

TABLE I
RELIABLE CONTROLLER DESIGN RESULTS

Outages Designed for	Actuator Outage	Sensor Outage
Controller	Σ_a	Σ_s
Design α	4.08	4.39
No Outage α_o	3.50	4.23
Controller Failure α_c	3.32	3.98

Similarly, by solving the ARE's in (42) and (43) in Corollaries 3.7, we have an output feedback controller Σ_s of the form (4) and (5) with

$$A_c = \begin{bmatrix} -23.9136 & 1.0316 & 1.0271 & 1.0169 \\ -21.1069 & 0 & 0 & 2.0000 \\ -53.4086 & 0 & -2.0000 & -3.0000 \\ 36.2765 & -2.1497 & 1.1114 & -1.5931 \end{bmatrix}$$

$$L = \begin{bmatrix} 21.9833 \\ 24.1069 \\ 52.4086 \\ -39.5770 \end{bmatrix}$$

$$K = [-1.3005 \quad -1.1497 \quad -0.8886 \quad -0.5931].$$

Both of these two controllers Σ_a and Σ_s can provide internal stability and guaranteed disturbance attenuation for the closed-loop system not only when both control channels are operational but also when any of these two control channels experiences an outage.

The design results are given in Table I. The two values of the closed-loop disturbance attenuation are computed for each of the two controllers. Namely:

α_o : when there is no outage;

α_c : when there is a controller failure.

The "Design α " in Table I is the value of α used in solving the two corresponding design equations.

The actual achievable values of α (namely α_o and α_c) for the closed-loop system are all less than and quite close to the value of α for which the design equations have solutions and the conditions in the Corollaries are satisfied. This indicates that degree of conservativeness in the design method is not very severe.

From Table I, it would seem that the actual system performance would be better when some controller failure occurs, contrary to the desirable property of graceful degradation of performance. This is so, however, because a controller failure (modeled as an actuator outage and/or sensor outage) effectively eliminates one column and/or one row of the closed-loop transfer function matrix. This is similar to an observation made in [3].

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Design of Performance Robustness for Uncertain Linear Systems with State and Control Delays

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Abstract—The linear systems considered in this paper are subject to uncertain perturbations of norm-bounded time-varying parameters and multiple time delays in system state and control. The time delays are uncertain, independent of each other, and allowed to be time-varying. The integral quadratic cost criterion is employed to measure system performance. Using solutions of Lyapunov and Riccati equations, a linear state feedback control law is proposed to stabilize the perturbed system and to guarantee an upper bound of system performance, which is applicable to arbitrary time delays.

Index Terms—Algebraic Riccati equation, delay effects, linear-quadratic control, Lyapunov matrix equation, robustness, stability, uncertain systems.

I. INTRODUCTION

The problem of stabilizing uncertain systems with time-varying and bounded parametric uncertainties has attracted a considerable amount of interest in recent years. Among different approaches, Lyapunov and Riccati equation descriptions of uncertainty are important ways to deal with the problem. Based on linear optimal control theory with quadratic cost criteria and using Lyapunov stability theory, many methods have been proposed for finding a linear state feedback law

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to stabilize uncertain systems. For example, sufficient conditions for quadratic stability of linear systems with time-varying structured uncertainties are formulated in [1] and [2]. For a class of norm-bounded time-varying uncertainties, the necessary and sufficient conditions are given in [3]–[5], involving both stability and performance robustness. A guaranteed cost control is proposed in [6]. In [7] a numerical method is given for constructing an optimal guaranteed cost control which minimizes the upper bound of the worst case performance for linear systems with structured uncertainties. In [8] an optimal guaranteed cost control is formulated for linear systems with a class of norm-bounded uncertainties. The results of [1]–[5] have been extended to time-delay systems [9]–[12]. However, the work in [9] focuses on the use of rank one decompositions of uncertainties. The work in [10] and [11] is limited to single time-delay systems. The matching condition for the time-delay system model used in [12] is restrictive, and the upper bound functions on uncertainty may be, in general, too conservative. In [14] a guaranteed cost control is proposed for linear systems with norm-bounded parametric uncertainty and with a single time delay in the states.

