

Optimal Agency Contracts for Delegated R&D

Joaquín Poblete*, Daniel Spulber†

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Abstract

A principal delegates R&D to an agent when the researcher's actions are unobservable. The optimal contract for delegated R&D is shown to take the form of an option. The principal and the agent are risk neutral and the agent is subject to limited liability. The principal makes an implementation decision after observing the quality of the invention. The discussion considers experimental design with simultaneous sampling, sequential sampling and a combination of the two. The analysis also applies when the agent designs the experiment by choosing the distribution from which to sample, when the principal imperfectly observes the outcome of R&D, and when the number of samples is random.

KEYWORDS: R&D, Invention, innovation, contract, principal, agent, incentives (JEL Codes: D82, D83, O3).

*London School of Economics and PUC, E-mail: joaco.poblete@gmail.com

†Kellogg School of Management, Northwestern University, Evanston, IL USA.
jems@kellogg.northwestern.edu Daniel F. Spulber gratefully acknowledges research support from Qualcomm and the Kellogg School of Management

1 Introduction

Research and development (R&D) generally is a *delegated* activity.¹ Companies conduct R&D in-house by employing specialized experts such as scientists, engineers, and statisticians. Companies also outsource R&D by contracting with research laboratories, specialized firms, universities, and independent researchers. Government agencies operate research laboratories and contract externally for R&D. Entrepreneurial technology startups, specialized research firms and independent researchers form contracts with investors to finance R&D. Thus, R&D is much more complex than a problem of efficient statistical decision making.² The delegation of R&D requires contracts that align the incentives of researchers with those of the firms and institutions that manage or sponsor the research. This paper considers how to choose optimal contracts for delegated R&D and shows that the optimal contract takes the form of an *option*.

Because R&D is delegated, it may be costly for managers or sponsors of R&D to observe or to monitor researchers' activities. Due to the specialized nature of R&D, managers or sponsors may lack the expertise to understand the design of experiments or scientific and technical efforts. R&D inputs may not be verifiable by third parties. Accordingly, it may not be possible or desirable to contract on the basis of investment in R&D activities. Delegated R&D thus is a principal-agent relationship that is subject to moral hazard problems. However, R&D differs from traditional principal-agent relationships involving contracts for productive effort or market intermediation. This paper attempts to extend the principal-agent framework to address experimental design and observation of scientific results of experiments. The discussion considers how firms should structure contractual incentives based on the quality of uncertain discoveries.

The choice of contracts for delegated R&D is important because of the significance of R&D. Invention and innovation are essential to the development of new products,

¹National and international agencies view R&D as a delegated economic function. The United Nations Statistical Commission (2009, p. 119) in its System of National Accounts (SNA) defines R&D as "creative work undertaken on a systematic basis to increase the stock of knowledge, and use this stock of knowledge for the purpose of discovering or developing new products, including improved versions or qualities of existing products, or discovering or developing new or more efficient processes of production." According to the the United Nations Statistical Commission (2009, p. 119) "The research and development undertaken by market producers on their own behalf should, in principle, be valued on the basis of the estimated basic prices that would be paid if the research were subcontracted commercially, but in practice is likely to have to be valued on the basis of the total production costs including the costs of fixed assets used in production."

²For a comprehensive overview of optimal statistical decisions, see DeGroot (1970)

production processes, and transaction methods. R&D is a major driver of the growth of firms and the growth of the economy.³ R&D expenditures are substantial; total US private, nonprofit and government investment in R&D exceeds \$400 billion per year with private R&D constituting over 10 percent of private fixed investment.⁴

We present a model in which a principal delegates R&D to an agent whose R&D efforts are unobservable. The agent designs an experiment by choosing the number of statistical samples or the population distribution from which to sample. The principal and the agent are risk neutral and the researcher has limited liability, so the principal cannot sell the R&D project to the researcher. The quality of the invention depends on the agent's research efforts, which involves sampling from a probability distribution over research outcomes.⁵ Because the researcher's efforts are unobservable, the incentive contract must be based on the quality of the outcome of R&D not the researcher's efforts. After observing the quality of the discovery, the principal makes an implementation decision. An important aspect of our analysis is that the probability distribution over research outcomes is fully general so that our results hold for any probability distribution. In this general setting, we find that the optimal incentive contract for R&D is an option contract because it is efficient to focus rewards for R&D effort on the best outcomes rather than diffusing incentives across all outcomes.

The main results of the analysis are as follows. First, we show that in equilibrium, the effect of the quality of the discovery on the payment to the agent is less than or equal to one. This follows from implementation of the research discovery by the principal after observing the outcome, which corresponds to statistical decision making. The payment to the agent also must be increasing in the quality of the discovery. This helps to extend the agency model in Poblete and Spulber (2012) by removing the assumption that the contract is monotonic and by endogenizing the assumption that the payment to the principal must be non-decreasing. We show that the results are robust to having a discrete decision variable. This result has general implications for agency models by replacing the contractual monotonicity assumption. Our analysis generalizes the basic agency setting to allow for statistical sampling and experimental design.

Second, we show that with simultaneous sampling from a probability distribution,

³See Lee and Schmidt (2010) on the contribution of R&D to economic growth.

⁴See Lee and Schmidt (2010). The National Income and product accounts have switched from treating public and private R&D as current expenditures to treating them as capital investments (Bureau of Economic Analysis, 2013).

⁵On search theory applied to R&D, see for example Weitzman (1979) and Spulber (1980).

the optimal incentive contract for R&D takes the form of an option. The researcher chooses the number of experiments or equivalently, the number of independent random draws from the distribution of potential results. The discovery that results from R&D is the highest value of the sample. By the theory of order statistics, this generates a probability distribution over the realizations of the best outcome. For any underlying probability distribution, the hazard rate of the distribution of the best outcome is decreasing in the size of the sample, thus satisfying the Decreasing Hazard Rate in Effort Property (DHREP). This implies a single-crossing result for the agent's utility that in turn implies that the optimal R&D contract takes the form of an option because it is efficient to motivate the researcher by targeting the best outcomes as rewards for performance. Optimal incentives for research therefore give the researcher no reward when the quality of the invention falls below a threshold and give the researcher all of the returns from R&D that are above a threshold.

Third, we show that with sequential sampling from any probability distribution, the optimal incentive contract for R&D again takes the form of an option. Although it is well known that optimal sequential search involves a stopping rule (DeGroot, 1970), we show that this result applies to delegated search. The agent engaged in costly sequential search chooses an optimal stopping rule based on the rewards for the quality of R&D. With sequential R&D, an option contract is optimal because it is efficient to motivate the researcher by targeting the best outcomes as rewards for performance. The optimal incentive contract guarantees that the researcher chooses the stopping rule that is optimal without delegation. We also show that the optimal contract again takes the form of an option when the research chooses a combination of simultaneous and sequential sampling. Then, both the number of samples per period and the stopping rule are first-best optimal, so that the combination of simultaneous and sequential sampling eliminates incentive distortions from delegated R&D.

Finally, we extend the analysis to consider some issues in experimental design. We also show that the optimal contract is an option when the principal cannot perfectly observe the agent's report of the outcome of the experiment. We further consider experimental design when the researcher chooses among distributions from which to sample by choosing the scale of the distribution. We show that an option contract also is optimal for inducing the agent to choose the scale of the distribution that will be sampled. We also consider an experiment where the agent's effort affects the distribution of the number of random observations and reports the sum of all observations. We provide conditions under which the optimal contract is an option if the outcome is the

sum of a random number of observations.

Our paper focuses on how to motivate the invention process that precedes commercialization and innovation and thus differs from optimal incentive schemes to reward innovation. Manso (2011) looks at the optimum contract to induce an agent to engage in innovation. Innovation in his setting is a process of market experimentation in which the agent needs to choose between testing a new product in the market or selling an existing one. Manso suggests that the optimal innovation-motivating incentive scheme for managers can be implemented via a combination of stock options with other types of incentives. Weyl and Tirole (2012) consider optimal contracts that reward innovation and solve a multidimensional screening problem in which the innovator has private information about research cost, quality and market size. Our paper takes the rewards for innovation as exogenous and focuses on the choice of incentives for agents to devote effort in conducting R&D.

Our model is related to that of Ottaviani and Lewis (2008) who consider delegation of sequential search to an agent who devotes effort to increasing the likelihood of making an observation in each period. Their model differs from ours in that they consider various issues regarding monitoring and revelation of information. They find that payments can increase or decrease over time and the agent devotes less effort than is optimal due to moral hazard effects. In our setting, the agent chooses the number of observations rather than the likelihood of making an observation. We also focus on characterizing the form of the incentive contract.

