The Transcendental Character of Determinism

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DETERMINISM

In philosophical discussions of determinism I believe it is fair to say that there is not ordinarily a sharp separation of a process being deterministic and a process being predictable. Much of the philosophical talk about determinism proceeds as if it is understood that a deterministic process is necessarily predictable. Here is a typical quotation, taken from A. J. Ayer's essay "Freedom and Necessity:"

"Nevertheless, it may be said, if the postulate of determinism is valid, then the future can be explained in terms of the past: and this means that if one knew enough about the past one would be able to predict the future." Without attempting quite general definitions for arbitrary systems, it will still be useful to examine in somewhat more detail how we ordinarily think about these two related but different concepts of determinism in prediction.

One formulation of determinism grows naturally out of the theory of differential equations. If a set of differential equations is given for a phenomenon, then we say that the phenomenon is deterministic if there is exactly one solution as a function of time of the differential equations satisfying the given initial and boundary conditions. There is no general conceptual reason, of course, for restricting ourselves to differential equations. We can easily say that a system of equations for discrete time intervals is deterministic in the same sense.

Sometimes in formulating what we mean by deterministic systems we put the emphasis rather differently. For example, in asserting the claim that classical mechanics is deterministic we may formulate the condition along the following lines. The history of an isolated system of particle mechanics is determined by the masses and forces acting on the particles, together with appropriate initial conditions. As is well known, these appropriate initial conditions give for some particular instant of time the position and velocity of each of the particles. When
we approach determinism from this more general system standpoint there are
difficulties, however, that are immediately encountered. For example, the proof
of the deterministic character of the system will rest on additional assumptions.
In the case of mechanics, for example, in ordinary formulations we will need
to assume no collisions occur, or we will on the other hand need to introduce a
number of special assumptions about the nature of the collisions. The kinds of
differential equations or systems just discussed might be said to be ones that
are completely deterministic in character.

I introduce the modifying adjective completely because systems that we
do not think of as deterministic still have many deterministic features, and
it has been an important part of their analysis to clarify the nature of these
features. A simple sort of example avoiding any complexity is provided by
elementary stochastic processes. We certainly think of such processes as not
being deterministic because the sample path, as we ordinarily talk about the
path of a particle in a stochastic process, is certainly not uniquely determined
by the initial and boundary conditions together with a set of equations. It is just
the feature of such processes that many different sample paths can result from
exactly the same initial circumstances. On the other hand, many features of the
process do have a deterministic character. The simplest example of all is coin
flipping with a fair coin, which has a probability half of heads on each flip, with
the flips being conducted in such a way that they are independent. If we flip a
coin a thousand times then we have with equal possibility $2^{1000}$ sample paths.
To give some idea of the magnitude of this number indicating the diversity of
possible sample paths in such a simple process, consider the following: If we
were to complete the equivalent of $2^{1000}$ flips, with each thousand flips taking
one hour, then to have a sample equal to the number of possibilities (not that, of
course, each possibility would show up in the sample) would itself take a time
about $10^{28}$ times longer than the present universe has been in existence, which
I am assuming roughly to be thirty billion years. So, at first thought, nothing
could be further removed from determinism than such a simple stochastic
process of coin flipping. On the other hand, a major thrust of theoretical analysis
of such processes has been to establish what results hold with certainty, i.e.,
with probability one, which in our present discussion is equivalent to being
deterministic. The various central limit theorems of probability theory are a
good example of the kinds of results that can be obtained.

I am not for a moment suggesting that the whole theory is deterministic.
What is important is that any standard stochastic theory, simple or complex,
has important deterministic elements and a major effort of theory is to find
them. Note that I have widened the use of deterministic to some extent in this
discussion, for I am treating a probabilistic law that we know with certainty,
or with probability one, as being in a general sense deterministic. What I want
to claim is that such laws are just as deterministic as are laws that arise from
theories that are fully or completely deterministic.

It is important also to separate the statistical data that we use to test
the correctness of such a deterministic distribution law and the deterministic
character of the law itself. In no sense do the finite sample data satisfy the law with probability one. The data are finite, and a statistical analysis is appropriate to determine the degree to which the law is satisfied. But there is nothing special about this because the underlying theory is stochastic. We test in the same general way completely deterministic theories, for the data here too are finite, subject to experimental errors of measurement and the inevitable finiteness of observation. I see, in principle, no strong difference between completely deterministic theories and stochastic theories that are only partially deterministic, from the standpoint of testing particular deterministic laws when they arise from either of the two kinds of general theories.