Most of the work that has appeared in the literature concerning time-delay systems, such as the work presented in [9]–[12], contributed to stability robustness. In this paper a guaranteed cost control is proposed for linear uncertain systems which are subject to both norm-bounded, time-varying parametric uncertainty and multiple arbitrary time delays in system state and control. A linear feedback control law is given to guarantee not only quadratic stability, but also performance robustness in terms of a linear integral-quadratic cost criterion. Since a general uncertain time-delay system model is adopted in this paper, some interesting problems can be treated as special cases. The following notation is used throughout the paper: A' denotes the transpose of matrix A ; for a real symmetric matrix we write $P > 0$ (≥ 0) if P is positive definite (positive semidefinite); a norm $\|A\|$ stands for the spectral norm of matrix A , and I is the identity matrix of any dimension.

II. PROBLEM STATEMENT AND PRELIMINARIES

Consider the class of uncertain linear time-delay systems described by state-space equations of the form

$$\begin{cases} \dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]u(t) \\ \quad + Ex(t - \tau_1(t)) + \Delta E(t)x(t - \tau_2(t)) \\ \quad + Du(t - \tau_3(t)) + \Delta D(t)u(t - \tau_4(t)), & t > 0 \\ x(t) = \phi(t), & t \leq 0 \end{cases} \quad (1)$$

where $x(t) \in \mathcal{R}^n$ is the state and $u(t) \in \mathcal{R}^m$ is the control. $A, E \in \mathcal{R}^{n \times n}$ and $B, D \in \mathcal{R}^{n \times m}$ are known real matrices. The real-valued functions $\Delta A(t), \Delta E(t) \in \mathcal{R}^{n \times n}$, and $\Delta B(t), \Delta D(t) \in \mathcal{R}^{n \times m}$ represent time-varying uncertainties. $\tau_i(t)$ ($i = 1, \dots, 4$) are arbitrary differentiable functions satisfying

$$0 \leq \tau_i(t) \leq d_i < \infty, \quad \dot{\tau}_i(t) \leq \xi_i < 1, \quad i = 1, \dots, 4 \quad (2)$$

for all $t > 0$. For simplicity and clarity of expression, henceforth the time-variation t for $\Delta A(t), \Delta B(t), \Delta D(t), \Delta E(t)$, and $\tau_i(t)$ ($i = 1, \dots, 4$) is dropped. ϕ is a continuous vector-valued function defined in $[-d, 0]$ with $d = \max\{d_i, i = 1, \dots, 4\}$. The norm-bounded time-varying parametric uncertainties are described by

$$\begin{cases} \Delta A(t) = M_1 F_1(t) N_1, & \Delta B(t) = M_2 F_2(t) N_2 \\ \Delta E(t) = M_3 F_3(t) N_3, & \Delta D(t) = M_4 F_4(t) N_4 \end{cases} \quad (3)$$

where $M_i \in \mathcal{R}^{n \times s_i}$ ($i = 1, \dots, 4$), $N_i \in \mathcal{R}^{q_i \times n}$ ($i = 1, 3$), or $N_i \in \mathcal{R}^{q_i \times m}$ ($i = 2, 4$) are known constant matrices, and

$F_i(t) \in \mathcal{R}^{s_i \times q_i}$ ($i = 1, \dots, 4$) are unknown matrix functions satisfying

$$F_i'(t)F_i(t) \leq I \quad \forall i = 1, \dots, 4, \forall t > 0 \quad (4)$$

with the elements of $F_i(\cdot)$ ($i = 1, \dots, 4$) being Lebesgue measurable.

