Pulse Shaping for Differential Offset-QPSK

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Abstract—Pulse shaping is examined as a means to improve the performance of a differential offset quadrature phase-shift keying system in a bandwidth-constrained environment. Through optimization with respect to a composite Nyquist criterion, the derived pulse shapes have comparable performance to a $\pi/4$-differential quadrature phase-shift keying in an additive white Gaussian noise (AWGN) channel and better performance in a hard-limited AWGN channel.

Index Terms—Differential offset quadrature phase-shift keying (DOQPSK), pulse shaping.

I. INTRODUCTION

DIFFERENTIAL offset quadrature phase-shift keying (DOQPSK) is well suited to nonlinear bandlimited channels [1]. By limiting phase transitions between symbols to $\pi/2$, DOQPSK has less out-of-band power than either differential quadrature phase-shift keying (DQPSK) or $\pi/4$-DQPSK [2]. Furthermore, DOQPSK has less envelope fluctuation than $\pi/4$-DQPSK, which leads to less spectral spreading if a nonlinear amplifier is used [3]. Since the in-phase (I) and quadrature-phase (Q) symbols in DOQPSK are half a symbol apart, the differentially demodulated output generally contains intersymbol interference (ISI). If the transmitter and receiver pulse shaping filters obey the combined Nyquist 1 (N1) and Nyquist 2 (N2) criteria, ISI is limited to the previous and next symbols, and can be resolved by an eight-state Viterbi decoder [3]. The combined N1 and N2 criteria cannot be jointly satisfied for a pulse shape having a roll-off factor less than unity. This ISI can be mitigated by truncating the filter impulse response and applying a trellis decoder, whose complexity grows exponentially with the truncation length. Alternatively, multiple-bit differential detection has also been applied to DOQPSK [4], and reduced-complexity algorithms have been proposed [5].

In this letter, we consider a DOQPSK system with design roll-off factor of $\alpha = 0.35$, and use pulse shaping as a means to improve performance without increasing complexity. Instead of expanding the number of trellis states, as in [2], to resolve ISI, we use a small trellis and examine how different pulse shapes can be used to minimize ISI. Using the closeness measure proposed in [6], different pulses are considered in terms of how closely they satisfy the N1 and N2 criteria. The pulse-shaping strategy developed in this letter can improve BER performance with no increase in complexity.

II. SYSTEM MODEL

We consider the DOQPSK in [2] and [3] that is shown in Fig. 1. At the transmitter, antipodally modulated bits are split into two bit streams $\{a_k, b_k\}$ that are differentially encoded into antipodal bit streams $\{x_{2k} = a_k + x_{2k-2}, y_{2k+1} = b_k + y_{2k-1}\}$, such that $x_k = 0$ at odd intervals of $k$ and $y_k = 0$ at even intervals of $k$. The quadrature-phase stream is delayed by $T_s/2$, where $T_s$ is the symbol time, and both streams are passed through pulse-shaping filters and modulated. At the receiver, the received signal is demodulated, passed through a pulse-shaping matched filter, and sampled at intervals of $T_s/2$.

In discrete time, the received signal can be represented as

$$r_k = \sum_{j=-\infty}^{\infty} h_k(x_{k-j} + jy_{k-j}) e^{j\phi_k} + n_k$$

(1)

where $\{h_k\}$ is the impulse of the combined transmitter and receiver pulse-shaping filters, $\phi_k$ is the slowly time-varying phase, and $n_k$ is a noise source that is additive white Gaussian noise (AWGN), provided that the pulse shaping satisfies N1. Although N1 is not strictly satisfied for the candidate pulse, we assume that correlation between noise samples is negligible for the purpose of the receiver design.

The pulse-shaping filter is typically designed to satisfy the N1 criterion to eliminate ISI and the N2 criterion, to ensure accurate timing recovery. The unique minimum bandwidth pulse satisfying the combined N1 and N2 criteria is the full raised cosine (FRC) pulse that has $\alpha = 1$ [7]. The N1 criterion implies $h_1 = 1$ and $h_i = 0$, $i = \pm 2, \pm 4, \ldots$, in (1), and the N2 criterion implies $h_{k-1} = (1/2)$ and $h_i = 0$, $i = \pm 3, \pm 5, \ldots$. With the N1 and N2 criteria satisfied, the received signal is $r_{2k} = [x_{2k} + j0.5(y_{2k-1} + y_{2k+1})]e^{j\phi} + n_{2k}$, $r_{2k+1} = [0.5(x_{2k} + x_{2k+2}) + jy_{2k+1}]e^{j\phi} + n_{2k+1}$. Since the I and Q channel symbols are offset by $T_s/2$, each symbol is interfered with by the previous and next symbols on the opposite channel.

The received signal is passed through a differential demodulator $(r_{2k}^* r_{k-2}^*)$ in order to remove the unknown phase $\phi$. The demodulator output contains ISI that can be resolved using the Viterbi algorithm. Following the work in [2] and [3], a trellis can be constructed based on the noiseless demodulator outputs that are given by

$$\tilde{X}_{2k} = R(r_{2k} r_{k-2}^*)$$

$$\tilde{Y}_{2k} = 3(r_{2k} r_{k-2}^*)$$

$$= x_{2k}x_{2k-2} + \frac{1}{4}(y_{2k-1} + y_{2k+1})(y_{2k-3} + y_{2k-1})$$

$$= \frac{1}{2}x_{2k}x_{2k-2}(y_{2k-1} + y_{2k+1}) - \frac{1}{2}x_{2k}(y_{2k-3} + y_{2k-1})$$