Our result that technology transfer contracts can take the form of options has implications for ownership of intellectual property (IP) and financing of R&D that will be discussed in a later section. Our analysis may be useful in understanding incentive contracts associated with transfers of IP. Such IP transactions are subject to problems of asymmetric information including moral hazard (Arrow, 1962, Zeckhauser, 1996, Spulber, 2008, 2010). IP transfers including patent assignments and licensing can require additional effort by the inventor to transfer tacit knowledge (Spulber, 2012). Macho-Stadler et al. (1996) present a moral hazard model of technology transfer in which the inventor decides whether or not to transfer additional complementary know-how, although the level of know-how is given. The recipient of IP transfers often must compensate the inventor for the R&D outcome so as to provide incentives for knowledge transfers (Lowe, 2006, Olsen, 1993). Jensen and Thursby (2001) present a principal-agent model of invention with moral hazard in which the licensee's probability of commercial success depends on the inventor's effort. In practice, "star scientists" form

contractual arrangements with firms that apply their research discoveries; see Zucker, Darby, and Armstrong (2002).

The paper is organized as follows. Section 2 presents the basic framework. Section 3 derives the optimal contract for delegated R&D for simultaneous search and Section 4 derives the optimal contract for sequential search. Section 5 obtains the optimal contract when the researcher can choose a combination of simultaneous and sequential search. Section 6 extends the analysis to consider some applications of the basic model to experimental design. Section 7 examines the property rights and financing implications of the model and Section 8 concludes the discussion.

2 The basic model

This section presents a model of delegated R&D that produces an invention. Consider contracting between an agent who engages in R&D and a principal who implements the invention. The agent's R&D activities consist of designing an experiment that generates multiple realizations of a random variable. The principal cannot observe the agent's experimental design or the realizations of the random variable. The agent observes the realizations of the random variable and accurately discloses a statistic to the principal. The statistic is based on the realizations of the random variable and represents the *invention*. After observing the invention, the principal introduces an innovation to the market by making a decision regarding implementation of the invention. The principal obtains a benefit that depends on the agent's invention and the principal's implementation decision. The contract between the principal and the agent is a payment that depends on the principal's benefit.

2.1 The Agent

The agent engaged in R&D is an expert researcher such as a statistician, engineer, scientist, social scientist, or inventor. The agent may be an employee of the principal or an independent contractor. The agent is risk neutral and has limited liability, which is normalized to zero. The agent has an opportunity cost $u_0 > 0$.

The agent conducts R&D by designing an experiment and observing experimental data.⁶ The agent's *experimental design* consists of determining the number of inde-

⁶See Blackwell (1951, 1953) on the general definition of an experiment. Adcock (1997) surveys the problem of sample size determination as a means of estimating unknown values of a parameter of a

pendent draws n from the distribution $F(x)$ defined on the positive real line. The *experimental data* consists of n independent and identically distributed random variables $X = (X_1, X_2, \dots, X_n)$ with realizations x_1, x_2, \dots, x_n . The agent's experimental design and experimental data are unobservable to the principal. The distribution $F(x)$ is common knowledge for the agent and the principal.

The experimental data provides information about the quality of the invention. After completing R&D, the agent reports the maximum order statistic to the principal that summarizes the realizations of the random variable,

$$t(X) = \max\{x_1, x_2, \dots, x_n\}. \quad (1)$$

The realization of the maximum order statistic $t(X)$ is the quality of the agent's invention. The statistic also corresponds to a signal to the principal that describes the realizations of the random variable. This can be generalized to allow imperfect observation of the statistic by the principal, see Section 6.2.

The value of the statistic that the agent communicates to the principal is observable but *non-verifiable* by the principal. The distribution of the maximum statistic t for a given sample size n is

$$H(t, n) = F^n(t). \quad (2)$$

The density of the statistic t for a given sample size n equals

$$h(t, n) = \frac{\partial H(t, n)}{\partial t} = nF^{n-1}(t)f(t). \quad (3)$$

Although the sample size n is an integer, the function $H(t, n)$ is well defined, decreasing, and differentiable for any positive real value n .

The agent incurs a cost c for each draw, so that the agent's total costs cn depend on the sample size. The number of samples taken by the agent represents the agent's effort. There is a moral hazard problem because the agent's R&D effort is costly and because the principal cannot observe the agent's experimental design. This may result in an experimental design that is not optimal. Generally, larger samples improve the performance of statistics so that a smaller sample may reduce the effectiveness of the experiment. The agent may shirk by taking a sample that is smaller than the optimal sample size.

distribution function for both frequentist and Bayesian approaches.

We consider both simultaneous and sequential sampling. If R&D consists of simultaneous sampling, then the agent's experimental design is the choice of the number of realizations, n , of the random variable. If R&D consists of sequential sampling, then the agent's experimental design is the choice of a stopping rule x^* for sampling that in turn determines the expected number of observations of the random variable.

The decreasing hazard rate in effort property (DHREP) is an important condition for characterizing the form of principal agent contracts (Poblete and Spulber, 2012). Define the hazard rate as a function of the sufficient statistic and the number of observations,

$$r(t, n) = \frac{h(t, n)}{1 - H(t, n)} = \frac{nF^{n-1}(t)f(t)}{1 - F^n(t)}.$$

The reduced-form distribution $H(t, n)$ is said to satisfy DHREP when its hazard rate for $t < 1$ is decreasing in the number of observations n ,

$$r_n(t, n) < 0.$$

where $r_n(t, n) = \frac{\partial}{\partial n}r(t, n)$. It is readily shown that the reduced-form distribution satisfies this property for any distribution F . The sample size decreases the hazard rate of the maximum order statistic

$$r_n(t, n) = F^{n-1}(t)f(t)\frac{1 - F^n(t) + n \ln F(t)}{(1 - F^n(t))^2} < 0. \quad (4)$$

The numerator is negative for $n \geq 1$. Therefore, the hazard rate of the reduced-form distribution satisfies DHREP.

To provide economic intuition for the DHREP property, we can define a critical ratio that identifies the rate of return from incentives for delegated R&D,

$$\gamma(t, n) = -\frac{H_n(t, n)}{1 - H(t, n)}. \quad (5)$$

This critical ratio provides a useful way to verify that the reduced-form distribution $H(t, n)$ satisfies DHREP. Intuitively, the benefit of increasing the slope of the payment to the agent at some t , is $-H_n(t, n)$. The cost of increasing the payment to agent at some t is $1 - H(t, n)$ if $\gamma_t(t, n) > 0$ because higher output levels are more efficient to induce the agent to increase the sampling effort. The effect of the outcome on the critical ratio is as follows

$$\frac{\partial \gamma(t, n)}{\partial t} = -\frac{[1 - H(t, n)]h_n(t, n) + h(t, n)H_n(t, n)}{[1 - H(t, n)]^2}. \quad (6)$$

This can be written as

$$\frac{\partial \gamma(t, n)}{\partial t} = -F^{n-1}(t)f(t) \frac{1 - F^n(t) + n \ln F(t)}{(1 - F^n(t))^2} = -\frac{\partial}{\partial n} \frac{h(t, n)}{1 - H(t, n)}. \quad (7)$$

It follows that a critical ratio that is increasing in the statistic t is equivalent to the hazard rate of the reduced-form distribution satisfying DHREP.

The DHREP property is a weaker requirement than the monotone likelihood ratio property (MLRP), which is said to hold when $\frac{\partial}{\partial t} \frac{h_n(t, n)}{h(t, n)} > 0$. MLRP is widely used in agency models.⁷ Although DHREP is sufficient for our results, note that the reduced form distribution $H(t, n)$ also satisfies MLRP.⁸ The hazard rate order is a sufficient but not necessary condition for the reduced-form distribution to satisfy first-order stochastic dominance (FOSD) (Shaked and Shanthikumar, 2007). This can be readily verified in our setting, $H_n(t, n) = F^n(t) \ln F(t) < 0$.

2.2 The Principal

The principal is risk neutral and owns a firm that implements the agent's invention, t . The invention provides a benefit to the principal's firm such as a new product design, a new production process, or a new transaction method. The principal introduces an *innovation* to the market by taking an action z that implements the invention, t . The principal's implementation decision z can represent prices, outputs, products, or investments. The value of the innovation to the principal depends on the invention and its implementation,

$$\pi = \Pi(t, z). \quad (10)$$

The value of the innovation π represents the principal's return from introducing the innovation net of the costs of implementation, excluding the payment to the agent

⁷This is because the likelihood ratio order is sufficient but not necessary for the hazard rate order (Shaked and Shanthikumar, 2007).

⁸The likelihood ratio equals

$$\frac{h_n(t, n)}{h(t, n)} = \frac{F^{n-1}(t)f(t)[1 + n \ln F(t)]}{nF^{n-1}(t)f(t)} = \frac{1}{n} + \ln F(t). \quad (8)$$

The effect of the outcome on the likelihood ratio is simply

$$\frac{\partial}{\partial t} \frac{h_n(t, n)}{h(t, n)} = \frac{f(t)}{F(t)}. \quad (9)$$

for R&D. With delegated R&D, the agent conducts the experiment and the principal chooses an action to maximize net benefits. This contrasts with the traditional characterization of experiments in which the statistician both conducts the experiment and chooses an action (Blackwell, 1951, 1952).⁹

The following assumptions are useful for characterizing the principal's equilibrium implementation decision.

