(t).
\]

Let \(y(t) = [\dot{e}^T(t), e^T(t), e^T(t - d_k(t)), F^T(x(t), t), \mu^T(t), v^T(t), \omega^T(t)]^T\). Then,
\[
\dot{V}(t) \leq y^T(t) \left( \Omega \otimes I_n \right) y(t),
\]
(18)

where
\[
\Omega = \begin{bmatrix}
    h^2Q & O & O & O & O & O & O \\
    O & \rho I_N & -\alpha PL & P (1 - I_N \xi^T) & O & O & O \\
    O & -\alpha L^T P^T & O & O & O & O & O \\
    O & [P (1 - I_N \xi^T)]^T & O & -I & O & O & O \\
    O & O & O & O & -Q & O & O \\
    O & O & O & O & O & -Q & O \\
    O & O & O & O & O & O & -Q
\end{bmatrix}.
\]

Furthermore, it follows from (5) and the fact \([ (1_T \otimes \xi^T) \otimes I_n ] y(t) = 0\) that
\[
Ay(t) = 0_{7Nn},
\]
(19)

where \(A = \begin{bmatrix} S & T \end{bmatrix} \otimes I_n\), \(S = \begin{bmatrix} I_N & O_N & \alpha L & -(I - I_N \xi^T) & O_N & O_N & O_N \end{bmatrix} \in \mathbb{R}^{2N \times 7N}\), and \(T = \begin{bmatrix} I_T & \xi^T \end{bmatrix} \in \mathbb{R}^{T \times 7N}\). System (5) is asymptotically stable if for all \(y(t)\) satisfying \(Ay(t) = 0\), so one has
\[
y^T(t) \left( \Omega \otimes I_n \right) y(t) < 0.
\]
(20)

According to Lemma 3, \(y^T(t) \left( \Omega \otimes I_n \right) y(t) < 0\) is equivalent to
\[
A^T \left( \Omega \otimes I_n \right) A < 0,
\]
(21)
where

\[ A^\perp = \begin{bmatrix}
    E & O & O & -\alpha LE & O \\
    O & E & O & O & O \\
    O & O & E & O & O \\
    E & O & O & O & O \\
    O & E & E & O & O \\
    O & E & E & E & O \\
    O & O & O & O & E
\end{bmatrix} \otimes I_n. \tag{22} \]

Thus, inequality (21) can be rewritten as

\[
\begin{bmatrix}
    \Phi_{11} & * & * & * & * \\
    E^T PE & \Phi_{22} & * & * & * \\
    O & -E^T QE & \Phi_{33} & * & * \\
    -\alpha h^2 E^T L^T QE & -\alpha E^T L^T PE & -E^T QE & \Phi_{44} & * \\
    O & O & O & O & -E^T QE
\end{bmatrix} < 0, \tag{23}
\]

where

\[
\begin{align*}
\Phi_{11} &= h^2 E^T QE - E^T E, \\
\Phi_{22} &= \rho E^T E - E^T QE, \\
\Phi_{33} &= -2E^T QE, \\
\Phi_{44} &= \alpha^2 h^2 E^T L^T QLE - E^T QE,
\end{align*}
\]

which altogether, in turn, are equivalent to (11) because \( E^T QE > 0 \). Therefore, \( \dot{V}(t) < -\varepsilon \| e(t) \|^2 \) for some sufficiently small \( \varepsilon > 0 \), which ensures the achievement of consensus in the multi-agent system (5), see for example [39]. This completes the proof. \( \square \)

**Remark 5**

Note that the symmetric matrices \( P \) and \( Q \) in (13) may not be positive definite. Each of them could have a simple nonpositive eigenvalue by \( E^T PE > 0 \), \( E^T QE > 0 \) according to the Sylvester’s law of inertia [30]. The inequality constraints \( E^T PE > 0 \) and \( E^T QE > 0 \) make the quadratic functional \( V(t) \) defined by (13) a valid Lyapunov–Krasovskii functional for system (10) and also make it possible to derive consensus conditions in terms of strict linear matrix inequalities. Compared with constructing Lyapunov–Krasovskii functional \( V(t) \) with positive-definite matrices \( P \) and \( Q \), the construction of \( V(t) \) adopted in this paper has an advantage of reducing the conservativeness of the consensus conditions obtained by solving linear matrix inequalities.