Definition 1: The uncertain time-delay system (1)–(4) is *quadratically stabilizable independent of delay* if there exists a static linear feedback control $u(t) = Kx(t)$, a matrix $P > 0$, a constant $\theta > 0$, and positive semidefinite matrices $R_i \in \mathcal{R}^{n \times n}$ ($i = 1, \dots, 4$) such that

$$\begin{aligned} V(x, t) = & x'(t)Px(t) + \int_{t-\tau_1}^t x'(s)R_1x(s) ds \\ & + \int_{t-\tau_2}^t x'(s)R_2x(s) ds + \int_{t-\tau_3}^t x'(s)R_3x(s) ds \\ & + \int_{t-\tau_4}^t x'(s)R_4x(s) ds \end{aligned} \quad (5)$$

satisfies

$$\frac{dV(x, t)}{dt} \leq -\theta \|x(t)\|^2 \quad (6)$$

along solutions $x(t)$ of [(1)–(4), $u = Kx$] for any admissible parametric uncertainties [13, p. 87]. The resulting closed-loop system is called quadratically stable. If such a function exists, then V is called a quadratic Lyapunov function for the uncertain system (1)–(4) and $u = Kx$ is a stabilizing control law.

III. MAIN RESULTS

Theorem 1: Let the uncertain time-delay system be described by (1)–(4). Suppose that there exist positive definite matrices $Q \in \mathcal{R}^{n \times n}$, $R \in \mathcal{R}^{m \times m}$ and constants $\mu_i > 0$, $i = 1, \dots, 6$ such that the following Riccati equation:

$$PA + A'P - PB\tilde{R}^{-1}B'P + \tilde{Q} + PHP = 0 \quad (7)$$

has a solution $P > 0$, where in (7)

$$\tilde{Q} = Q + \frac{N_1'N_1}{\mu_1} + \frac{I_n}{\mu_3(1-\xi_1)} + \frac{N_3'N_3}{\mu_4(1-\xi_2)} \quad (8)$$

$$\tilde{R} = R + \frac{N_2'N_2}{\mu_2} + \frac{I_m}{\mu_5(1-\xi_3)} + \frac{N_4'N_4}{\mu_6(1-\xi_4)} \quad (9)$$

$$\begin{aligned} H = & \mu_1 M_1 M_1' + \mu_2 M_2 M_2' + \mu_3 E E' + \mu_4 M_3 M_3' \\ & + \mu_5 D D' + \mu_6 M_4 M_4'. \end{aligned} \quad (10)$$

Then:

- 1) (1)–(4) is quadratically stabilizable independent of delay.
- 2) A guaranteed cost control of linear state feedback $u = Kx$ is given by

$$K = -\tilde{R}^{-1}B'P \quad (11)$$

which guarantees that the system performance in terms of the linear integral-quadratic cost criterion

$$J = \int_0^\infty x'(t)Qx(t) + u'(t)Ru(t) dt \quad (12)$$

is upper bounded by

$$\begin{aligned} J \leq & \phi'(0)P\phi(0) + \frac{1}{\mu_3(1-\xi_1)} \int_{-d_1}^0 \phi'(t)\phi(t) dt \\ & + \frac{1}{\mu_4(1-\xi_2)} \int_{-d_2}^0 \phi'(t)N_3'N_3\phi(t) dt \\ & + \frac{1}{\mu_5(1-\xi_3)} \int_{-d_3}^0 \phi'(t)K'K\phi(t) dt \\ & + \frac{1}{\mu_6(1-\xi_4)} \int_{-d_4}^0 \phi'(t)K'N_4'N_4K\phi(t) dt. \end{aligned} \quad (13)$$

Proof: Let $Q > 0$, $R > 0$ and constants $\mu_i > 0$, $i = 1, \dots, 6$ be given such that (7) has a positive definite solution P . Define K' as in (11), let $R_i \in \mathcal{R}^{n \times n}$ ($i = 1, \dots, 4$) be given by

$$R_1 = \frac{1}{\mu_3(1-\xi_1)} I_n, \quad R_2 = \frac{1}{\mu_4(1-\xi_2)} N_3' N_3 \quad (14)$$

$$R_3 = \frac{1}{\mu_5(1-\xi_3)} K' K, \quad R_4 = \frac{1}{\mu_6(1-\xi_4)} K' N_4' N_4 K \quad (15)$$