Assumption 1. *The value of the innovation to the principal $\Pi(t, z)$ is strictly supermodular and continuous in (t, z) .*

Assumption 2. *There exists a unique finite efficient implementation decision $z^0(t) > 0$ that maximizes $\Pi(t, z)$ for every t .*

By monotone comparative statics (Topkis, 1998), these two assumptions imply that the efficient implementation decision $z^0(t)$ is strictly increasing in t .

The contract between the principal and the agent is fully described by a payment from the principal to the agent, w . The contract is based only on the outcome of innovation, π ,

$$w = w(\pi). \tag{11}$$

The contract needs to provide incentives both for the agent to invent and for the principal to implement the invention. The agent's liability constraint implies that the payment to the agent must be nonnegative, $w \geq 0$. The agent's limited liability rules out the principal selling the task to the agent and achieving an optimal experimental design. For ease of presentation, we restrict attention to contracts with $w(0) = 0$. This restriction is without loss of optimality of the contract.

For any realization of π , the principal's net benefit from innovation is equal to the outcome minus the payment to the agent,

$$v(w, \pi) = \pi - w(\pi). \tag{12}$$

The agent's net benefit is given by the payment net of the cost of conducting R&D,

$$u(w, \pi) = w(\pi) - cn - u_0. \tag{13}$$

After observing the reported value of the maximum order statistic, t , the principal chooses an implementation level z to maximize net benefits. The implementation level is assumed not to be contractible, and therefore the equilibrium implementation level must be incentive compatible according to the following definition.

⁹The statistician can choose an action that minimizes a loss function (Blackwell, 1951, 1953).

Definition 1 *An implementation $z^*(t)$ is said to be incentive compatible for the principal at t under contract $w(\cdot)$ if*

$$z^*(t) \in \arg \max_z \Pi(t, z) - w(\Pi(t, z))$$

The agent anticipates the effect of the realization of the statistic on the principal's implementation decision $z^*(T(X))$.

The timing of the innovation game is as follows:

1. The principal offers a contract $w(\pi)$ to the agent.
2. The agent conducts R&D by choosing a sample of size n and observing a sample X .
3. The agent makes a non-verifiable disclosure $t(X) = \max\{x_1, x_2, \dots, x_n\}$ regarding the maximum order statistic t to the principal.
4. The principal chooses an implementation level z .
5. The principal and agent observe the innovative outcome $\pi = \Pi(t, z(t))$ and the principal makes a payment $w(\pi)$ to the agent.

In this paper, we focus on a situation in which the principal and agent can renegotiate the contract after the invention is revealed but before implementation is performed. This is the natural assumption to make because we have assumed that the value of the invention is observable by both the agent and the principal but not contractible. In the literature on repeated bilateral contracting, a contract is usually considered to be robust to renegotiation if "at every contracting date, the continuation contract is an optimal solution to the continuation contracting problem for the remaining periods" (see Bolton and Dewatripoint, 2004). The principal and the agent may renegotiate the contract after the innovation t has been communicated but before the principal implements the innovation.

As a result, without loss of generality we can restrict attention to contracts that are robust to renegotiation at the interim stage according to the following definition.

Definition 2 *A contract $w(\Pi(t, z))$ is robust to renegotiation at \tilde{t} if given \tilde{z} incentive compatible under $w(\cdot)$ and \hat{z} incentive compatible under an arbitrary contract $\hat{w}(\Pi(\tilde{t}, z))$, then $\hat{w}(\Pi(\tilde{t}, \hat{z})) > w(\Pi(\tilde{t}, \hat{z}))$ implies $\Pi(\tilde{t}, \hat{z}) - \hat{w}(\Pi(\tilde{t}, \hat{z})) \leq \Pi(\tilde{t}, \tilde{z}) - w(\Pi(\tilde{t}, \tilde{z}))$. A contract is robust to renegotiation if it is robust to renegotiation for every t .*

In this setting of contracting with symmetric but unverifiable information, a contract will be robust to renegotiation if and only if the efficient implementation $z^0(t)$ is chosen.

Intuitively if a contract does not satisfy this condition, the principal and agent will renegotiate the contract to induce $z^0(t)$ and divide the proceeds among them.

Lemma 1. *A contract is robust to renegotiation if and only if $[w(\pi) - w(\pi - \varepsilon)]/\varepsilon \leq 1$ for every π and $\varepsilon > 0$*

Proof: As we argue above a contract is robust to renegotiation if and only if it induces an implementation level $z^*(t) = z^0(t)$ for every t . So proving the lemma is equivalent to showing that a contract induces an implementation level $z^*(t) = z^0(t)$ for every t if and only if $[w(\pi) - w(\pi - \varepsilon)]/\varepsilon \leq 1$ for every π and $\varepsilon > 0$. The principal chooses z to maximize $v(t, z) = \pi(t, z) - w(\Pi(t, z))$.

Suppose to the contrary that $z^0(t)$ is not incentive compatible. Then, because $z^0(t)$ is the unique maximizer of $\Pi(t, z)$ for any realization t , we can define $\varepsilon = \Pi(t, z^0(t)) - \Pi(t, z^*(t)) > 0$ for any $z^*(t)$ incentive compatible under $w(\cdot)$. Then, $[w(\pi) - w(\pi - \varepsilon)]/\varepsilon \leq 1$ for every π and $\varepsilon > 0$ implies that $\Pi(t, z^0(t)) - w(\Pi(t, z^0(t))) = \Pi(t, z^*(t)) + \varepsilon - w(\Pi(t, z^*(t)) + \varepsilon) \geq \Pi(t, z^*(t)) - w(\Pi(t, z^*(t)))$. This contradicts the non-optimality of $z^0(t)$, so that $z^0(t)$ is incentive compatible for every t . Conversely, let $z^*(t) = z^0(t)$ for every t . Then, by similar arguments, the optimality of $z^0(t)$ implies $[w(\pi) - w(\pi - \varepsilon)]/\varepsilon \leq 1$. \square

Intuitively contracts are robust to renegotiation if the principal's net return $\pi - w(\pi)$ is non-decreasing in the output π . In what follows, we consider contracts such that the principal's net return is nondecreasing. By Lemma 1, given $z^*(t) = z^0(t)$, we restrict attention to the innovative outcome $\Pi(t) = \Pi(t, z^0(t))$ where by standard monotone comparative statics $\Pi(t)$ is an increasing function.

The principal benefits from taking a greater number of samples. Intuitively, this occurs because more samples increase the expected value of highest value order statistics.¹⁰ We characterize the form of the optimal incentive contract for R&D and show that in this setting the optimal incentive contract for delegated R&D takes the form of

¹⁰Using integration by parts, the expected outcome equals

$$\int_0^\infty \Pi(t)h(t, n)dt = \Pi(0) + \int_0^\infty \Pi'(t)[1 - H(t, n)]dt.$$

Because $\Pi(t)$ is an increasing function and $H(t, n)$ is decreasing in n , the expected outcome is increasing in the number of samples,

$$\frac{\partial}{\partial n} \int_0^\infty \Pi(t)h(t, n)dt = - \int_0^\infty \Pi'(t)H_n(t, n)dt > 0.$$

a call option.¹¹ The principal offers the agent a contract that specifies a threshold level of the outcome, $\pi = R$. If the outcome rises above the threshold, the agent obtains all of the benefits of the outcome net of the threshold and if the outcome is less than or equal to the threshold, the agent does not obtain any benefits,

$$w(\pi) = \max\{0, \pi - R\}. \quad (14)$$

Formally, a contract $w(\pi)$ is an *option* if satisfies the condition

$$\frac{\partial w(\pi)}{\partial \pi} = \begin{cases} 0 & \text{if } \pi < R, \\ 1 & \text{if } \pi > R. \end{cases} \quad (15)$$

To obtain our main results, we do not need to impose any further restrictions on the set of feasible contracts.

3 Simultaneous Sampling

This section considers the situation in which the agent designs the experiment by choosing how many experiments to perform. Simultaneous sampling is used in many types of R&D such as clinical trials in pharmaceuticals and agricultural experiments. Researchers employ simultaneous sampling when the costs of taking multiple samples are less than the costs of waiting for additional observations that are associated with sequential sampling. Simultaneous sampling also is useful in obtaining comparable observations when some experimental conditions such as weather are not subject to control by the researcher. R&D models in which an economic actor chooses the number of sequential samples include Tan (1992), Fullerton and McAfee (1999), Baye and Hoppe (2003), Fu and Lu (2011), and Fu et al. (2012).

As a benchmark for evaluating delegated R&D, consider the number of samples that maximizes expected net benefits

$$n^0 \in \arg \max_n \int_0^\infty \Pi(t)h(t, n)dt - cn.$$

¹¹In finance, a call option is a contract that gives the owner the right but not the obligation to purchase a security at a given price, known as the strike price. The owner of the option will purchase the security only when the actual price rises above the strike price. The realized value of the call option is positive if and only if the actual price of the security rises above the strike price.