**Remark 6**

The maximal allowable \( h_{\text{max}} \) guaranteeing consensus in Theorem 1 can be obtained following the two-step procedure:

1. Set \( h_{\text{max}} = h_0 \) and step size \( \tau = \tau_0 \), where \( h_0 \) and \( \tau_0 \) are specified positive constants.
2. Search symmetric matrices \( P \) and \( Q \) such that \( E^T PE > 0 \), \( E^T QE > 0 \), and (11) holds. If the conditions are satisfied, set \( h_{\text{max}} = h_{\text{max}} + \tau_0 \) and return to step 2. Otherwise, stop \( h \) is the maximal allowable sampling interval.

3.2. Consensus in networks containing a directed spanning tree with sampled-data information

In this subsection, consensus in multi-agent systems (5) whose topology contains a directed spanning tree is studied.

According to Lemma 2, without loss of generality, it is assumed that the Laplacian matrix \( L \) is in its Frobenius normal form. Furthermore, let \( \overline{L}_{ii} = \overline{L}_i + A_i \), where \( \overline{L}_i \) is a zero-row-sum matrix, and \( A_i \geq 0 \) is a diagonal matrix, \( i = 1, \ldots, m \). By Lemma 1, there exists a positive vector \( \overline{\xi}_1 = (\overline{\xi}_{1i}, \ldots, \overline{\xi}_{1q_1})^T \) such that \( \overline{\xi}_1^T \overline{L}_1 = 0 \) and \( \xi_1^T \xi_{q_1} = 1 \).
Theorem 2
Suppose that the communication topology $G(\mathcal{A})$ contains a directed spanning tree and Assumption 1 holds. Then, consensus in system (5) is achieved if there exist symmetric matrices $P_1, Q_i \in \mathbb{R}^{q_1}$, and positive-definite matrices $P_i, Q_i \in \mathbb{R}^{q_1}, i = 2, \ldots, m$, such that $E_1^T P_1 E_1 > 0, E_1^T Q_i E_1 > 0$, and the following LMIs hold:

\[
\begin{bmatrix}
    h^2E_1^T Q_i E_1 - E_1^T E_1 & * & * & * \\
    E_1^T P_i E_1 & \rho E_1^T Q_i E_1 - E_1^T Q_i E_1 & * & * \\
    0 & -E_1^T Q_1 E_1 & -2E_1^T Q_1 E_1 & * \\
    -\alpha h^2E_1^T L_{11}^T Q_i E_1 - \alpha E_1^T L_{11}^T P_i E_1 & -E_1^T Q_1 E_1 & \alpha^2 h^2 E_1^T L_{11}^T Q_1 L_{11} E_1 - E_1^T Q_1 E_1 & *
\end{bmatrix} < 0,
\]

\[
\begin{bmatrix}
    h^2Q_i - I & * & * & * \\
    P_i & \rho I - Q_i & * & * \\
    0 & -Q_i & -2Q_i & * \\
    -\alpha h^2 L_{ii}^T Q_i - \alpha L_{ii}^T P_i & -Q_i & \alpha^2 h^2 L_{ii}^T Q_i L_{ii} - Q_i & *
\end{bmatrix} < 0,
\]

where

\[
E_1 = \begin{bmatrix}
I_{q_1-1} \\
0
\end{bmatrix} \in \mathbb{R}^{q_1 \times (q_1-1)}, \quad \eta = \begin{bmatrix}
\bar{\xi}_1, \ldots, \bar{\xi}_{q_1-1}
\end{bmatrix}^T \in \mathbb{R}^{q_1-1},
\]

and $\bar{\xi}_1 = \begin{bmatrix}
\bar{\xi}_1, \ldots, \bar{\xi}_{q_1}
\end{bmatrix}^T$ is the positive left eigenvector of Laplacian matrix $L_{11}$ associated with its zero eigenvalue, satisfying $\bar{\xi}_1^T 1_{q_1} = 1$.