and let

$$\theta = \lambda_{\min}(Q + K' R K). \quad (16)$$

Then $R_i > 0$, $i = 1, \dots, 4$, $\theta > 0$ (as $Q > 0$) and \tilde{R} is invertible (as $R > 0$). Now consider the derivative of $V(x, t)$ as defined in (5). With (2) it follows that $-(1 - \dot{\tau}_i) \leq -(1 - \xi_i) < 0$, $i = 1, 4$. Consequently

$$\begin{aligned} \dot{V}(x, t) \leq & x'(t) \left(PA + A'P + PBK + K'B'P + \sum_{i=1}^4 R_i \right) x(t) \\ & + 2x'(t)P\Delta Ax(t) + 2x'(t)P\Delta BKx(t) \\ & + 2x'(t)PEx(t - \tau_1) + 2x'(t)P\Delta Ex(t - \tau_2) \\ & + 2x'(t)PDKx(t - \tau_3) + 2x'(t)P\Delta DKx(t - \tau_4) \\ & - \sum_{i=1}^4 (1 - \xi_i)x'(t - \tau_i)R_i x(t - \tau_i). \end{aligned} \quad (17)$$

Since for any $\alpha > 0$ and real vectors y and z we have

$$\left[\sqrt{\alpha}y - \frac{1}{\sqrt{\alpha}}z \right]' \left[\sqrt{\alpha}y - \frac{1}{\sqrt{\alpha}}z \right] \geq 0$$

it follows that

$$2y'z \leq \alpha y'y + \frac{1}{\alpha} z'z. \quad (18)$$

Applying (3), (4), and (18) to the terms on the right-hand side of (17) with appropriate choices for α , $y(t)$, and $z(t)$, the following inequalities are obtained:

$$\begin{aligned} & 2x'(t)P\Delta Ax(t) \\ & \leq x'(t) \left[\mu_1 P M_1 M_1' P + \frac{1}{\mu_1} N_1' N_1 \right] x(t) \end{aligned} \quad (19)$$

$$\begin{aligned} & 2x'(t)P\Delta BKx(t) \\ & \leq x'(t) \left[\mu_2 P M_2 M_2' P + \frac{1}{\mu_2} K' N_2' N_2 K \right] x(t) \end{aligned} \quad (20)$$

$$\begin{aligned} & 2x'(t)PEx(t - \tau_1) \\ & \leq \mu_3 x'(t)PEE'Px(t) + \frac{1}{\mu_3} x'(t - \tau_1)x(t - \tau_1) \end{aligned} \quad (21)$$

$$\begin{aligned} & 2x'(t)P\Delta Ex(t - \tau_2) \\ & \leq \mu_4 x'(t)P M_3 M_3' P x(t) + \frac{1}{\mu_4} x'(t - \tau_2)N_3' N_3 x(t - \tau_2) \end{aligned} \quad (22)$$

$$\begin{aligned} & 2x'(t)PDKx(t - \tau_3) \\ & \leq \mu_5 x'(t)PDD'Px(t) + \frac{1}{\mu_5} x'(t - \tau_3)K' K x(t - \tau_3) \end{aligned} \quad (23)$$

$$\begin{aligned} & 2x'(t)P\Delta DKx(t - \tau_4) \\ & \leq \mu_6 x'(t)P M_4 M_4' P x(t) \\ & \quad + \frac{1}{\mu_6} x'(t - \tau_4)K' N_4' N_4 K x(t - \tau_4). \end{aligned} \quad (24)$$

Taking (19)–(24) into consideration and using $\xi_i - 1 < 0$, (17) yields

$$\begin{aligned} \dot{V}(x, t) \leq & x'(t) \left[PA + A'P + PBK + K'B'P \right. \\ & + \left(Q + \frac{N_1' N_1}{\mu_1} + \frac{I_n}{\mu_3(1-\xi_1)} + \frac{N_3' N_3}{\mu_4(1-\xi_2)} \right) \\ & + K' \left(R + \frac{N_2' N_2}{\mu_2} + \frac{I}{\mu_5(1-\xi_3)} + \frac{N_4' N_4}{\mu_6(1-\xi_4)} \right) K \\ & + P(\mu_1 M_1 M_1' + \mu_2 M_2 M_2' + \mu_3 E E' + \mu_4 M_3 M_3' \\ & \left. + \mu_5 D D' + \mu_6 M_4 M_4') P - (Q + K' R K) \right] x(t) \end{aligned}$$