The marginal return to increasing the number of samples is given by $-\int_0^\infty \Pi'(t)H_n(t, n)dt - c$. By the definition of the critical ratio $\gamma(\Pi, n)$, this is equivalent to the expectation ratio of the marginal effect on profit divided by the hazard rate times the critical ratio,

$$\int_0^\infty \frac{\Pi'(t)}{r(t, n)} \gamma(\Pi(t), n) h(t, n) dt - c.$$

With delegated R&D, the principal faces a trade-off between the benefits of additional samples and the costs of giving incentives to the agent. Given a contract $w(\pi)$, the expected utility of the agent and the principal are given by

$$U(w, n) = \int_0^\infty w(\Pi(t)) h(t, n) dt - cn - u_0, \quad (16)$$

$$V(w, n) = \int_0^\infty [\Pi(t) - w(\Pi(t))] h(t, n) dt. \quad (17)$$

The principal's problem of choosing an optimal contract, w , subject to feasibility restrictions can be stated as follows,

$$\max_{w, n} V(w, n) \quad (18)$$

subject to

$$n \in \arg \max_a U(w, a), \quad (19)$$

$$U(w, n) \geq 0, \quad (20)$$

$$w(\pi) \geq 0, \text{ for all } \pi, \quad (21)$$

$$[w(\pi + \varepsilon) - w(\pi)]/\varepsilon \leq 1, \text{ for all } \pi. \quad (22)$$

The first constraint is the agent's incentive compatibility condition for the agent's choice of effort, n , the second constraint is the agent's individual rationality condition, and the third constraint represents the agent's limited liability. The last constraint follows from the requirement of robustness to renegotiation that limits the slope of the contract, w .

Proposition 1. *The optimal incentive contract for delegated R&D with simultaneous search takes the form of an option.*

Proof: We first show that given any continuous contract, there always exist a better contract that takes the form of an option. Take an arbitrary continuous contract w that is not an option that induces the agent to choose $\hat{n} \geq 1$ draws. Consider an

option contract $\widehat{w}(\pi)$ that yields the same net expected benefits for the agent for that \widehat{n} , $U(\widehat{w}, \widehat{n}) = U(w, \widehat{n})$. Because $\Pi(t)$ is increasing, the value of the maximum \widehat{t} that satisfies $\Pi(\widehat{t}) = \widehat{\pi}$ is well defined. Remember that

$$U(w, n) = \int_0^\infty w(\Pi(t))h(t, n)dt - cn - u_0.$$

Integrating by parts gives

$$U(w, n) = \int_0^\infty w'(\Pi(t))\Pi'(t)[1 - H(t, n)]dt - cn - u_0. \quad (23)$$

Differentiating with respect to n implies

$$\frac{\partial U(w, n)}{\partial n} = - \int_0^\infty w'(\Pi(t))\Pi'(t)H_n(t, n)dt - c.$$

Noting that $U(\widehat{w}, \widehat{n}) = U(w(\bullet), \widehat{n})$, adding $U(\widehat{w}, \widehat{n}) - U(w, \widehat{n})$ implies

$$\frac{dU(\widehat{w}, \widehat{n})}{dn} - \frac{dU(w, \widehat{n})}{dn} = \int_0^\infty \Pi'(t)[\widehat{w}'(\Pi(t)) - w'(\Pi(t))] \left[\frac{H_n(t, \widehat{n})}{1 - H(t, \widehat{n})} - \frac{H_n(\widehat{t}, \widehat{n})}{1 - H(\widehat{t}, \widehat{n})} \right] (1 - H(t, \widehat{n}))dt.$$

Therefore, we can write this as

$$\begin{aligned} \frac{dU(\widehat{w}, \widehat{n})}{dn} - \frac{dU(w, \widehat{n})}{dn} &= \int_0^{\widehat{t}} -w'(\Pi(t))\Pi'(t) [\gamma(\Pi(t), \widehat{n}) - \gamma(\Pi(\widehat{t}), \widehat{n})] (1 - H(t, \widehat{n}))dt \\ &\quad + \int_{\widehat{t}}^\infty [1 - w'(\Pi(t))]\Pi'(t) [\gamma(\Pi(t), \widehat{n}) - \gamma(\Pi(\widehat{t}), \widehat{n})] (1 - H(t, \widehat{n}))dt \end{aligned}$$

The second term $\int_{\widehat{t}}^\infty [1 - w'(\Pi(t))]\Pi'(t) [\gamma(\Pi(t), \widehat{n}) - \gamma(\Pi(\widehat{t}), \widehat{n})] (1 - H(t, \widehat{n}))dt$ is positive

because $w'(\bullet)$ is restricted to be less than one. To prove that the first term is positive, notice that

$$\frac{\partial}{\partial t} [(\gamma(\Pi(t), \widehat{n}) - \gamma(\Pi(\widehat{t}), \widehat{n})) (1 - H(t, \widehat{n}))] = \frac{\partial \gamma(\Pi(t), \widehat{n})}{\partial t} (1 - H(t, \widehat{n})) - h(t, \widehat{n}) [\gamma(\Pi(t), \widehat{n}) - \gamma(\Pi(\widehat{t}), \widehat{n})].$$

So, $[(\gamma(\Pi(t), \widehat{n}) - \gamma(\Pi(\widehat{t}), \widehat{n})) (1 - H(t, \widehat{n}))]$ is negative and increasing in $(0, \widehat{t})$. Inte-

grating by parts and noting that at \hat{t} the value of the function is 0,

$$\begin{aligned} & \int_0^{\hat{t}} -w'(\Pi)\Pi'(t) [\gamma(\Pi(t), \hat{n}) - \gamma(\Pi(\hat{t}), \hat{n})] (1 - H(t, \hat{n}))dt \\ &= \int_0^{\hat{t}} w(\Pi) \frac{\partial}{\partial t} [\gamma(\Pi(t), \hat{n}) - \gamma(\Pi(\hat{t}), \hat{n})] (1 - H(t, \hat{n}))dt. \end{aligned}$$

Clearly this integral will be positive since $w(\Pi) > 0$ by the limited liability constraint and we already showed that $\frac{\partial}{\partial t} [\gamma(\Pi(t), \hat{n}) - \gamma(\Pi(\hat{t}), \hat{n})] (1 - H(t, \hat{n}))$ is positive. We therefore know that $\frac{dU(\hat{w}(\pi), n)}{dn} > \frac{dU(w, n)}{dn}$ at \hat{n} .

Consider now the case when $1 \leq \tilde{n} \neq \hat{n}$ (not necessarily an integer) and suppose that $U(w, \tilde{n}) = U(\hat{\pi}, \tilde{n})$ holds. We can rewrite the equation as

$$\frac{dU(\hat{w}, \tilde{n})}{dn} - \frac{dU(w, \tilde{n})}{dn} = \int_0^{\infty} \Pi'(t) [\hat{w}'(\Pi(t)) - w'(\Pi(t))] [\gamma(\Pi(t), \tilde{n}) - \gamma(\Pi(\hat{t}), \tilde{n})] (1 - H(t, \tilde{n})) dt. \quad (24)$$

Given the definition of \hat{t} , this expression is positive when evaluated at \tilde{n} . Therefore, whenever a contract gives the same net benefits as an option we have that $\frac{dU(\hat{w}, n)}{dn} > \frac{dU(w, n)}{dn}$. It is well-known that if two functions satisfy the property that whenever they are equal, the slope of one of them is larger, then they cross at most once. The single-crossing property implies that $U(\hat{w}, n) = U(w, n)$ can only hold at \hat{n} . The single-crossing property of U has several implications. For these implications consider that we now restrict attention to integers greater than or equal to one. (i) If $\hat{n} \in \arg \max_x U(w, x)$ and $n' \in \arg \max_x U(\hat{w}, x)$, then $n' \geq \hat{n}$. (ii) Letting

$$U(w) = \max_{n \in \mathbb{Z}_{\geq 1}} \int_0^{\infty} w(\Pi(t)) h(t, n) dt - cn - u_0 \quad (25)$$

single-crossing at \hat{n} implies $U(\hat{w}) \geq U(w)$. (iii) If $\hat{n} \in \arg \max_a U(w, a)$ and $n' \in \arg \max_a U(\hat{\pi}, a)$, letting $V(w) = V(w, \hat{n})$ and $V(\hat{w}) = V(\hat{w}, n')$ implies that $V(\hat{w}) \geq V(w)$. This is because from (i), $n' \geq \hat{n}$, which implies that $V(w, \hat{n}) = V(\hat{w}, \hat{n}) \leq V(\hat{w}, n')$.

From (ii) and (iii), the contract \hat{w} generates no less benefits for the principal than does the contract w , and satisfies the agent's individual rationality constraint. This implies that there always exists an option contract that outperforms any continuous contract.