Of course, I am not attempting to amalgamate under a common heading completely deterministic and partially deterministic theories. The great thrust of classical physics for the development of completely deterministic theories has been and continues to be of importance. My point is rather to emphasize that we often have deterministic laws arising from partially deterministic theories.

**PREDICTION**

As already mentioned, in much general talk, including much by philosophers, there is a confounding of determinism and prediction. If a theory is completely deterministic, it is often talked about as if phenomena governed by the theory are completely predictable. It is of course important and fundamental to modern science that many aspects of phenomena are predictable. It is also fundamental that in restricted cases of deterministic theories we think of phenomena as completely predictable, except for observational errors of initial conditions or other parameters. But the existence of predictable phenomena, which constitute some of the most important results of science, by no means guarantees anything like universal predictability. In fact, the conceptual separation of determinism and predictability is one of the fundamental themes of this article.

A good example to illustrate this separation is the famous three-body problem of classical mechanics—or more generally the N-body problem, which is easy to describe. It is that of determining the trajectories of three (or more) bodies interacting only under the force of gravity, and with given initial conditions. It is of course assumed that the three-body system is isolated, that is, no forces external to the system are affecting the motions of the three bodies. Already in the nineteenth century the problem of making long-term predictions of the three bodies was investigated in a deep way by Poincaré. It had been shown by Bruns in 1877 that essentially quantitative methods other than series expansions could not settle the three-body problem. There is no closed analytical solution in general. Poincaré then showed that the series expansions developed earlier in the work of Laplace, Lagrange, and others diverged rather than converged, as required in order to have a proper long-term solution. Methods of numerical approximation must be used, and when the solution, which may in principle be proved to exist, is sufficiently pathological in character, no extrapolations based on various methods of series expansion
will give accurate predictions for extended periods of time. I shall return later to still other fundamental problems of prediction for simple three-body systems, problems which exist quite apart from any question of methods of numerical approximation in the construction of solutions.

It is of great importance to emphasize that the three-body problem is not at all special in the sense of the limited results one can obtain. For most mechanical problems that one would naturally write down, it is fair to say that we can neither solve the resulting system of differential equations nor give a sufficiently detailed description of the solutions to claim that we have complete predictability of behavior. It is a fact not sufficiently emphasized in foundational discussions of these matters that we actually can obtain explicit solutions or very accurate approximations for only a very small number of types of problems. To be more specific about these matters, it is important in mechanics to analyze problems in terms of a potential or a potential energy function. In a mechanical system with one degree of freedom it is always possible to introduce the potential energy of the solution of the differential equation. For a system with more than one degree of freedom, this is not always possible, and in general our analysis by current mathematical methods of potential systems with more than one degree of freedom is quite incomplete.

A skeptical reader may want to reply that of course I have not proved any of these results about the absence of predictability. I have been discussing general classes of models, for example, classical mechanical systems with a potential of two degrees or more of freedom. I shall shortly turn to more formal, results that can be stated in a clear mathematical form about predictability. But it is important to emphasize as well the broad scientific experience with the use of mathematical methods in the sciences. Only for special classes of problems are solutions possible. Predictability of general system behavior is limited. Numerical computations can be carried out for particular values of parameters for a certain distance, but such methods will not provide a satisfactory general approach to predictability and will not even be practical for many kinds of applications. What I consider important to stress here is the chasm that separates many deterministic theories from being theories that are satisfactorily predictive in character.

**Turing Machines**

I turn now to some more specific results, first about Turing machines and then about cellular automata. A Turing machine, like any computing machine, can only really compute when it has a physical embodiment. It is therefore natural to think of Turing machines as finite physical processes or, in other terminology, the embodiment of a Turing machine is a physical process of computation. Note that I have said "embodiment of a Turing machine," because there is a good argument for treating Turing machines themselves as abstract objects, but it is not pertinent here to belabor the distinction. The important point is rather that physical processes that embody Turing machines provide excellent examples of physical processes that are not predictable when initial conditions are given.
The important objective of our analysis is to show that such simple discrete elementary mechanical devices as Turing machines already have behavior in general that is unpredictable. What I have in mind, of course, is the well-known unsolvability of the halting problem for Turing machines. Given a Turing machine in an arbitrary configuration with a finite length string of nonblank tape symbols, will the Turing machine eventually halt? The well-known result is that the answer is not decidable. In terminology useful here, the following important theorem is provable.