which can be rewritten as

$$\begin{aligned} \dot{V}(x, t) \leq & x'(t)(PA + A'P - PB\tilde{R}^{-1}B'P + \tilde{Q} + PHP)x(t) \\ & - x'(t)(Q + K' R K)x(t) + x'(t)(\tilde{R}^{-1}B'P + K)' \\ & \cdot \tilde{R}(\tilde{R}^{-1}B'P + K)x(t) \end{aligned} \quad (25)$$

where \tilde{Q} , \tilde{R} , and H are given by (8)–(10). With (7), the first term on the right-hand side of (25) becomes zero, and with (11) the third term equals zero. So, (25) yields

$$\dot{V}(x, t) \leq -x'(t)(Q + K' R K)x(t). \quad (26)$$

It yields

$$\dot{V}(x, t) \leq -\theta \|x(t)\|^2. \quad (27)$$

Therefore, the uncertain time-delay system described by (1)–(4) is quadratically stabilizable independent of delay. Secondly, we prove that the system performance is upper bounded by (13). From (26) follows:

$$\int_0^\infty \dot{V}(x, t) dt \leq - \int_0^\infty \{x'(t)Qx(t) + u'(t)Ru(t)\} dt \quad (28)$$

which yields that

$$\lim_{t \rightarrow \infty} V(x, t) - V(x, 0) \leq - \int_0^\infty x'(t)Qx(t) + u'(t)Ru(t) dt. \quad (29)$$

Since $V(x, t) > 0$ and $\dot{V}(x, t) \leq -\theta \|x(t)\|^2$ along solutions $x(\cdot)$ of (1)–(4) with $u = Kx$, it follows that $V(x, t) \rightarrow 0$ as $t \rightarrow \infty$. Consequently, (29) gives

$$\int_0^\infty x'(t)Qx(t) + u'(t)Ru(t) dt \leq V(x, 0). \quad (30)$$

With (2), (5), (14), and (15)

$$\begin{aligned} V(x, 0) = & \phi'(0)P\phi(0) + \frac{1}{\mu_3(1-\xi_1)} \int_{-\tau_1}^0 \phi'(t)\phi(t) dt \\ & + \frac{1}{\mu_4(1-\xi_2)} \int_{-\tau_2}^0 \phi'(t)N_3' N_3 \phi(t) dt \\ & + \frac{1}{\mu_5(1-\xi_3)} \int_{-\tau_3}^0 \phi'(t)K' K \phi(t) dt \\ & + \frac{1}{\mu_6(1-\xi_4)} \int_{-\tau_4}^0 \phi'(t)K' N_4' N_4 K \phi(t) dt \\ \leq & \phi'(0)P\phi(0) + \frac{1}{\mu_3(1-\xi_1)} \int_{-d_1}^0 \phi'(t)\phi(t) dt \\ & + \frac{1}{\mu_4(1-\xi_2)} \int_{-d_2}^0 \phi'(t)N_3' N_3 \phi(t) dt \\ & + \frac{1}{\mu_5(1-\xi_3)} \int_{-d_3}^0 \phi'(t)K' K \phi(t) dt \\ & + \frac{1}{\mu_6(1-\xi_4)} \int_{-d_4}^0 \phi'(t)K' N_4' N_4 K \phi(t) dt \end{aligned} \quad (31)$$

which is (13). This completes the proof. \square

It can be seen from Theorem 1 that quadratic stability is independent of the size of time delays and that the integral-quadratic measure (12) of system performance is related with the size of time delays as well as with the system initial function $\phi(t)$, $t \in [-d, 0]$. For any given $\phi(t)$, $t \in [-d, 0]$ the system performance can be evaluated quantitatively using (13). However, due to the existence of parametric uncertainties and uncertain time delays, in general $\phi(t)$, $t \in [-d, 0]$ cannot be known precisely. Some simple upper bound functions on system performance are obtained from (13) with different assumptions on $\phi(t)$, $t \in [-d, 0]$. For example, suppose that