Finally we claim that no discontinuous contract can strictly outperform all option contracts, since discontinuous contracts can have only discontinuities of the first type

and can be approximated with continuous contract. So, if there exists an optimal contract, then there must exist an optimal option contract. Existence follows because the space of option contracts that satisfy the participation constraint is compact. \square

The intuition behind the result is that the induced outcome distribution satisfies DHREP, which as discussed before suggests that it is better to induce effort by increasing the slope of the contract for higher output values. This follows from the properties of the maximum order statistic and the increasing inverse of the outcome function. Define $t = \tau(\Pi)$ as the inverse of $\Pi = \Pi(t)$. The distribution of the maximum order statistic, induces a reduced-form distribution of the outcome Π for any given sample size

$$G(\pi, n) = H(\tau(\pi), n). \quad (26)$$

Letting $g(\pi, n) = \frac{\partial G(\pi, n)}{\partial \pi}$ denote the density of the reduced-form distribution, define the hazard rate of the reduced-form distribution $\frac{g(\pi, n)}{1 - G(\pi, n)}$. The hazard rate of the reduced-form distribution has a useful multiplicative decomposition. The hazard rate equals the slope of the inverse of the outcome function times the hazard rate of the maximum order statistic,

$$\frac{g(\pi, n)}{1 - G(\pi, n)} = \tau'(\pi)r(\tau(\pi), n). \quad (27)$$

As we showed above, the distribution $h(\tau, n)$ always satisfies DHREP and so the induced output distribution also satisfies DHREP.

With delegated R&D the agent chooses n^* samples. The agent's marginal incentive for effort excluding effort costs can be rewritten using the critical ratio,

$$\frac{\partial U(w, n)}{\partial n} = \int_0^\infty w'(\Pi(t))\Pi'(t)\gamma(t, n)(1 - H(t, n))dt - c. \quad (28)$$

This can be rewritten using the hazard rate,

$$\frac{\partial U(w, n)}{\partial n} = \int_{\tau(\hat{\pi})}^\infty \frac{\Pi'(t)}{r(t, n)}\gamma(\Pi(t), n)h(t, n)dt - c. \quad (29)$$

Comparing this expression with the first-order condition for the optimal number of samples confirms that the marginal return to effort is decreased by delegation. This implies that with delegated R&D the agent shirks by taking fewer than the optimal number of samples, $n^* \leq n^0$. This is due to the agent not receiving returns when the order statistic falls below the threshold.

The marginal return to sampling for the agent net of sampling costs can be written using the covariance between the marginal effect of the sufficient statistic on the agent's reward divided by the hazard rate and the critical ratio,

$$\frac{\partial U(w, n)}{\partial n} = cov\left(\frac{w'(\Pi(t))\Pi'(t)}{r(t, n)}, \gamma(\Pi(t), n)\right) + E\left[\frac{w'(\Pi(t))\Pi'(t)}{r(t, n)}\right]E\gamma(\Pi(t), n) - c.$$

When the agent's participation constraint is binding, this implies that the marginal return to sampling is greater than the covariance net of unit sampling costs,

$$\frac{\partial U(w, n)}{\partial n} - [cov\left(\frac{w'(\Pi(t))\Pi'(t)}{r(t, n)}, \gamma(\Pi(t), n)\right) - c] = (cn + u_0)E\gamma(\Pi(t), n).$$

4 Sequential Sampling

This section considers the optimal incentive contract for R&D when the agent engages in sequential sampling. The experimenter uses sequential sampling when the costs of taking additional samples outweigh the costs of waiting for additional samples. Sequential sampling is used in many types of repeated scientific experiments. Sequential sampling corresponds to economic models of searching for the highest sample from a distribution.

Let t be the realization of a draw from the distribution $F(x)$ and assume that the distribution has an increasing hazard rate $\frac{f(x)}{1-F(x)}$. This property is widely used in economics and statistics.¹² As a benchmark for evaluating delegated R&D, consider the stopping rule that maximizes the principal's expected net benefits without delegation. The researcher stops the R&D process when the realized benefit $\Pi(t)$ exceeds a critical level. Because the benefit is an increasing function, the stopping rule can be expressed in terms of t . The optimal stopping rule t^0 satisfies the standard recursive equation,

$$\Pi(t^0) = -c + \Pi(t^0)F(t^0) + \int_{t^0}^{\infty} \Pi(t)f(t)dt. \quad (30)$$

¹²For applications in statistics involving an increasing hazard rate, see for example Ross et al. (2005) who consider "sums of a random number of random variables, the time at which a Markov chain crosses a random threshold; the time until a random number of events have occurred in an inhomogeneous Poisson process; and the number of events of a renewal process, and of a general counting process, that have occurred by a randomly distributed time."

Rearranging terms we have

$$\Pi(t^0) = \frac{1}{1 - F(t^0)}[-c + \int_{t^0}^{\infty} \Pi(t)f(t)dt]. \quad (31)$$

With delegated R&D, the optimal incentive contract is designed to induce the agent to choose the best possible stopping rule. Given a contract $w(\bullet)$, the agent chooses a stopping rule t^* that satisfies the recursive condition

$$w(\Pi(t^*)) = -c + w(\Pi(t^*))F(t^*) + \int_{t^*}^{\infty} w(\Pi(t))f(t)dt. \quad (32)$$

We can write the expected utility of the agent as a function of the stopping rule t^* ,

$$U(w, t^*) = w(\Pi(t^*)) - u_0 = \frac{1}{1 - F(t^*)} \left[\int_{t^*}^{\infty} w(\Pi(t))f(t)dt - c \right] - u_0. \quad (33)$$

The principal's expected utility also can be written as a function of the stopping rule,

$$V(w, t^*) = \int_0^{t^*} [\Pi(t) - w(\Pi(t))]dF(t) + [\Pi(t^*) - w(\Pi(t^*))](1 - F(t^*)). \quad (34)$$

The principal's problem of choosing an optimal contract, w , subject to feasibility restrictions then takes the form,

$$\max_{w, t^*} V(w, t^*) \quad (35)$$

subject to

$$t^* \in \arg \max_{\zeta} U(w, \zeta), \quad (36)$$

$$U(w, t^*) \geq 0, \quad (37)$$

$$w(\pi) \geq 0, \text{ for all } \pi,$$

$$[w(\pi + \varepsilon) - w(\pi)]/\varepsilon \leq 1, \text{ for all } \pi. \quad (38)$$

If the reported statistic were verifiable and there was not a need to implement the invention, the contracting problem could be readily solved using a basic forcing contract. The least-cost way to implement a stopping rule, say t^0 , would be to choose a contract $\zeta(t)$ of the form

$$\zeta(t) = \begin{cases} u_0 + \frac{c}{1 - F(t^0)} & \text{if } t > t^0, \\ 0 & \text{otherwise.} \end{cases}$$

This contract would induce the agent to stop the search at t^0 . However, the need to implement the invention does not allow for the contract to have upward jumps or a slope greater than one. The proof as before shows that any continuous contract can be outperformed by an option.

Proposition 2. *The optimal incentive contract for delegated R&D with sequential search takes the form of an option.*

Proof: Suppose to the contrary that the optimal contract, w , is not an option and that it induces the agent to choose a stopping rule \tilde{t} . Consider an option contract $\hat{\pi}(\Pi)$, that yields the same net benefits for the agent for that stopping rule \tilde{t} , $U(\hat{\pi}, \tilde{t}) = U(w, \tilde{t})$. This implies that

$$\int_{\tilde{t}}^{\infty} w(\Pi(t))f(t)dt = \int_{\tilde{t}}^{\infty} [\Pi(t) - R]^+ f(t)dt.$$

Integrating by parts this means that

$$\int_{\tilde{t}}^{\infty} w'(\Pi)\Pi'(t) [1 - F(t)] dt + w(\tilde{\Pi})[1 - F(\tilde{t})] = \int_{\tilde{t}}^{\infty} \Pi'(t) [1 - F(t)] dt + [\tilde{\Pi} - R]^+ [1 - F(\tilde{t})].$$

We know that

$$[\tilde{\pi} - R]^+ < w(\tilde{\pi}). \quad (39)$$

Differentiating the agent's utility with respect to the stopping rule \tilde{t} gives

$$\frac{\partial U(w, \tilde{t})}{\partial \tilde{t}} = \frac{f(\tilde{t})}{1 - F(\tilde{t})} [U(w, \tilde{t}) + u_0 - w(\tilde{\pi})].$$

Noting that $U(\hat{w}, \tilde{t}) = U(w, \tilde{t})$, implies

$$\frac{dU(\hat{w}, \tilde{t})}{d\tilde{t}} - \frac{dU(w, \tilde{t})}{d\tilde{t}} = \frac{f(\tilde{t})}{1 - F(\tilde{t})} [w(\tilde{\pi}) - [\tilde{\pi} - R]^+].$$

This term is positive at \tilde{t} . Moreover we can repeat the argument whenever $U(\hat{w}, \tilde{t}) = U(w, \tilde{t})$. This means that whenever $U(\hat{w}, t) = U(w, t)$, $\frac{dU(\hat{w}, t)}{dt} > \frac{dU(w, t)}{dt}$ and it is well known that if two functions are such that whenever they are equal the slope of one of them is higher, they cross at most once.