**Theorem 1:** *(Halting problem):* There is no algorithm to determine if an arbitrary Turing machine in an arbitrary configuration will eventually halt.

Put in other words, given the finite nonblank string and the configuration, which correspond to initial conditions of a mechanical system, the theorem says that there is no predictive algorithm as a function of these initial conditions such that it can be predicted whether or not the Turing machine will eventually halt. Notice, of course, that a particular configuration and a particular string of nonblank tape symbols may possibly be analyzed and a prediction made. But as in the case of Newton’s celebrated solution of the two-body problem, what we are interested in is a general solution predicting the behavior of systems in closed form, that is, algorithmically, as a function of initial conditions. What is conceptually surprising is that such an elementary and simple physical machine as a Turing machine has such fundamentally unpredictable behavior. Note that there is no question of determinism here, or errors of measurement of continuous quantities, because the setup is essentially discrete. The behavior of the machine is deterministic.

It is worth mentioning in this connection that if we do not require that a Turing machine be deterministic but permit the machine to have several choices for the next move, then we have what is called a nondeterministic Turing machine. It is natural to ask if this weakening of the restrictions on the moves of the machine increases its power. The answer is negative, as formulated in the following theorem:

**Theorem 2:** Any nondeterministic Turing machine can be simulated by a deterministic Turing machine.

By the way, I am not suggesting that the notion of a nondeterministic automaton of a given class catches, in any sense, the philosophical sense of nondeterminism.

**Cellular Automata**

The formal definition of Turing machines is rather complicated. I want now to turn to some discrete systems that are closer to a wide range of physical systems and that are, above all, easy to characterize.

To illustrate ideas, here is a simple example of a one-dimensional cellular automaton. In the initial state, only the discrete position represented by the
coordinate 0 has the value 1. All other integer coordinates, positive or negative, have the value 0. The automaton can in successive “moves” or steps produce only the values 0 or 1 at any given location. The rule of change is given by a function that depends on the value of the given position and the values for the simple case to be considered here of the adjacent values on either side. Thus, using \( a_i' \) as the value at site \( i \), after supplying the function for change, we can represent the rule of change as follows:

\[
a_i' = f(a_{i-1}, a_i, a_{i+1}).
\]  

(1)

Note that because of the restriction in the values at any site this function takes just eight arguments, the eight possibilities for strings of three 0’s and 1’s. The automata being discussed here are at the very lowest rank on the number \( k \) of possible values: \( k = 2 \) is the number of possible values and \( r = 1 \) is the distance away from \( a_i \). The characterization here in terms of the infinite line can be replaced by a finite characterization, for example, in terms of a discrete set of points on a circle. The same kind of function as that of equation (1) applies.

Cellular automata have the following obvious characteristics: discrete in space; discrete in time; discrete state values; homogeneous—in the sense all cells are identical; synchronous updating; deterministic rule of change corresponding to a deterministic differential equation of motion in the case of classical mechanics; locality of change rule: the rule of change at a site depends only on a local neighborhood of the site; temporal locality: the rule of change depends only on values for a fixed number of preceding steps—in the present example just one step.

Notice that one characteristic of ordinary mechanical systems has not been mentioned, and that is reversibility. It is also easy to characterize reversible automata but I shall not enter into the details here. (For information on these and many other aspects of cellular automata, see Wolfram 1986a.)

The basic problem of prediction of interest here for cellular automata can be given a simple formulation. Knowing the transition rule for the cellular automata and given an initial configuration, can one predict in closed form the configuration after \( n \) steps? Once again I emphasize that determinism is not an issue. Clearly the process is deterministic. Predictability is another matter. By putting stress on closed-form expressions, I do not mean to imply that this is the only form of prediction. It is just a good criterion of our understanding of a phenomenon.

In the case of cellular automaton we can, of course, get predictability by directly simulating the automaton, but this is not what is ordinarily meant by predictability. There is, in fact, a concept that is useful to introduce at this point in these discussions. This is the concept of computationally irreducible (Wolfram 1985). A system is computationally irreducible if a prediction about the system cannot be made essentially by shorter methods than simulating the system or running the system itself. Wolfram (1986b) has shown that the
following $k = 2^r = 1$ cellular automaton generates highly complex sequences that pass many tests for randomness, in spite of its totally elementary character. The automaton is defined by the equivalent equations, one in terms of exclusive
or the other in terms of mod 2 arithmetic.

$$a'_i = a_{i-1} \text{XOR}(a_i OR a_{i+1}) \quad (2)$$
$$a'_i = (a_{i-1} + a_i + a_{i+1} + a_i a_{i+1}) \mod 2 \quad (3)$$

Already the 256 cellular automata with $k = 2$ and $r = 1$ fall into four natural classes: (1) the pattern becomes homogenous, (2) the pattern degenerates into a simple periodic structure, (3) the pattern is aperiodic, and (4) the structure is complicated and localized.