- 1) if $d_i = 0$, $i = 1, \dots, 4$, then (13) becomes

$$J \leq \phi'(0)P\phi(0) \quad (32)$$

- 2) if $\phi(t) = c$, for all $t \in [-d, 0]$ then (13) yields

$$J \leq c' \left\{ P + \frac{d_1}{\mu_3(1-\xi_1)} + \frac{d_2 N_3' N_3}{\mu_4(1-\xi_2)} + \frac{d_3 K' K}{\mu_5(1-\xi_3)} + \frac{d_4 K' N_4' N_4 K}{\mu_6(1-\xi_4)} \right\} c \quad (33)$$

- 3) if $d_1 = d_2 = d_3 = d_4 = d$, $\|\phi(t)\| \leq \gamma$ for all $t \in [-d, 0]$ then (13) reads

$$J \leq \gamma^2 \left[\|P\| + d \left\| \frac{1}{\mu_3(1-\xi_1)} + \frac{N_3' N_3}{\mu_4(1-\xi_2)} + \frac{K' K}{\mu_5(1-\xi_3)} + \frac{K' N_4' N_4 K}{\mu_6(1-\xi_4)} \right\| \right] \quad (34)$$

- 4) if $d_1 = d_2 = d_3 = d_4 = d$, $\phi(0)$ and $\psi = (1/d) \int_{-d}^0 \phi'(t)\phi(t) dt$ are available then (13) yields

$$J \leq \phi'(0)P\phi(0) + \psi d \left\| \frac{1}{\mu_3(1-\xi_1)} + \frac{N_3' N_3}{\mu_4(1-\xi_2)} + \frac{K' K}{\mu_5(1-\xi_3)} + \frac{K' N_4' N_4 K}{\mu_6(1-\xi_4)} \right\| \quad (35)$$

- 5) with the upper bound $\|\phi(t)\| \leq \gamma$, for all $t \in [-d, 0]$, (13) gives

$$J \leq \gamma^2 \left\{ \|P\| + \frac{1}{\mu_3(1-\xi_1)} + \frac{\|N_3\|^2}{\mu_4(1-\xi_2)} + \frac{\|K\|^2}{\mu_5(1-\xi_3)} + \frac{\|N_4 K\|^2}{\mu_6(1-\xi_4)} \right\} \quad (36)$$

Obviously, the matrices P and K satisfy (7) and (11) and the upper bound functions of (13) and (32)–(36) are dependent on the choices of the design parameters μ_i ($i = 1, \dots, 6$). As $J \geq 0$, the minimum of upper bound functions on J in terms of optimal μ_i ($i = 1, \dots, 6$) exists. Therefore, in each specific application a feasible numerical optimization program can be employed to get the optimal μ_i ($i = 1, \dots, 6$) which minimize the upper bound of J given by (13) or minimize a suitable upper bound function given by (32)–(36) for any given pair of Q and R matrices.

Remark 1: For simplicity, the design parameters $\mu_i > 0$ ($i = 1, \dots, 6$) could be reduced to a single parameter $\mu > 0$. However, note that if the uncertain time-delay system is quadratically stabilizable for a set $\mu_i > 0$ ($i = 1, \dots, 6$), there may not exist a suitable solution $P > 0$ for Riccati equation (7) with (8)–(10) in which one common μ is used to replace μ_i ($i = 1, \dots, 6$).

Remark 2: A numerical optimization program could be utilized to find appropriate values for μ_i , $i = 1, \dots, 6$ to reduce the performance upper bound.

Remark 3: The main result is valid for nonnegative bounded time delays whose derivatives satisfy an upper bound. It is interesting to note that the rate of changes in the time delays is not bounded from below.

IV. CONCLUSION

A guaranteed cost control law with linear state feedback is formulated to stabilize uncertain linear continuous-time systems which are subject to norm-bounded time-varying parametric uncertainties and time-varying delays in the state and control. The gain of the controller is obtained from the solution of an algebraic Riccati equation. The system performance in terms of integral-quadratic cost criteria is guaranteed to be less than an upper bound.

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