The single-crossing property of U has several implications. (i) Letting

$$U(w) = \max_t U(w, t) \quad (40)$$

single-crossing at \tilde{t} implies $U(\hat{w}) \geq U(w)$. (ii) If $t^w \in \arg \max_t U(w, t)$ and $t^{\hat{w}} \in \arg \max_t U(\hat{w}, t)$, single crossing implies that $t^{\hat{w}} > t^w$. (iii) Letting $V(w) = V(w, t^w)$ and $V(\hat{w}) = V(\hat{w}, t^{\hat{w}})$ implies that $V(\hat{w}) \geq V(w)$. This is because from (ii), $t^{\hat{w}} \geq t^w$, which implies that $V(w, t^w) = V(\hat{w}, t^w) < V(\hat{w}, t^{\hat{w}})$.

From (i) (ii) and (iii), the contract w generates benefits for the principal that are not less than those generated by the contract \hat{w} and satisfies the agent's individual rationality constraint. For the same arguments as in Proposition 1, this implies that if there exists an optimal contract, there also exists an optimal option contract. Existence follows because the space of option contracts that satisfy the participation constraint is compact. \square

The result shows that the optimality of the call option is robust to the kind of search performed. Recall that a contract $w(\pi)$ is an *option* if $w(\pi) = \max\{0, \pi - R\}$. It must be the case that $\Pi(t^*) > R$ because otherwise the agent would not expect to receive benefits from search, $w(\Pi(t^*)) = 0$, and the recursive equation would not hold. Note that $\Pi(t)$ increasing implies that $\Pi(t) > R$ for $t > t^*$. The recursive equation thus implies that at the optimal strike price R , the critical value of the stopping rule t^* solves

$$\Pi(t^*) - R = \frac{1}{1 - F(t^*)} [-c + \int_{t^*}^{\infty} (\Pi(t) - R) f(t) dt]. \quad (41)$$

The strike price cancels from this expression which immediately implies that the option contract attains the first best with sequential search.

Proposition 3. *The stopping rule with delegated R&D is the first best, $t^* = t^0$.*

With delegated R&D, the agent chooses the optimal value of the stopping rule because the strike price does not affect the stopping rule. This may be counterintuitive because it means that the agent pursues the same quality of the discovery as optimal R&D without delegation, $\Pi(t^*) = \Pi(t^0)$. The researcher devotes the first-best effort to observation not because the researcher enjoys research or receives social and personal rewards from higher-quality discoveries, but because of the structure of the contract.

The optimal stopping rule is greater than the inverse of the outcome function evaluated at the strike price, $t^* > \tau(R)$. Because the optimal contract is an option, the principal's benefit is simply

$$V(w, t^*) = R \quad (42)$$

The principal has an incentive to increase the strike price until the agent's participation constraint is binding,

$$U(w, t^*) = \Pi(t^*) - R - u_0 = 0. \quad (43)$$

So, the strike price exactly equals the difference between profit at the stopping rule and the agent's opportunity cost, $R = \Pi(t^*) - u_0$. The principal's benefit equals profit at the first-best stopping rule net of the agent's opportunity cost,

$$V(w, t^*) = \Pi(t^*) - u_0. \quad (44)$$

5 Mixture of Simultaneous and Sequential Sampling

This section considers the optimal incentive contract for R&D when the agent chooses a mixture of simultaneous and sequential sampling.¹³ Suppose that the agent incurs a cost k at each stage of sequential sampling in addition to the costs of simultaneous sampling, cN . Assume the number of samples must be the same at every stage. There is a tradeoff between taking additional samples at each stage and continuing to search. The value of the experiment t at each stage depends only on the value of the highest draw, which has a distribution $H(t, N) = F^N(t)$. The density $h(t, N) = \frac{\partial H(t, N)}{\partial t}$ is well defined.

As a benchmark consider the principal's problem without delegated R&D. The optimal stopping rule $t^0 = t^0(N)$ satisfies the recursive condition

$$\Pi(t^0) = -k - cN + \Pi(t^0)H(t^0, N) + \int_{t^0}^{\infty} \Pi(t)h(t, N)dt. \quad (45)$$

The net benefit $\Pi(t^0)$ can be written as follows,

$$\Pi(t^0) = \frac{1}{1 - H(t^0, N)}[-k - cN + \int_{t^0}^{\infty} \Pi(t)h(t, N)dt]. \quad (46)$$

The optimal number of samples per period N^0 maximizes the net benefit $\Pi(t^0(N))$.

With delegated R&D, it is useful to break up the agent's problem into two parts. First, given a contract $w(\bullet)$ and the number of samples at each stage N , the agent's stopping rule $t^* = t^*(w, N)$ satisfies the recursive condition

$$w(\Pi(t^*)) = -k - cN + w(\Pi(t^*))H(t^*, N) + \int_{t^*}^{\infty} w(\Pi(t))h(t, N)dt. \quad (47)$$

Solving for the agent's net benefit gives

$$w(\Pi(t^*)) = \frac{1}{1 - H(t^*, N)}[-k - cN + \int_{t^*}^{\infty} w(\Pi(t))h(t, N)dt]. \quad (48)$$

¹³See for example Morgan and Manning (1985) on general search rules.

The agent chooses the number of samples in each period to maximize the $w(\Pi(t^*(w, N)))$ given the stopping rule $t^* = t^*(w, N)$. This implies that the agent chooses N^* such that

$$N^* \in \arg \max w(\Pi(t^*(w, N))) \text{ subject to (48).}$$

This allows us to reduce the principal's problem to the choice of the optimal contract and the number of samples at each stage.

Given a contract $w(\pi)$, and the stopping rule $t^*(w, N)$ the expected utility of the agent and the principal are given by

$$U(w, N) = w(\Pi(t^*(w, N))) - u_0, \quad (49)$$

$$V(w, N) = \int_{t^*(w, N)}^{\infty} [\Pi(t) - w(\Pi(t))] h(t, n) dt. \quad (50)$$

The principal's problem of choosing an optimal contract, w , subject to feasibility restrictions can be stated as follows,

$$\max_{w, N} V(w, N) \quad (51)$$

subject to

$$N \in \arg \max_a U(w, a), \quad (52)$$

$$U(w, N) \geq 0, \quad (53)$$

$$w(\pi) \geq 0, \text{ for all } \pi, \quad (54)$$

$$[w(\pi + \varepsilon) - w(\pi)]/\varepsilon \leq 1, \text{ for all } \pi. \quad (55)$$

Proposition 4: The optimal incentive contract for delegated R&D with a mixture of simultaneous and sequential search takes the form of an option.

Proof: We first show that given any continuous contract, there always exist a better contract that takes the form of an option. Take an arbitrary continuous contract w that is not an option that induces the agent to choose $\hat{N} \geq 1$ draws. Consider an option contract $\hat{w}(\pi)$ that yields the same net expected benefits for the agent for that \hat{N} , $U(\hat{w}, \hat{N}) = U(w, \hat{N})$. Because $\Pi(t)$ is increasing, the value of the maximum \hat{t} that satisfies $\Pi(\hat{t}) = \hat{\pi}$ is well defined. Differentiating with respect to N implies

$$\frac{\partial U(w, N)}{\partial N} = w'(\Pi(t^*(w, N)))\Pi'(t^*(w, N))\frac{\partial t^*(w, N)}{\partial N}.$$

Noting that $U(\widehat{w}, \widehat{N}) = U(w(\bullet), \widehat{N})$, which implies that $t^*(\widehat{w}, \widehat{N}) = t^*(w, \widehat{N})$, we have

$$\frac{dU(\widehat{w}, \widehat{N})}{dN} - \frac{dU(w, \widehat{N})}{dN} = w'(\Pi(t^*(\widehat{w}, \widehat{N})))\Pi'(t^*(\widehat{w}, \widehat{N}))\left[\frac{\partial t^*(\widehat{w}, \widehat{N})}{\partial N} - \frac{\partial t^*(w, \widehat{N})}{\partial N}\right]$$

From (48), the derivative of the stopping rule is

$$\frac{\partial t^*(w, N)}{\partial N} = \frac{w(\Pi(t^*))H_N(t^*, N) - c + \int_{t^*}^{\infty} w(\Pi(t))h_N(t, N)dt}{w'(\Pi(t^*))\Pi'(t^*)(1 - H(t^*, N))}.$$

Given $\widehat{w}'(\Pi(t^*)) \geq w'(\Pi(t^*))$, it follows that

$$\frac{\partial t^*(\widehat{w}, \widehat{N})}{\partial N} - \frac{\partial t^*(w, \widehat{N})}{\partial N} \geq 0.$$

Therefore, whenever a contract gives the same net benefits as an option we have that $\frac{dU(\widehat{w}, n)}{dn} > \frac{dU(w, n)}{dn}$, so that the single-crossing property holds. By arguments similar to those in the proof of Proposition 1, the contract \widehat{w} generates no less benefits for the principal than does the contract w , and satisfies the agent's individual rationality constraint. As before, the optimal contract is an option. \square

Substituting for the optimal contract $w(\pi) = \max\{0, \pi - R\}$ in the agent's recursive equation (48), observe that the strike price R cancels from both sides of the agent's recursive equation,

$$\Pi(t^*) - R = \frac{1}{1 - H(t^*, N)}[-k - cN + \int_{t^*}^{\infty} (\Pi(t) - R)h(t, N)dt], \quad (56)$$

which is exactly the recursive equation without delegated R&D (46). This implies that for a given number of samples in each period, the stopping rule equals the first best stopping rule for any given number of samples, $t^*(w, N) = t^0(N)$. The strike price also does not affect the agent's choice of N ,

$$N^* \in \arg \max \Pi(t^*) - R \text{ subject to (46).}$$

This implies that the agent chooses the optimal number of samples in each period, $N^* = N^0$.