Another way of thinking about the complexity of cellular automata is that it seems intuitively clear that cellular automata can be constructed which simulate universal Turing machines, and thus can compute any partial recursive function. In fact, a quite reasonable one-dimensional cellular automaton with only fourteen states has been shown by Albert and Culik (1987) to be a universal computational device, that is, it can simulate a universal Turing machine. The interesting conjecture, due to Wolfram, is that we have naturally occurring physical processes that are universal computing devices, so that a way to think about the complexity of much natural phenomena is that it is identical with the complexity we associate with computation. Such ideas, which are still to some extent speculative, help to drive a further wedge between determinism and predictability.

*Chaos*

I now turn to another closely related topic which provides many illustrations of systems that are deterministic but not in any practical sense predictable in their behavior. Such systems possess the property of chaos, a concept now much studied for physical systems of a great variety.

To illustrate ideas, let us begin with a simple example that is not really physical in content, but shows how a really simple case can still go a long way toward illustrating the basic ideas. Let $f$ be the doubling function mapping the unit interval into itself.

$$x_{n+1} = f(x_n) = 2x_n \pmod{1} \quad (4)$$

where mod 1 means taking away the integer part so that $x_{n+1}$ lies in the unit interval. So if $x_1 = 2/3, x_2 = 1/3, x_3 = 2/3, x_4 = 1/3$ and so on periodically. The explicit solution of equation (4) is immediate:

$$x_{n+1} = 2^n x_1 \pmod{1} \quad (5)$$

With random sequences in mind, let us represent $x_1$ in binary decimal notation, i.e., as a sequence of 1's and 0's. Equation (4) now can be expressed
as the rule: for each iteration from \( n \) to \( n + 1 \) move the decimal point one position to the right, and drop whatever is to the left of the decimal point:

\[
.1011 \ldots \rightarrow .0111 \ldots
\]

We think of each \( x_n \) as a point in the discrete trajectory of this apparently simple system. The remarks just made show immediately that the distance between successive discrete points of the trajectory cannot be predicted in general without complete knowledge of \( x_1 \). If \( x_1 \) is a random number, i.e., a number between 0 and 1 whose binary decimal expansion is a random sequence, then nontrivial prediction will be out of the question. Moreover, any error in knowing \( x_1 \) spreads exponentially—the doubling system defined by equation (4) is highly unstable.

Surprisingly simple natural physical systems can exhibit chaos. Lighthill (1986) gives a very nice example of a pendulum which is forced to move back and forth in a sinusoidal oscillation with a period just slightly greater than the natural period of the pendulum. The pattern of motion followed by the pendulum is quite unpredictable by standard methods. It may be shown that the dependence on initial conditions is even deeper in chaotic systems than it is in the classical unstable systems that have been studied for a very long time. Perhaps one way of describing the characteristic feature of chaotic systems is the existence of what has been termed a predictability horizon. Such a horizon is the time after which solutions of very close initial conditions, close to the accuracy of specification being used, become quite remote from one another, but even more, the solutions vary in a discontinuous fashion in response to last decimal place rather than in any smooth continuous fashion. In short, chaotic systems exhibit the property that solutions that are close to each other for given initial conditions of a given number of decimal places diverge exponentially from one another.

One of the earliest and most influential chaotic models has been the Lorenz model, which was drawn from the theory of the atmosphere and was developed by Lorenz to persuade meteorologists that simple deterministic models of the atmosphere could lead to chaotic behavior, that is, to the weather being unpredictable even under the most accurate set of observations. Lorenz’s model was drawn more particularly from the theory of convection in a fluid heated from below (Lorenz 1963). What is most surprising about Lorenz’s system is that he abstracted and simplified so much that he ended up with a system of three ordinary differential equations with no more than a polynomial of second degree in the unknowns. A full-scale study of the Lorenz equations is to be found in Sparrow (1982).

