Distributed Power Control for Interference-Limited Cooperative Relay Networks

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Abstract—In this paper, a distributed power control algorithm is proposed for wireless relay networks in interference-limited environments. The objective is to minimize the total transmission power while satisfying the signal-to-interference-plus-noise ratio (SINR) requirements. Two forwarding techniques, i.e., decode-and-forward (DF) and amplify-and-forward (AF), are considered. The proposed algorithm only requires locally measured SINR on the relay nodes (RNs) and the destination nodes (DNs), based on which each cooperation unit (defined as one source node (SN) and DN pair with the RN associated to it) iteratively updates the transmission power of the SN and the RN by solving a local optimization problem. We prove that the convergence is guaranteed when the parameters adopted in the algorithm are sufficiently large, and then a parameter adjusting method is also designed. Simulation results indicate that the proposed algorithm converges fast and leads to only 7% more power consumption than the optimal power allocation in the considered scenarios. It is also shown that even in interference-limited environments, relaying can still improve system performance substantially in terms of outage and power consumption. 1

I. INTRODUCTION

Cooperative transmission has been receiving much attention as a promising technology for the future wireless networks [1] [2]. Since relay nodes (RNs) can provide additional spatial diversity, the advantage of relaying is twofold: reducing signal transmission power and improving data rate. Extensive researches have been made on resource allocation for cooperative relay communications, such as power control to guarantee outage performance [4], relay node selection combined with power control to enhance capacity [5], etc. All these efforts have helped understanding the benefits of cooperative relays and inspired using relaying in practical systems, for instance, multi-hop cellular networks [6].

However, most of the existing work is based on isolated relay transmission, i.e., only one source node (SN) and destination node (DN) pair is presented, or transmissions are orthogonal, where interference is not taken into account. On the other hand, to adopt relaying in practical systems, such as wireless cellular networks or wireless sensor networks, interference is an inevitable issue to deal with. In these systems, the interference exists not only on SN to DN links, but may also exist on SN to RN links and RN to DN links. In [8], the authors show that, the performance of relaying in large-scale wireless networks is penalized by the elevated level of interference induced by the RNs. Hence, a nature problem has arisen: How much gain can we get from relaying in interference-limited environments [3], and how can we design resource allocation schemes to achieve the gain?

One way to investigate this problem is to consider power control. As a basic way to facilitate spatial reuse, power control plays a fundamental role in system design and performance evaluation. There are few papers addressing power control for interference-limited relay networks, and most of them provide centralized solutions. In [8], a channel allocation mechanism is proposed to mitigate the impact of interference, where the power control is in fact binary (on-off) on the allocated channels. The authors in [6] design power control and RN selection schemes for multihop cellular networks considering inter-cell interference, and based on the assumption that the second hop is sufficiently good, the power control is similar to the ones for single-hop scenario described in [10]. The most related work to ours is Ref. [7], which studies power allocation problem for the uplink in cooperative CDMA networks. Interference induced by all the nodes in the system (including RNs and SNs) is considered, and a type of complementary geometric programming (GP) method [9] is used to solve the power optimization problem, which requires global channel state information (CSI) thus can only be used in a centralized way. Moreover, when the number of simultaneous transmissions increases, the GP method will require very long computational time. The difficulty of collecting global CSI and the high realization complexity limit the usage of centralized solutions in practical systems, such as cellular systems when multiple Base Stations (BSs) are considered. Therefore, effective distributed power control using local measurements is of more practical significance.

In this paper, we propose a distributed yet effective power control algorithm for interference-limited cooperative relay networks. In particular, the algorithm aims to minimize the total transmission power subject to the signal-to-interference-plus-noise ratio (SINR) requirements. Each cooperation unit (defined as one SN-DN pair with the RN associated to it) utilizes the locally measured SINR to solve a simple optimization problem, of which the solution can be written in close-form. The algorithm requires low computational over-
head, and together with the distributed feature, it can be easily adopted in practical large-scale wireless networks. The remaining of the paper is organized as follows. The system model is introduced in Section II. In Section III, the proposed algorithm is described and its convergence property is studied, based on which the parameter adjustment method to guarantee convergence is also detailed. Numerical results are provided and the performance of the proposed algorithm is evaluated in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a 2-hop relay network. As shown in Fig.1, there are $N$ SN-DN pairs, and each pair is assisted by one RN to form a cooperation unit (CU). Two orthogonal channels (time slots or frequency bands, etc.) are available: the SNs transmit on channel 1 and the RNs transmit on channel 2 (i.e., out-of-band relay). The RNs can receive on channel 1, while the DNs can combine the signals received on both two channels. Although the transmissions from the SNs and RNs are separated on the two channels, the simultaneous transmissions on channel 1 from different SNs will cause interference to each other, and so do the RNs on channel 2. On channel 1, we denote $G_{j,d_i}$ as the channel gain from SN $j$ to DN $i$, and $G_{j,r_i}$ as the channel gain from SN $j$ to RN $i$. On channel 2 we denote $G_{r_j,d_i}$ as the channel gain from RN $j$ to DN $i$. For simplicity, we omit the channel indices on the notations of the channel gains.