Proposition 5: *With delegated R&D and a mixture of simultaneous and sequential search, the stopping rule and the number of samples per period attain the first best.*

The combination of simultaneous and sequential sampling serves to eliminate the moral hazard effects of delegated R&D on simultaneous sampling. The optimal stopping

rule is greater than the inverse of the outcome function evaluated at the strike price, $t^* > \tau(R)$. Because the optimal contract is an option, the principal's benefit is simply $V(w, N^0) = R$. The principal has an incentive to increase the strike price subject to the agent's participation constraint. The agent's participation constraint is binding,

$$R = \Pi(t^0(N^0)) - u_0.$$

The principal's benefit with delegated R&D and a mixture of search types equals the value of optimal search with optimal sampling per period net of the agent's opportunity cost,

$$V(w, N^0) = \Pi(t^0(N^0)) - u_0. \tag{57}$$

6 Extensions

This section considers two extensions of the analysis. We first consider the choice of experimental design in terms of selecting distributions from which to sample. We next consider the choice of the number of samples when the principal cannot perfectly observe the sample statistics. Then, we consider the researcher's effort when the number of samples is a random variable.

6.1 Experimental Design

Delegated R&D involves other considerations besides sample size. In particular, the agent must choose among different experimental designs. Blackwell (1951,1953) considers the choice of how to sample within a population and suggests that the statistician should use the most informative experimental design. Our analysis suggests that in addition to considerations of informative distributions, a statistician should take into account the costs of a more informative experimental design. These costs might include the costs of more precise equipment and the costs of additional data gathering and processing associated with sampling from different distributions. Our analysis further suggests that when a principal delegates R&D, the choice of an experimental design also depends on the agency costs of inducing effort from the researcher.

The experimental design problem can be seen as choosing among m different distributions from which to sample, $H_1(t, n)$, $H_2(t, n)$, ... , $H_m(t, n)$. To illustrate this problem separately from the sampling, let the sample size n be fixed and common knowledge. As before, the researcher reports the maximum value of the sample,

$t(X) = \max\{x_1, x_2, \dots, x_n\}$. Suppose that the distributions differ in terms of the scale of the distribution, s_j ,

$$H_j(t, n) = H(t, s_j, n) = F^n\left(\frac{t}{s_j}\right). \quad (58)$$

The scale of the distribution is a measure of dispersion. For example, in the normal distribution the scale is equal to the standard deviation σ . Because the principal benefits from the largest observation, the principal derives benefits from distributions with a higher scale.

Order the family of distributions in terms of their scale and treat effort as a continuous variable. Suppose that the researcher's effort determines the scale of the distribution s at a cost of cs . Then, the researcher's choice of effort s corresponds to choosing among distributions and represents the experimental design. The hazard rate is then

$$r(t, s, n) = \frac{h(t, s, n)}{1 - H(t, s, n)}. \quad (59)$$

As before, the principal's profit is $\Pi = \pi(t, z)$ and satisfies Assumptions 1 and 2, so that $\Pi(t)$ is an increasing function. The cumulative distribution is decreasing in the scale of the distribution,

$$H_s(t, s, n) = -nF^{n-s}\left(\frac{t}{s}\right)\frac{t}{s^2}f\left(\frac{t}{s}\right) < 0.$$

Therefore, effort devoted to increasing the scale of the distribution increases the principal's expected return

$$\frac{\partial}{\partial s} \int_0^\infty \Pi(t)h(t, s, n)dt = - \int_0^\infty \Pi'(t)H_s(t, s, n)dt > 0$$

Shirking by the researcher affects the choice of the distribution. With delegated R&D, the scale of the distribution will be less than the full-information optimal scale.

As an illustration, consider the design of experiments to determine reliability. Suppose that a researcher conducts an experiment to determine the failure rate of a product or technology and reports the reliability to the principal. The family of distributions is given by the Weibull distribution, which is often used to model experimental failure rates. Let $x \geq 0$ represent the observed date of failure. Normalize the shape of the distribution to equal one so that the failure rate is constant over time.¹⁴ Then, the

¹⁴The Weibull distribution is generally written as $F(x, s) = 1 - e^{-\frac{x^\xi}{s}}$ where $\xi > 0$ represents the shape of the distribution. If ξ is greater than, equal to, or less than one, then the failure rate is increasing, constant, or decreasing over time.

distribution of the reliability x is given by $F(x, s) = 1 - e^{-\frac{x}{s}}$. The hazard rate satisfies DHREP in s . For $n = 1$ we have $r_s(t, s, n) < 0$ and for $n > 1$, we have

$$r_s(t, s, n) = r(t, s, n) \frac{t}{s^2} \left[-\frac{s}{t} - sr(t, s, n) - (n-1) \frac{e^{-\frac{t}{s}}}{1 - e^{-\frac{t}{s}}} + 1 \right] < 0.$$

From Proposition 1, this implies the following.

Corollary 1. *The optimal incentive contract for delegated R&D with simultaneous search when the researcher's effort increases the scale of the distribution takes the form of an option.*

This implies that an option contract can be used to induce the agent to choose the experimental design.

The analysis of experimental design can be extended to the sequential choice of distinct distributions from which to sample. One approach to the design of the experiment is to order the distributions and to sample from each of them in turn with the possibility of stopping if a desirable outcome is observed. This generates a stopping rule that is similar to that for sequential sampling from the same distribution, see Weitzman (1979) and Spulber (1980). The present analysis, including Propositions 2 and 3, suggests that with delegated R&D the researcher will choose to sample from fewer distributions than is optimal without delegation.

6.2 Imperfect observation of experimental results

Suppose that the principal cannot perfectly observe the outcome of the experiment. The researcher reports $t(X)$ and the principal observes

$$y = \theta + t, \tag{60}$$

where θ is a shock with cumulative distribution function $\Psi(\theta)$ and density $\psi(\theta)$ on $[0, \infty)$. The shock θ is independent of t . Assume that the shock θ has a log-concave failure rate $1 - \Psi(\theta)$, which implies that it has an increasing hazard rate $\frac{\psi(\theta)}{1 - \Psi(\theta)}$.

The shock is realized when the researcher reports t and is unknown to both the principal and the agent. The principal can only contract with the agent on the basis of the observed outcome y . An implementation $z^*(y)$ is incentive compatible for the principal at y under contract $w(\pi)$ if $z^*(y) \in \arg \max_z \Pi(y, z) - w(\Pi(y, z))$. The agent anticipates the effect of the realization of the statistic on the principal's implementation

decision $z^*(\theta + t(X))$. It follows that the principal's return $\Pi(y)$ is increasing in the outcome y .

The density of the sum y is given by the convolution of the density of the shock $\psi(\theta)$ and the density of the statistic $h(t, n)$,

$$b(y, n) = \int_0^y \psi(y-x)h(x, n)dx. \quad (61)$$

Let $B(y, n) = \int_0^y [\int_0^z \psi(z-x)h(x, n)dx]dz$ be the cumulative density of y . Define the hazard rate function for the convolution, $r(y, n) = \frac{b(y, n)}{1-B(y, n)}$. Because the shock θ is independent of t and has a logconcave failure rate, it follows that the convolution has a hazard rate that is decreasing in n , $r(y, n) > r(y, n+1)$. This is obtained by noting from Lemma 1.B.3 in Shaked and Shantikumar (2007, p. 18) that if a random variable dominates another random variable in the hazard rate order, the order is preserved by adding a shock with an increasing hazard rate. Therefore, the distribution $B(y, n)$ satisfies DHREP.