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where \( \tilde{X}_{2k+1} \) and \( \tilde{Y}_{2k+1} \) are the noiseless outputs of the differentially demodulated received signal and \( X_t \) and \( Y_t \) are the real and imaginary parts of the received signal, respectively. Finally, differential decoding is performed according to 
\[
\tilde{\delta}_k = x_{2k}x_{2k-2} \quad \text{and} \quad \tilde{b}_k = y_{2k+1}y_{2k-1} \quad \text{for the I and Q channels, respectively.}
\]

As a low-complexity alternative to the trellis-state expansion in [2], we fix the trellis size and propose using pulse shaping to minimize ISI. Since there no pulse that satisfies the combined N1 and N2 criterion for \( \alpha = 0.35 \), we strive to design pulses that minimize errors in both criteria. Employing the measure of closeness proposed in [6], the mean-squared error with respect to the N1 and N2 criteria is defined as
\[
e_f^2 = \sum_k \left[ h(k) - \delta_{nk} \right]^2 \quad (3)
\]
\[
e_{II}^2 = \sum_k \left[ h \left( \frac{2k - 1}{2} \right) - \frac{1}{2} (\delta_{k-1} + \delta_{k+1}) \right]^2 \quad (4)
\]
where \( \delta_{ij} \) is the Kronecker delta. In (3) and (4), it has been assumed that \( T_n = 1 \) for notational simplicity. The composite Nyquist error given by [6]
\[
e^2 = \gamma e_f^2 + \zeta e_{II}^2 \quad (5)
\]
where \( \gamma \) and \( \zeta \) are the weights given to the first and second criteria, respectively. A family of pulses can be derived by setting the parameters \( \gamma, \zeta \) and optimizing the filter design with respect to the criterion in (5). Minimizing (5) for a particular \( \gamma, \zeta \) yields an overall filter response given by [6], [8].
\[
H_{\gamma\zeta}(f) = \left\{ \begin{array}{ll}
\frac{\zeta \cos(\pi f \Delta) - \lambda}{\gamma \cos(\pi f \Delta) - \lambda}, & 0 \leq |f| \leq \frac{1-\alpha}{2} \\
0, & \frac{1-\alpha}{2} \leq |f| \leq \frac{1+\alpha}{2} \\
\end{array} \right.
\quad (6)
\]
where \( \lambda \) is used to normalize the pulse, and is such that \( \int_{-\frac{1+\alpha}{2}}^{\frac{1+\alpha}{2}} H_{\gamma\zeta}(f) df = 1 \). The rationale for choosing these measures of closeness and the additive composite error is twofold. First, the derivation becomes mathematically tractable,
and second, the FRC pulse result for $\gamma = \zeta = 1$ with a unity roll-off. We next derive three pulses by controlling $\gamma$ and $\zeta$.

1) Modified Raised Cosine: The modified raised cosine (MRC) (Table I) is a pulse that minimizes (4) subject to the N1 constraint. Thus, it satisfies N1, and is as close as one can get to N2. It is obtained by setting $\gamma$ and $\zeta$ such that $\gamma = \zeta = 1$.

2) Truncated Raised Cosine: If $\gamma = \zeta = 1$, the truncated raised cosine (TRC) (Table I) results. It simultaneously minimizes (3) and (4), but satisfies neither N1 nor N2. Since its area is dependent on $\gamma$, it has to be normalized by setting $\gamma$. In terms of the criterion, the RC has the most error.

3) Continuous Filters: The three filters previously discussed have discontinuities in their spectra which can lead to difficulties in practical implementation. We consider two continuous spectral pulses both satisfying N1, namely: the linear rolloff (LROLL) [6] and the better than raised cosine (BTRC) [9] pulses (Table I). These pulses are not derived with respect to composite error criteria (5), although they have errors in N2 that are comparable to the MRC and TRC pulses.

### IV. Simulation Results

In this section, the effectiveness of the different pulse-shaping filters in a DOQPSK transmission is examined, with the $\pi/4$-DQPSK and 128-state DOQPSK [2], [3] systems used as a reference. We consider the AWGN channel and the hard-limited AWGN channel, the latter approximating a saturated Class C power amplifier.

For the different pulse shapes, we compared the root-mean-square error with respect to the N1 and N2 criteria as a function of the excess bandwidth in Fig. 2. The RC2 and TRC are the only pulses having errors in $e_{11}$, with the TRC having less error. In terms of the $e_{12}$ criterion, the the RC has the most error, the
channel in Fig. 3(b), using different pulse shapes at a roll-off factor of $\alpha = 0.35$. Shown for reference are the $\pi/4$-DQPSK system with an RC pulse having $\alpha = 0.35$, the DOQPSK system with $\alpha = 1$, and the 128-state DOQPSK [2], [3] receiver. The $\alpha = 1$ DOQPSK receiver represents the best achievable performance, and the $\alpha = 0.35$ DOQPSK RC pulse has the worst performance. Of the candidate pulse shapes, MRC and TRC pulses had the best performance: 0.1 dB worse than the $\pi/4$-DQPSK system in the AWGN channel, and 0.5 dB better in the hard-limited AWGN channel at a nominal BER of $10^{-3}$. The 128-state DOQPSK system in [2] and [3] had worse performance than the MRC and TRC pulses, followed by the BTRC, LROLL, and RC2 pulses, with the spread in performance being larger in the hard-limited channel. The relatively poor performance of the RC2 pulse stems from correlation in the noise samples that results when N1 is not satisfied.

V. SUMMARY

Pulse shaping is an effective means to improve performance in DOQPSK system with constrained bandwidth. In both the AWGN and hard-limited AWGN channels, the MRC and TRC pulses with an 8-state trellis outperformed the more complex 128-state solution in [2] and [3]. These pulses show similar performance to the $\pi/4$-DQPSK system in the AWGN channel, and a 0.5 dB performance improvement over the $\pi/4$-DQPSK system in the hard-limited AWGN channel.

REFERENCES