The received SINR at DN $i$ on channel 1 is given by

$$\gamma_{sd,i} = \frac{G_{i,d_i} P_{s_i}}{\sum_{j=1}^{N} G_{j,d_i} P_{s_j} - G_{i,d_i} P_{s_i} + \eta},$$

(1)

where $P_{s_i}$ is the transmission power of SN $i$, and $\eta$ is the power of the background additive white Gaussian noise. Similarly, the received SINR at RN $i$ is

$$\gamma_{sr,i} = \frac{G_{i,r_i} P_{s_i}}{\sum_{j=1}^{N} G_{j,r_i} P_{s_j} - G_{i,r_i} P_{s_i} + \eta}.$$  

(2)

We further denote $P_{r_i}$ as the transmission power of RN $i$, which will interfere with the transmissions of other RNs. The received SINR of the forwarded signal from RN $i$ at DN $i$ on channel 2 is given by

$$\gamma_{rd,i} = \frac{G_{r_i,d_i} P_{r_i}}{\sum_{j=1}^{N} G_{r_j,d_i} P_{r_j} - G_{r_i,d_i} P_{r_i} + \eta}.$$  

(3)

At the DNs, maximum ratio combining (MRC) is used to combine the signals received from the two channels. The equivalent SINR after MRC is the sum of the direct link SINR and the relay link SINR. For decode-and-forward (DF) relaying, the equivalent SINR is

$$\gamma_{df,i} = \gamma_{sd,i} + \min\{\gamma_{sr,i}, \gamma_{rd,i}\},$$

(4)

where the second term means that the quality of the relay link is constrained by the quality of the SN-RN and RN-DN links.

For amplify-and-forward (AF) relaying, the RN amplifies the received signal together with interference and noise and forwards to the DN. The equivalent relay link SINR in terms of the transmitted signal from the SN is given by $\gamma_{af,i} = \gamma_{sr,i} + \gamma_{rd,i} + 1$. Hence, the equivalent SINR after combining is

$$\gamma_{af,i} = \gamma_{sd,i} + \gamma_{sr,i} + \gamma_{rd,i} + 1.$$  

(5)

In order to guarantee successful transmission, the combined SINR should be larger than a threshold: $\gamma^*$. There is also a maximum transmission power limit $P_{max}$ for each SN/RN. Here the power control problem can be described as to minimize the total transmission power on the SNs and RNs subject to the combined SINR constraints. For DF relaying, in order for the RN to decode the signal from the SN successfully, we also have $\gamma_{sr,i} \geq \gamma^*$. Then the power optimization problem can be formulated as follows for DF relaying:

```
\text{minimize} \quad \sum_{i=1}^{N} (P_{s_i} + P_{r_i})
\text{subject to} \quad \gamma_{sd,i} + \gamma_{rd,i} \geq \gamma^*, \quad \forall i,
\max\{\gamma_{sd,i}, \gamma_{sr,i}\} \geq \gamma^*, \quad \forall i,
0 \leq P_{s_i} \leq P_{max}, \quad \forall i,
0 \leq P_{r_i} \leq P_{max}, \quad \forall i,
```

where the the second constraint indicates that if the direct link satisfies the SINR requirement, the RN does not need to provide help. Otherwise, $\gamma_{sr,i} \geq \gamma^*$, so $\gamma_{rd,i}$ can simply be no larger than $\gamma_{sr,i}$, then from (4), we get the first constraint.