Lorenz equations depend on just three real positive parameters. Variation in parameters, as is to be expected, changes the behavior of the solutions of the equations. In particular, for some parameter values, numerically computed solutions oscillate, apparently forever, in a way that appears random and which is now called chaotic. I emphasize that the oscillation is not random but seems
extremely complex and is what is sometimes termed pseudorandom. For other
parameter values there is what has been "preturbulence," which is a phe-
omenon in which trajectories behave chaotically for long periods of time but
finally settle down to stable behavior. There are still other values of parameters
in which intermittent chaos is observed. In these cases the trajectories alternate
between chaotic and apparently stable behavior. It is not appropriate here to
enter into technical details concerning the solutions of the Lorenz equations.
What is important is that the equations represent an extremely good example
of a simple physical system that is obviously completely deterministic in character
but is, for all practical purposes, unpredictable for large sets of values of the pa-
rameters. To the best of our knowledge, the system represents for such values of
the parameters a computationally irreducible system in the sense defined earlier.

It is important to emphasize that I have only touched the surface of the
now enormous literature on chaotic systems. What is central here is simply
to cite them as examples of deterministic systems that are unpredictable in
behavior.

RANDOMNESS AND DETERMINISM

It is characteristic of chaotic systems and also of the deterministic behavior
of cellular automata that actual randomness in the behavior has not been
proved, where I have in mind using a very strict definition of random behavior
in the sense, for example, of Kolmogorov for infinite sequences. Remember
that Kolmogorov's sense of randomness is that of sequences of maximum
complexity, where complexity of a sequence is defined in terms of the length
of the program required to describe the sequence.

It might be thought that strict randomness is inconsistent, in a formal
sense, with determinism. It is a major point of this article to emphasize that
this is not the case. Strict randomness and strict determinism are mutually
compatible, contrary to much philosophical and even physical thought of the
past. I shall use an example that I have used previously in discussions of
randomness of deterministic systems, namely, a certain special case of the three-
body problem, and I shall end by indicating what deeper questions it would be
time to have answers to concerning the relationship between randomness and
determinism (cf. Suppes 1987).

Our special case is this. There are two particles of equal mass \( m_1 \) and \( m_2 \)
moving according to Newton's inverse-square law of gravitation in an elliptic
orbit relative to their common center of mass, which is at rest. The third particle
has a nearly negligible mass, so it does not affect the motion of the other two
particles, but they affect its motion. This third particle is moving along a line
perpendicular to the plane of motion of the first two particles and intersecting
the plane at the center of their mass—let this be the \( z \) axis. From symmetry
considerations, we can see that the third particle will not move off the line.
The restricted problem is to describe the motion of the third particle.

With these restrictive assumptions it is easy to derive an ordinary differ-
tential equation governing the motion of the third particle. The analysis of this
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It is easily described situation is quite complicated and technical, but some of the results are simple to state in informal terms. Near the escape velocity for the third particle—the velocity at which it leaves and does not periodically return, the periodic motion is very irregular. In particular, the following remarkable theorem can be proved. Let \( t_1, t_2, \ldots \) be the times at which the particle intersects the plane of motion of the other two particles. Let \( s_k \) be the largest integer equal to or less than the difference between \( t_{k+1} \) and \( t_k \). Variation in the \( s_k \)'s obviously measures the irregularity in the periodic motion. The theorem, due to the Russian mathematicians Sitnikov (1960) and Alekseev (1968a, b, 1969a, b), as formulated in Moser (1973), is this:

**Theorem 3:** Given that the eccentricity of the elliptic orbits is positive but not too large, there exists an integer, say \( \alpha \), such that any infinite sequence of terms \( s_k \) with \( s_k \geq \alpha \), corresponds to a solution of the deterministic differential equation governing the motion of the third particle.

The correspondence between a solution of the differential equation and a sequence of integers is the source of the term symbolic dynamics. The idea of such a correspondence originated with G. D. Birkhoff in the 1930s. A corollary about random sequences immediately follows. Consider any random sequence of heads and tails—for this purpose we can use any of the several variant definitions—Church, Kolmogorov, Martin L"of, etc. We pick two integers greater than \( \alpha \) to represent the random sequence, the lesser of the two representing heads, say, and the other tails. We then have:

**Corollary 1:** Any random sequence of heads and tails corresponds to a solution of the deterministic differential equation governing the motion of the third particle.