Proof
It is obvious that the condensation network of $G(\mathcal{A})$, indicated by $G^+(\mathcal{A}^*)$, is itself a directed spanning tree. The dynamics of the agents belonging to the vertex set of the root of $G^+(\mathcal{A}^*)$ will not be affected by the others, and the local topology among them is strongly connected. According to (24) and by Theorem 1, the states of these agents will reach consensus with an asymptotic decay rate, that is, there exists $\epsilon_1 > 0$, such that $x_{ij}(t) = \bar{x}(t) + O(t^{-\epsilon_1}), i = 1, 2, \ldots, q_1$, where $\bar{x}(t)$ represents the asymptotic consensus state satisfying $\dot{\bar{x}}(t) = f(\bar{x}(t), t) + O(t^{-\epsilon_1})$.

Based on the above analysis, consider the dynamics of the agents denoted by $u_{i1}, u_{i2}, \ldots, u_{iq_i}$, $2 \leq i \leq m$ belonging to the $i$th vertex in $G^+(\mathcal{A}^*)$. It is only affected by these vertices, such that there exist directed paths from them to $u_{ik}, s = 1, 2, \ldots, q_i$. Suppose that such agents excluding $u_{ik}, s = 1, 2, \ldots, q_i$, are $u_{i1}, u_{i2}, \ldots, u_{iq_i}$. Furthermore, assume that the states of agents $u_{i1}, u_{i2}, \ldots, u_{iq_i}$ have already achieved consensus, and the consensus position state is $\bar{x}(t) = \bar{x}_i(t)$. For $t \in [t_k, t_{k+1})$ and arbitrarily given $k$, simple calculations give the following error dynamical system:

\[
\dot{x}_{ir}(t) = f(x_{ir}(t), t) - f(\bar{x}_i(t), t) - \alpha \sum_{j=1}^{q_i} l_{irij}(x_{ij}(t - d_k(t)) - x_{ir}(t - d_k(t)))
\]

\[
- \alpha \sum_{j=1}^{q_i} l_{irij}(\bar{x}_i(t - d_k(t)) - x_{ir}(t - d_k(t))) + O(e^{-\epsilon_1}), \quad r = 1, 2, \ldots, q_i.
\]
for some $\epsilon > 0$. Let $\hat{x}_{i_r}(t) = x_{i_r}(t) - \bar{x}(t)$, $r = 1, 2, \cdots, q_i$. It then follows from (26) that

$$
\dot{\hat{x}}_{i_r}(t) = f(x_{i_r}(t), t) - f(\bar{x}(t), t) - \alpha \sum_{j=1}^{q_i} l_{i_r j} (\hat{x}_{i_j}(t) - d_k(t)) \\
- \hat{x}_{i_r}(t) - d_k(t)) - \alpha \sum_{p=1}^{q_i} l_{i_r p} \hat{x}_{i_r}(t) - d_k(t)) + O(e^{-\epsilon t}),
$$

(27)

t \in [t_k, t_{k+1}], \ r = 1, 2, \cdots, q_i.