For AF relaying, the optimization problem can be formulated as:

```
\text{minimize} \quad \sum_{i=1}^{N} (P_{s_i} + P_{r_i})
\text{subject to} \quad \gamma_{sd,i} + \gamma_{sr,i} + \gamma_{rd,i} \geq \gamma^*, \quad \forall i,
0 \leq P_{s_i} \leq P_{max}, \quad \forall i,
0 \leq P_{r_i} \leq P_{max}, \quad \forall i,
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(7)

Due to the existence of the first constraint in (6) and (7), it can been seen that they are not convex optimization problems [7], therefore hard to be efficiently solved. In the next section, a distributed algorithm to solve the above power control problems is described.
III. PROPOSED DISTRIBUTED ALGORITHM

In this section, we provide a distributed power control algorithm for both DF and AF relaying. In our work, **distributed means** that signaling exists only between the RN, DN and the SN that belongs to one CU. No signaling is exchanged between different CUs. This assumption is reasonable because the DN always needs to feedback the SINR information to its RN and SN to accomplish power control (for example, the uplink power control in cellular networks), meanwhile the signaling can make use of the feedback channel. We first make the following notations

\[
\begin{align*}
    a_i &= \frac{\sum_{j \neq i} G_{i,j} s_j t_j P_{j,i} - G_{i,i} s_i t_i P_{i,i}}{G_{i,i}}, \\
    b_i &= \frac{\sum_{j \neq i} G_{i,j} s_j t_j P_{j,i} - G_{i,i} s_i t_i P_{i,i}}{G_{i,i}}, \\
    c_i &= \frac{\sum_{j \neq i} G_{i,j} s_j t_j P_{j,i} - G_{i,i} s_i t_i P_{i,i}}{G_{i,i}}.
\end{align*}
\]

(8)

Since \(\gamma_{sd,i}\), \(\gamma_{sr,i}\) and \(\gamma_{rd,i}\) can be locally measured by RN \(i\)/DN \(i\), all \(a_i\), \(b_i\) and \(c_i\) can be acquired locally by each CU \(i\). We further introduce a power control weight vector \(\omega = \{\omega_i\}\), which actually represents the different impacts on the system power consumption from \(P_s\) and \(P_r\). The proposed iterative distributed power control algorithm can be described in a slot (indexed by \(t\)) by slot manner:

**Step 0** Set all \(P_{s,i}^{(0)}\) and \(P_{r,i}^{(0)}\) with an initial value, eg. zero, and set \(t = 1\);

**Step 1** Each CU \(i\) solves a local optimization problem based on \(a_i^{(t-1)}, b_i^{(t-1)}\) and \(c_i^{(t-1)}\) measured at the end of the last time slot

\[
\begin{align*}
    \min_{P_s} & \omega_i P_s + P_{r,i} \\
    \text{s.t.} & \quad 0 \leq P_s \leq P_{max}, \\
    & \quad 0 \leq P_{r,i} \leq P_{max}, \\
    (DF) & \quad a_t^{(t-1)} P_s + b_t^{(t-1)} P_{r,i} \geq \gamma_i^t, \\
    (AF) & \quad a_t^{(t-1)} P_s + b_t^{(t-1)} P_{r,i} + c_t^{(t-1)} P_{r,i} \geq \gamma_i^t.
\end{align*}
\]

(9)

Notice that for DF relaying, the SINR constraint should also include the second constraint in (6):

\[
\max\{a_t^{(t-1)} P_s, b_t^{(t-1)} P_{r,i}\} \geq \gamma_i^t.
\]

Suppose the solution to (9) is \(P_s^{(t)}\) and \(P_{r,i}^{(t)}\), then \(\omega\) is also updated;

**Step 2** SN \(i\) transmits with \(P_s^{(t)}\) and RN \(i\) transmits with \(P_{r,i}^{(t)}\). Increase \(t\) by 1 and go back to Step 1 until converges. The convergence criteria is \(\sum_t |(P_s^{(t)} - P_s^{(t-1)})| + |(P_{r,i}^{(t)} - P_{r,i}^{(t-1)})| < \epsilon\), where \(\epsilon\) is the error tolerance for exit condition.

Next, the close-form solution to (9) will be detailed for DF and AF relaying respectively, and the method of tuning \(\omega_i\) will be provided. We will replace \(a_t^{(t-1)}, b_t^{(t-1)}\) and \(c_t^{(t-1)}\) by \(a_i\), \(b_i\) and \(c_i\) respectively without ambiguity.

A. Detailed Algorithm for DF Relaying

For DF relaying, the local optimization problem (9) is a simple linear programming problem. We change the SINR constraint in (9) to an equality, and the optimization problem remains the same. Then \(P_r\) can be represented by a function of \(P_s\) (if \(b_i P_s \geq \gamma_i^t\) holds):

\[
P_r = \max\{0, (\gamma_i^t - a_i P_s)/c_i\}.
\]

(10)

Hence (9) can be easily solved as:

\[
\begin{align*}
P_{s,i}^{(t)} &= \min\{P_{max}, \frac{\gamma_i^t}{a_i} \}, \\