In other words, for each random sequence there exists a set of initial conditions that determines the corresponding solution. Notice that in essential ways the motion of the particle is completely unpredictable even though deterministic. This is a consequence at once of the associated sequence being random. In this case the notion of unpredictable is a sharp one, for surely the strongest sense of unpredictability is that of randomness.

Of course, we also have the following corollary, which some may find disturbing.

**Corollary 2:** Let the books of the Library of Congress and the Bibliothèque Nationale be encoded according to the current sequential arrangement of the books, with the letter "a" being encoded by the first integer larger than \( \alpha \), "b" the next integer, etc. Then the sequence of integers whose initial segment encodes the contents of these libraries corresponds to a solution of the deterministic differential equation governing the motion of the third particle.

Upon reflection, this second corollary, given the first one, is not too surprising. Any information desired can be expressed by the continuum of initial conditions.
possible for this restricted three-body problem. It just took time and analysis for the idea to surface that indeed any sequence could be obtained in this fashion with only the mild restriction stated in the theorem.

It certainly seems right that there are a large number of other simple physical systems that are deterministic in character and that can generate random sequences, but I know of no other examples that have been worked out in sufficient detail to lead to Corollary 1 stated above. It is to be emphasized that the proof of the theorem stated above by Sitnikov, Alekseev, and Moser is quite long, technical, and difficult. Almost all the results in the theory of chaos have not yet received a similar intense scrutiny. I do emphasize that there is a definite step requiring specific mathematical argument to pass from a system's being chaotic to its exhibiting strict randomness, a step very likely not possible for many familiar chaotic systems.

A striking feature of randomness is complexity. So what are random sequences? Under one view, they are the limiting case of increasingly complex deterministic sequences. And the most complex deterministic systems are completely unpredictable in their behavior.

INDETERMINISM

The richness and complexity of deterministic systems suggest that any phenomena can be accounted for by a suitable choice of system type and suitable parameters. But there is a long scientific and philosophical tradition of urging a role for indeterminism. One of the problems has been deciding what indeterminism should mean, a problem made more difficult by the inclusion of chaos and randomness within the framework of determinism.

There is, I am sure, no definitive analysis of indeterminism, even for the limited purpose of this article. For a variety of reasons, the most prominent concept of indeterminism is that derived from quantum mechanics. But exactly what this view is is not transparent. There are at least three different ideas advanced in the literature.

(i) Quantum mechanics is indeterministic because it implies the existence of objective probabilistic phenomena in nature. But, as we have already seen, this argument by itself is not decisive because such phenomena are also implied by classical mechanics.

(ii) Quantum mechanics is indeterministic because it implies the Heisenberg uncertainty principle, which does not have an analogue in classical physics. But, it may be argued, this is misleading, because the classical theory of measurement, embodying a theory of accidental or random errors, also yields an uncertainty principle. The difference is that the theory of measurement is not part of classical physical theory but stands alongside it as a necessary supplement to provide a theoretical basis for the actual measurement of classical physical quantities.
To begin with, there is a re~eval~u~ta~tion~ point to be made and determinism and indeterminism. If we could make a case for the universe consisting entirely of stable deterministic or indeterministic systems, it would be easy to show that there could be no instantaneous action at a distance. The latest arguments have centered on the inequalities first formulated in 1964 by John Bell. The inequalities formulate a simple condition that must be satisfied by any local hidden variable theory, i.e., any satisfying locality, for a wide class of experiments involving simultaneous measurement of two spin-1/2 particles from a single source, or similar phenomena. Quantum mechanics is inconsistent with the Bell inequalities. This seems straightforward and a clear triumph for indeterminism, given the strong experimental support of quantum mechanics.

In fact, from the standpoint of the most natural intuition that indeterminism implies probabilistic phenomena, the Bell results are ironic rather than straightforward. For the nonexistence of a local hidden variable, i.e., local common cause, is exactly equivalent to the nonexistence of a joint probability distribution of the spin or other experimental observables. As Laplace would have appreciated, causes exist if and only if probabilities exist.

As I have argued elsewhere (Suppes 1990), quantum mechanics in general only generates average probabilities. Hidden-variable extensions, possibly nonlocal in nature, can be made that are either deterministic or indeterministic in character. Quantum mechanics by itself does not provide a knockdown argument for indeterminism, contrary to much popular thought.