With simultaneous search, the expected utility of the agent and the principal are given by

$$\begin{aligned} U(w, n) &= \int_0^\infty w(\Pi(y))b(y, n)dy - cn - u_0, \\ V(w, n) &= \int_0^\infty [\Pi(y) - w(\Pi(y))]b(y, n)dy. \end{aligned} \quad (62)$$

The imperfect observability of experimental results by the principal does not change the form of the incentive contract. By arguments similar to those in Proposition 1, we obtain the following result.

Corollary 2. *The optimal incentive contract for delegated R&D with simultaneous search when the agent's report is imperfectly observed by the principal takes the form of an option.*

6.3 Random Number of Observations

The number of observations may be random in various types of experiments and statistical analysis. This occurs, for example, when studying random processes for a fixed period of time (Anscombe, 1952, Gut, 2012). Summing a random number of observations yields random sums, which are used in the random-sum strong law of large numbers and the random-sum central limit theorem (Gut, 2012). Random sums are

useful in various types of experiments for determining "fatigue damage" in mechanical and electrical systems (Mallor and Omev, 2001).

Suppose that the statistic consists of a random sum,

$$t(X, m) = \sum_{i=1}^m x_i,$$

where m is a discrete random variable. Assume that the random variables X_i are independently and identically distributed with cumulative distribution $F(x)$. Suppose that the failure rate $1 - F(x)$ is log-concave so that the hazard rate $\frac{f(x)}{1-F(x)}$ is increasing. Suppose that the conditional likelihood of obtaining m observations $\rho(m, a)$ is decreasing in the researcher's effort a , where

$$\rho(m, a) = \frac{P(Y = m, a)}{1 - P(Y \leq m, a)}$$

The researcher's effort is continuous and has a cost ca . The probability distribution of the random sum $h(t, a)$ satisfies DHREP (based on an application of Shaked and Shanthikumar, 2007, Theorem 1.B.7, p. 20). This implies the following result

Corollary 3: *The optimal incentive contract for delegated R&D with random numbers of observations takes the form of an option.*

Due to moral hazard effects, the researcher will devote less than optimal effort to generating a random number of observations.

7 Ownership of IP and Financing of R&D

Our finding that the optimal contract for delegated R&D takes the form of an option has some useful implications for ownership of IP. Suppose first that a firm hires an employee to conduct R&D. Based on employment contracts that assign ownership of patents to employers and default rules in state laws, this generally implies that the firm owns inventions generated by the employee; see Merges (1999) for an overview. The IP question is how to provide incentives to the agent to conduct delegated R&D when the principal will own the invention. Unlike copyright, there is no work-for-hire doctrine in patent law (Merges, 1999, Burk, 2004).

Our analysis of the optimal agency contract provides insights into employee incentives when the firm owns the invention. The optimal contract can be interpreted as a call option in which an employee has a right but not an obligation to purchase the

invention from the firm. Inventions belong to the principal, but the employee owns an option to buy the invention at a price R . The option contract specifies the lump-sum payment to the firm, which corresponds to the strike price of an option contract. The optimal contract provides no additional rewards for invention to an employee for small inventions that fall below a certain threshold. The contract provides rewards to the employee for significant inventions that exceed that threshold. Thus, the employer gives the employee an option to purchase the invention at a given strike price. After purchasing the invention, the employee can enter the market for inventions and sell or license the invention to others. Alternatively, the employee can become an entrepreneur and seek financing for a new firm that will implement the invention.

The option contract in our setting also corresponds to spin-offs in which the employee starts a new firm and compensates the employer for knowledge acquired at the parent company. The firm creating the spin-off provides financing to the new venture. In this setting, the firm and the employee implement the invention by establishing the new firm. This corresponds to developing the invention in the present model represented by the principal's choice of z . The optimal agency contract provides the employee with incentives to devote effort to invention and also gives the principal incentives to invest efficiently in the venture after the discovery is made. Our analysis suggests that small innovation will be performed by the firm (intrapreneurship) and significant innovation will result in a spin-off (entrepreneurship) so as to provide incentives to the researcher based on the best outcomes.

The use of option contracts as a means of providing incentives for R&D within the firm is related to IP in an interesting way. Setting aside agency issues, suppose that an independent inventor develops an invention. Although patents confer ownership to inventors, there are additional costs of commercializing the invention including costs of marketing, selling, and licensing the invention, costs of developing a product using the invention, legal and administrative costs of maintaining patent rights, and legal costs of defending against infringement. Suppose that these costs are represented by a lump sum R . Then, an inventor will receive a reward from owning a patent only if the market returns π exceed the costs R , so that a patent acts like an option contract. Pakes (1986) observes that when inventors must pay fees to maintain their patents, such IP rights function as options because inventors will only choose to renew the patents when their value exceeds the renewal fee. Erkal and Scotchmer (2009) suggest that innovation is an option that is exercised when the returns to developing an idea exceed the costs of investing in the idea. Ziedonis (2007) considers contracts offered by firms that sponsor

R&D conducted by university researchers that give the firm an option to purchase the invention.

Our agency analysis of R&D contrasts with the economics and law literatures, which have focused attention on relationship-specific investment and contract renegotiation in R&D.¹⁵ Notably, Nöldeke and Schmidt (1998) consider R&D joint ventures with relationship-specific investments and find that first-best investments can be achieved when one party owns the firm initially and the other party has an option to buy the firm at a set price. Options to buy the firm are robust to renegotiation and uncertainty. Nöldeke and Schmidt (1998) point out that contingent ownership structures such as warrants and convertible securities are often used in joint ventures. Effects on investment incentives of options to purchase are also considered by Demski and Sappington (1991). Aghion and Tirole examine how the allocation of property rights within the firm affects investment incentives in R&D, and observe that shop rights, property rights that are contingent on the nature of the innovation, and rules governing breakaway research are ways of sharing ownership of the innovation.

Our analysis sheds some light on technology transfer contracts including assignment and licensing of patents. Standard royalty contracts involve either a lump-sum payment, a payment per-unit of output, or some combination of the two, which provides a means of metering market demand for the technology. The combination of lump-sum and per-unit royalties is used to maximize the IP owner's returns from commercializing a given invention *after* R&D has taken place. However, our analysis does suggest that technology transfer contracts that involve additional R&D to further develop the invention should take the form of option contracts. Jensen and Thursby (2001) find that more than 70 percent of university inventions are sufficiently "embryonic" that after transferring the license to a firm, the inventor must provide additional effort to improve the probability of commercial success. Jensen and Thursby (2001) also examine payments included in university license transfers and find that fixed fees (license-issue or annual) and proportional royalties appear in roughly 80 percent of the license agreements, with fixed fees accounting for 13 percent of revenue received and royalties accounting for 75 percent, along with commonly-used milestone payments and patent reimbursement. They also find that agreements that include equity also tend to include fixed fees and proportional royalties.

¹⁵Merges (1999) and Burk (2004) consider investment incentives pertaining to allocation of ownership of inventions between employees and the firm. Merges (1999) also considers agency issues.

Option contracts for delegated R&D are consistent with incentives for innovative entrepreneurs who obtain financing from banks or venture capitalists. Entrepreneurs often obtain debt financing from banks. Debt is a type of option contract in which the strike price is the face value of the debt and the borrower retains earnings in excess of the face value of the debt. Kaplan and Strömberg (2003) observe that venture capitalists often use convertible preferred securities to finance startups that function like option contracts.

There is increasing financing for independent inventors and startup firms engaged in R&D. Our results suggest that debt contracts can be useful in financing R&D because they have the form of a call option and thus are optimal contracts for R&D when the researcher's effort is unobservable to investors. In addition, financial valuation techniques can be used to calculate the value of options, which suggests that there may be ways to value debt contracts used to finance R&D. Option contracts can be readily standardized, which should facilitate the exchange of contracts used to finance R&D in the same way that financial options are traded on organized exchanges.

8 Conclusion

Because R&D tends to be a delegated activity, it is necessary to understand the optimal design of incentives for invention. Our analysis shows that optimal contracts takes the form of an options because it is efficient to shift rewards for researchers towards the better outcomes. We found that option contracts are optimal in rewarding simultaneous and sequential sampling in data gathering. We also found that option contracts are optimal in rewarding experimental design in terms of the choice of distributions from which to sample and when the number of samples is random.

Our finding that optimal contracts for delegated R&D are options has a number of practical implications. Rewards for managers and employees through options can be applied to specialized research personnel. Additionally, option contracts for researchers that allow for entrepreneurial spin-offs efficiently promote invention within the firm. Financial contracts such as debt and convertible preferred securities are useful for financing independent inventors and startups that focus on R&D.

Delegation of R&D raises subtle and complex questions about experimental design and statistical inference. Although optimal statistical decisions are a highly desirable, the incentives of firms that manage or sponsor R&D may not be perfectly aligned with

those of researchers. Because it is costly and difficult to monitor scientific and technological research, it is necessary to rely on contractual incentives based on outcomes. Our analysis suggests that the study of contractual incentives for R&D can be extended address a wide range of problems encountered in conducting experiments, gathering data, and making statistical inferences.

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