Let $\hat{x}(t) = [\hat{x}_{i_1}(t), \cdots, \hat{x}_{i_q}(t)]^T$, $\check{x}(t) = [\hat{x}_{i_1}(t), \cdots, \hat{x}_{i_q}(t)]^T$. Then, system (27) can be written as

$$
\dot{\hat{x}}(t) = F(x(t), t) - \alpha (\bar{L} \otimes I_n) e(t - d_k(t)) + O(e^{-\epsilon t}), \ t \in [t_k, t_{k+1}],
$$

(28)

where $F(x(t), t) = f(x(t), t) - \bar{1}_{q_i} \otimes f(\bar{x}(t), t)$ and $f(x(t), t) = [f^T(x_{i_1}(t), t), \cdots, f^T(x_{i_q}(t), t)]^T$. According to (32) and by following the proof of Theorem 1, one can show that the states of agents $v_{i_1}, v_{i_2}, \cdots, v_{i_q}$, $2 \leq i \leq m$, will reach consensus asymptotically, converge to $\bar{x}(t)$. This completes the proof.

Remark 7
In the proof of Theorem 2, the multiple agents in the first strongly connected component of the network in the Frobenius normal form of the Laplacian matrix can be regarded as a leader while the rest are followers. If the consensus can be achieved in system (5), the position states of the followers approach that of the leader asymptotically.

3.3. Consensus in networks without any directed spanning tree but with sampled-data information

In this subsection, consider how to guarantee the position states of multi-agent system (5) to achieve consensus where the communication topology does not contain any directed spanning tree. One way to achieve this goal is to assume that there is a virtual leader and a fraction of agents that are informed agents that have the information of the leader so that a navigational feedback term could be added to each of them [40]. Similar to the analysis in [40], it is assumed that there is a dynamical virtual leader $s(t)$, satisfying

$$
\dot{s}(t) = f(s(t), t), \quad s(t) \in \mathbb{R}^n,
$$

(29)

in system (5).

For $t \in [t_k, t_{k+1})$ and arbitrarily given $k$, one has the following dynamical system:

$$
\dot{x}_i(t) = f(x_i(t), t) - \alpha \sum_{j=1}^{N} l_{ij} x_j(t - d_k(t)) - c_i \beta (x_i(t) - d_k(t)) - s(t - d_k(t))),
$$

(30)

i = 1, \cdots, N,

where $c_i = 1$ if agent $i$ is informed and 0 otherwise. Let $\check{x}_i(t) = x_i(t) - s(t)$, $i \in \{1, \cdots, N\}$. Then, by (30), one has

$$
\dot{\check{x}}(t) = F(x(t), t) - (\bar{L} \otimes I_n) \check{x}(t - d_k(t)), \ t \in [t_k, t_{k+1}),
$$

(31)

where $\check{x}(t) = (\check{x}_1^T(t), \cdots, \check{x}_N^T(t))^T$, $F(x(t), t) = f(x(t), t) - \bar{1}_N \otimes f(\bar{x}(t), t)$, $f(x(t), t) = [f^T(x_1(t), t), \cdots, f^T(x_1(t), t)]^T$, $\bar{L} = \alpha L + \beta \text{diag}(c_1, \cdots, c_N)$. 

Theorem 3
Suppose that there exists a directed path from the virtual leader to each agent \( i, i = 1, \ldots, N \), and Assumption 1 holds. Then, consensus in system (31) is achieved if there exist positive-definite matrices \( P \) and \( Q \in \mathbb{R}^N \), such that the following LMI holds:

\[
\begin{bmatrix}
    h^2Q - I & * & * & * \\
    P & \rho I - Q & * & * \\
    O & -Q & -2Q & * \\
    -\alpha h^2\tilde{L}^TQ & -\alpha\tilde{L}^TP & -Q & \alpha^2h^2\tilde{L}^TQ\tilde{L} - Q
\end{bmatrix} < 0. \tag{32}
\]

Proof
This theorem can be proved directly from that of Theorem 2 above, so the proof is omitted for brevity.