On the other hand, the general case for indeterministic theories of much natural phenomena seems highly persuasive, in view of the apparent hopelessness of formulating detailed deterministic theories that have serious scientific support. Examples abound, ranging from the communication of neurons in animal nervous systems to the motion of clouds and the formation of weather disturbances. It is not mere fashion or fancy that accounts for the use of probabilistic models, without deterministic underpinnings, in essentially every area of science. Deterministic models or theories cannot be made to work. But, as I shall now argue, the philosophical consequences of this robust fact about current science is not support for the absolutely indeterministic structure of nature, but rather the transcendental character of the issue.²

TRANSCENDENTAL ISSUE

To begin with, there is a relevant fundamental point to be made about both determinism and indeterminism. If we could make a case for the universe consisting entirely of stable deterministic or indeterministic systems, it would be easy to show that there could be no place for any biological species in such a universe. If the universe consisted of something like the eternal heavens
of ancient Greek astronomy with only a touch here and there of radioactive
decay or some similar phenomena, it would indeed be a lifeless, mostly very
ordrly place. But in fact nothing could be farther from the truth. The extension
of the standard ideas of determinism or indeterminism, as embodied in current
scientific theories, to the entire universe in its full blooming, buzzing confusion
is a metaphysical fantasy as extreme as any that can be retrieved from the
archives of past philosophical thought.

For a great variety of empirical phenomena there is no clear scientific way
of deciding whether the appropriate "ultimate" theory should be deterministic
or indeterministic. Philosophy would like a general answer, but fortunately
science is opportunistic—going for limited but highly constructive results. The
metaphysics of either determinism or indeterminism is transcendental, in the
sense that any general thesis about the nature of the universe must transcend
available scientific facts and theories by a very wide mark. I now turn to a
strong argument for this thesis.

The modern research on dynamical systems, whose lineage extends back
to the deep analysis of Poincaré of motion in celestial mechanics in the
nineteenth century, has produced a variety of philosophically significant results,
but none more so than that expressed in the following theorem.

**Theorem 4:** (Ornstein): There are processes which can equally well be
analyzed as deterministic systems of classical mechanics or as indeterministic semi-Markov processes, no matter how
many observations are made.

The theorem is due to Donald Ornstein. It depends on earlier work of Kol-
mogorov, T. G. Sinai, and others; an excellent detailed overview is provided in
Ornstein and Weiss (1991). It is this theorem that justifies the title of this paper.
The existence of physically realistic models of natural phenomena for which
such a theorem holds is the basis for skepticism about the empirical nature
of any general claims for determinism. The simplest concrete models for which
the theorem holds are those with a single billiard ball moving on a table on
which is placed a convex-shaped obstacle. As the theorem indicates, the motion
of the ball as it hits the obstacle from various angles is not predictable in detail,
but only in a stochastic fashion. Moreover, there are many reasons for believing
what has been proved for certain physical processes is also true of a great many
more. A conjecture would be that the isomorphism result of Theorem 4 holds
for most physical processes above a certain complexity level.

Deterministic metaphysicians can comfortably hold to their view knowing
they cannot be empirically refuted, but so can indeterministic ones as well.
Both schools of thought can embrace the presence of randomness and the great
variety of quantum phenomena with equanimity. The theorem stated above of
course does not cover the case of quantum phenomena, but there is a great
deal of current evidence that hidden-variable theories of either a deterministic
or indeterministic kind can be consistently introduced, even if they lead to no
new experimentally verifiable predictions.
As every philosopher knows, Kant's three great problems of pure reason were the existence of God, freedom of the will, and the immortality of the soul. All problems whose "solutions" necessarily transcend experience. Kant's Third Antinomy bears more directly on the analysis given here than the complex of ideas associated with freedom of the will would suggest. The Thesis is:

Causality, according to the laws of nature, is not the only causality from which all the phenomena of the world can be deduced. In order to account for these phenomena it is necessary also to admit another causality, that of freedom. (A444)

and the Antithesis is:

There is no freedom, but everything in the world takes place entirely according to the laws of nature. (A445)

The "proof" of the thesis makes clear that Kant has focused the argument on spontaneity of causes, a concept that is exactly right for indeterministic stochastic processes.