Remark 8
In Theorem 3, the condition that the virtual leader has a directed path from itself to each agent is very mild. Suppose that the communication topology \( \mathcal{G}(\mathcal{A}) \) contains \( m \) separated, strongly connected components. Then, the condition can be satisfied by informing \( m \) agents which are the roots of the corresponding spanning trees in the union of \( m \) strongly connected components.
4. NUMERICAL SIMULATIONS

In this section, some numerical simulations are provided to verify the effectiveness of the theoretical analysis and to discuss the dependence of the upper bound of maximal allowable sampling intervals on the coupling strength $\alpha$. In the simulations, the periodic sampling technique is adopted.

Example 1
Consider multi-agent system (5) with the topology $\mathcal{G}(A_1)$ as in Figure 1, where the weights are indicated on the edges. Figure 1 shows that the topology $\mathcal{G}(A_1)$ contains a directed spanning tree, where agents 1–3 and 4–7 belong to the first and the second strongly connected components, respectively. In simulations, the following two cases are considered:

(I) Let $f(x_i(t), t) = [0.15 \sin(x_{i1}(t)), 0.15 \cos(x_{i2}(t))]^T \in \mathbb{R}^2$, where $x_i(t) = (x_{i1}(t), x_{i2}(t))^T \in \mathbb{R}^2, i = 1, \ldots, N$.

(II) Let $f(x_i(t), t) = [\kappa(-x_{i1}(t) + x_{i2}(t) - \eta(x_{i1}(t))), x_{i1}(t) - x_{i2}(t) + x_{i3}(t), -\varrho x_{i2}(t)]^T \in \mathbb{R}^3$, where $\eta(x_{i1}(t)) = bx_{i1}(t) + 0.5(a - b)(|x_{i1}(t) + 1| - |x_{i1}(t) - 1|), i = 1, \ldots, N$. In this case, the isolated system is chaotic when $\kappa = 10, \varrho = 18, a = -4/3$, and $b = -3/4$, as shown in Figure 2.

![Figure 3](image1.png)  
Figure 3. Consensus is achieved with sampling interval $h = 0.2500$ for case (I).

![Figure 4](image2.png)  
Figure 4. Consensus cannot be achieved with sampling interval $h = 0.7500$ for case (I).
For case (I), let the coupling strength $\alpha = 0.30$. Some calculations give the maximal allowable sampling intervals $h_{\text{max}} = 0.2500$. Taking $h = 0.2450$ and $h = 0.7500$, the position states of all agents are obtained as shown in Figures 3 and 4, respectively, with the same initial conditions $x_1(0) = (1.25, 0.05)^T$, $x_2(0) = (-0.5, 0.175)^T$, $x_3(0) = (0, 0)^T$, $x_4(0) = (1.5, -0.75)^T$, $x_5(0) = (3.0, -0.65)^T$, $x_6(0) = (1.75, 0.45)^T$, and $x_7(0) = (0.55, 0.60)^T$. The results verify the theoretical analysis very well. For case (II), let the coupling strength $\alpha = 1.50$. Some calculations give the maximal allowable sampling intervals $h_{\text{max}} = 0.0500$. Taking $h = 0.0500$ and $h = 0.1550$, the position states of all agents are shown in Figures 5 and 6, respectively, with the same initial conditions $x_1(0) = (1.25, 0.05)^T$, $x_2(0) = (-0.5, 0.175)^T$, $x_3(0) = (0, 0)^T$, $x_4(0) = (1.5, -0.75)^T$, $x_5(0) = (3.0, -0.65)^T$, $x_6(0) = (1.75, 0.45)^T$, and $x_7(0) = (0.55, 0.60)^T$. The results verify the theoretical analysis very well. Furthermore, according to the multi-step procedure given in Remark 6, one can get a maximal allowable sampling interval $h_{\text{max}}(\alpha)$ for each $\alpha$ if the conditions of Theorem 2 are satisfied. In the following, the nontrivial relationship between the
Figure 7. Relation between the maximal allowable sampling interval $h_{\text{max}}$ and the coupling strength $\alpha$ for case (I).

Figure 8. Relation between the maximal allowable sampling interval $h_{\text{max}}$ and the coupling strength $\alpha$ for case (II).