We must therefore admit another causality, through which something takes place, without its cause being further determined according to necessary laws by a preceding cause, that is, an absolute spontaneity of causes, by which a series of phenomena, proceeding according to natural laws, begins by itself; we must consequently admit transcendental freedom, without which, even in the course of nature, the series of phenomena on the side of causes, can never be perfect. (A447)

Again, as every philosopher knows, Kant argued at great length to show why the thesis of spontaneous causes should be rejected by any empirical philosopher. Here is a characteristic passage:

If the empirical philosopher had no other purpose with his antithesis but to put down the rashness and presumption of reason in mistaking her true purpose, while boasting of insight and knowledge, where insight and knowledge come to an end, nay, while representing, what might have been allowed to pass on account of practical interests, as a real advancement of speculative enquiry, in order, when it is so disposed, either to tear the thread of physical enquiry, or to fasten it, under the pretence of enlarging our knowledge to those transcendental ideas, which really teach us only that we know nothing; if, I say, the empiricist were satisfied with this, then his principle would only serve to teach moderation in claims, modesty in assertions, and encourage the greatest possible enlargement of our understanding through the true teacher given to us, namely, experience. (A470)

At the same time Kant warns against the empirical philosopher becoming dogmatic and extending his ideas so as to inflict injury on the practical interests of reason (A471).
The fundamental reinterpretation of the Third Antinomy being proposed in this article is evident. Both Thesis and Antithesis can be supported empirically, not just the Antithesis. The choice must transcend experience. This idea of detailed theories of phenomena that are mathematically inconsistent but equally supported by any possible observational data is not one that is congenial to Kant's philosophy of science, but in other ways can, I think, be a central concept of a revised transcendental philosophy sympathetic to Kant's objective of exposing dogmatic metaphysics for what it is, even when traveling in modern guise. Modern arguments for the universal presence of determinism are of this kind. Arguments like those of Inwagen (1983) that the hypothesis of determinism implies absence of free will are of another sort, which do not belong to dogmatic metaphysics and with which I am quite sympathetic. On the other hand, I am quite unsympathetic with Inwagen's view (p. 209) that the existence of moral responsibility is the chief reason for believing in free will. Arguments against the empirical nature of any universal thesis of determinism are above all necessary to make room for an unlimited range of biological phenomena, much of it intentional, exhibited in animals long before there were even any humans to say "Gavagai," let alone reason about moral problems.

NOTES

Much of this article comes from the draft of the first two of four lectures given at the College de France in April 1988. A further revision was included in the First Ernest Nagel Lecture given at Columbia University in November 1988. Still further changes were made for the Fifth Evert Willem Beth Lecture given in Nijmegen in August 1989 and the Thirteenth Hausser Lecture at Montana State University in June 1991.

I have benefited from many critical comments made at these various lectures, but especially those of Jules Vuillemin, Isaac Levi, Sidney Morgenbesser, and Gordon G. Brittan, Jr., listed in the historical order of their remarks, and also Yair Guttmann who has made several useful criticisms of the final written draft.

1. In introducing various distinctions about determinism, I have, in the interest of avoiding many technical points, neither distinguished between theories and the structures (or models) satisfying them, nor, at the next level, distinguished between set-theoretical structures satisfying a theory and "real" physical phenomena from which the structures were abstracted. Important and useful distinctions are to be made about these matters, but can, I believe, be omitted without too much risk of confusion in an article like this meant to be informal in character. I have also used "system" and "process" in the sense of structure, without really saying so. The various theorems that are stated below are, as mathematical theorems, about set-theoretical structures that satisfy certain theories. However, these structures are not abstract fantasies but ones that approximate quite closely real phenomena that already exist in nature or that can be experimentally produced.

2. I stress that I have in no sense canvassed the many different theories of phenomena that should count as being nondeterministic. In terms of the emphasis I have placed on probability and randomness, it is worth noting the significant generalizations of probability used in various contexts when no proper concept of probability has sufficed. A well-defined concept of nondeterminacy that is of this ilk is discussed in Suppes and Zanotti (1977). The more general setting is the large literature on upper and lower probabilities. Of particular interest conceptually in the present context are upper probabilities that are not supported by any probability measure, because they are nonmonotonic, i.e., there are two events, say A and B, such that the occurrence of A implies the occurrence of B but the upper probability
of $A$ is greater than the upper probability of $B$. For an application of such upper probabilities to quantum mechanics, see Suppes and Zanotti (1991).

3. My discussion of Kant is too brief to hope to give a proper exegesis of his ideas, but the Third Antinomy is too important historically to be entirely omitted from my analysis. As Christoph Lehner has pointed out to me, I need to go on to distinguish between Kant’s concepts of transcendental freedom and of practical freedom, the latter concept being known through experience (B831). It is also clear that my general use of the concept of transcendental is not identical with Kant’s.

REFERENCES