The upper bound of the maximal allowable sampling intervals $h_{\text{max}}(\alpha)$ and the coupling strength $\alpha$ is numerically demonstrated (see Figures 7 and 8). Particularly, it is shown that there exists a positive constant $\alpha_0 > 0$, such that $h_{\text{max}}(\alpha)$ attains its maximum when $\alpha = \alpha_0$ and monotonically increases by enlarging $\alpha$ with $\alpha \leq \alpha_0$, whereas it monotonically decreases by enlarging $\alpha$ with $\alpha \geq \alpha_0$ for cases (I) and (II). This is contrary to the common view that the larger the coupling strength, the easier the consensus is achieved.

Example 2
Consider multi-agent system (5) with topology $G(\mathcal{A}_2)$ as in Figure 9, where the weights are indicated on the edges. Figure 9 shows that the topology $G(\mathcal{A}_2)$ does not contain any spanning tree. Suppose that there is a virtue leader labeled V in system (5), where agents 1 and 4 belonging to the first and the second strongly connected components are informed agents, as indicated in Figure 9. Let $x_v(t)$ represent the position states of the virtual leader, where $x_v(t) = (x_{v1}(t), x_{v2}(t))^T$ with $x_v(0) = (0.5, 0.5)^T$. In simulations, let $f(x_i(t), t) = [0.15 \sin(x_{i1}(t)), 0.10x_{i2}(t)]^T \in \mathbb{R}^2$, where $x_i(t) = (x_{i1}(t), x_{i2}(t))^T \in \mathbb{R}^2$, $i = 1, \ldots, N$. Let the coupling strength $\alpha = 0.45$. Copyright © 2012 John Wiley & Sons, Ltd.
Some calculations give that the maximal allowable sampling intervals $h_{\text{max}} = 0.1560$. Taking $h = 0.1500$ and $h = 0.4500$, the position state of all agents are shown in Figures 10 and 11, respectively, with initial conditions $x_1(0) = (1.25, 0.05)^T$, $x_2(0) = (-0.5, 0.175)^T$, $x_3(0) = (0, 0)^T$. 

Figure 12. Relation between the maximal allowable sampling interval $h_{\text{max}}$ and the coupling strength $\alpha$ in Example 2.

\[ x_4(0) = (1.5, -0.75)^T, \text{ and } x_5(0) = (3.0, -0.65)^T. \]

The results verify the theoretical analysis very well. The dependence of the upper bound of maximal allowable sampling intervals $h_{\text{max}}$ on the coupling strength $\alpha$ is summarized in Figure 12. It is numerically shown that there exists a positive constant $\alpha_1 > 0$, such that $h_{\text{max}}(\alpha)$ attains its maximum when $\alpha = \alpha_1$ and monotonically increases by enlarging $\alpha$ with $\alpha < \alpha_1$, whereas it monotonically decreases by enlarging $\alpha$ with $\alpha \geq \alpha_1$.

5. CONCLUDING REMARKS

In this paper, we have investigated the consensus problem of directed networks of multiple agents with intrinsic nonlinear dynamics and sampled-data information. By constructing a Lyapunov–Krasovskii functional and using Finsler’s lemma, we have theoretically proved that consensus with time-varying velocities in strongly connected networks can be achieved if the sampling interval is less than the maximal allowable sampling interval, which can be obtained by solving a feasible linear matrix inequality. The results are then extended to two scenarios where the topology contains and does not contain a directed spanning tree, respectively. Finally, two simulation examples have been reported to verify the effectiveness of the theoretical results. Particularly, it is numerically shown that the maximum allowable sampling interval may be reduced by enlarging the coupling strength, which is contrary to the common view that the larger the coupling strengths, the easier the consensus. Our future works will focus on the consensus behaviors of more practical models, such as second-order multi-agent systems with nonlinear dynamics and sampled-data information, among others.

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