P_{r,i}^{(t)} &= 0; \quad \text{if} \quad a_i > b_i, \\
P_{s,i}^{(t)} &= \min\{P_{max}, \frac{\gamma_i^t - a_i P_{r,i}^{(t)}}{b_i} \}, \quad \text{if} \quad a_i \leq b_i \text{ and } \omega_i c_i \geq a_i, \\
P_{r,i}^{(t)} &= \min\{P_{max}, \frac{\gamma_i^t - a_i P_{r,i}^{(t)}}{c_i} \}, \quad \text{if} \quad a_i \leq b_i \text{ and } \omega_i c_i < a_i.
\end{align*}
\]

(11)

In the above explanation, under the second condition, we actually get \(P_{r,i}^{(t)}\) first then decide \(P_{s,i}^{(t)}\), and vice versa under the third condition. Notice that (11) includes the power allocation results when the local problem is infeasible, which leads to \(P_{s,i}^{(t)}=(P_{max},0)\) or \((P_{max}, P_{max})\), while the SINR requirement is not satisfied in these circumstances.

B. Detailed Algorithm for AF Relaying

For AF relaying, we also change the SINR constraint in (9) to an equality, and the optimization problem remains the same. Then \(P_{r,i}\) can be represented by a function of \(P_{s,i}\) (if \(P_{s,i} \geq \gamma_i^t/(a_i + b_i)\) holds):

\[
P_{r,i} = \max\{0, f(P_{s,i})\},
\]

(12)

where we define \(f(x)\) as

\[
f(x) = \frac{1 + b_i x}{c_i (b_i x + a_i x - \gamma_i^t)}, \quad \text{for} \quad x > \frac{\gamma_i^t}{a_i + b_i}.
\]

(13)

The relation between \(P_{s,i}\) and \(P_{r,i}\) is shown in Fig. 2. Furthermore, we can easily verify that \(f''(x) > 0\) in its domain, thus the local optimization problem is now a convex optimization problem. If we define \(P_{s,i}^*\) as the solution to the formula

\[
f'(x) + \omega_i = 0,
\]

(14)
which leads to (as shown in Fig.2)

\[ P_{s_i}^* = \frac{1}{a_i + b_i} \left[ \frac{\gamma_i^s (a_i b_i + b_i^2 + \gamma_i^s b_i^2)}{c_i \omega_i (a_i + b_i) - a_i b_i} + \gamma_i^s \right]. \]  (15)

Note that \( P_{s_i}^* \) may not exist when \( c_i \omega_i (a_i + b_i) - a_i b_i \leq 0 \). Then the solution to (9) without maximum power constraint is given by

\[ P_{s_i}(t) = \begin{cases} \frac{\gamma_i^s}{a_i} & \text{if } P_{s_i}^* \geq \frac{\gamma_i^s}{a_i} \text{ or } P_{s_i}^* \text{ does not exist} \\ P_{s_i}^* & \text{if } P_{s_i}^* < \frac{\gamma_i^s}{a_i} \end{cases} \]

\[ P_{r_i}(t) = f(P_{s_i}(t)). \]  (16)

If \( P_{\text{max}} \leq \gamma_i^s / (a_i + b_i) \), then the local problem is infeasible; otherwise, if any one of \( P_{s_i}(t) \) and \( P_{r_i}(t) \) violates the \( P_{\text{max}} \) constraint, it will be set as \( P_{\text{max}} \), and the other one will be decided by (12) or its inverse. Moreover, if the result violates the power constraint again, we will set \( (P_{s_i}(t), P_{r_i}(t)) = (P_{\text{max}}, P_{\text{max}}) \), but the SINR requirement is not satisfied.

\section*{C. Adjusting Parameter \( \omega_i \)}

As mentioned previously, weight parameter \( \omega_i \) actually represents the different impacts on the system power consumption from \( P_{s_i} \) and \( P_{r_i} \). It has two functions:

1) To improve the performance of the proposed algorithm. We note that the proposed algorithm is sub-optimal, however, by balancing the impacts from \( P_{s_i} \) and \( P_{r_i} \) on the system power consumption, the result can be very close to the global optimum. This can be done by optimizing \( \omega_i \) for each CU.

2) To guarantee convergence. For an arbitrary \( \omega \), the proposed algorithm is not guaranteed to converge. However, by appropriately tuning \( \omega \), we can actually keep the algorithm converge, which is described in the following proposition:

**Proposition 1:** If the power control problem is feasible, there exists an \( \omega^* \geq 0 \), for any \( \omega \geq \omega^* \), the proposed algorithm with parameter \( \omega \) will converge to an equilibrium.

See Appendix for the proof.

The above propositions indicate that, when the proposed algorithm does not converge, by increasing the elements of \( \omega \), the algorithm will finally converge. Thus, the dynamic adaptation of \( \omega \) should detect oscillations and appropriately increase each \( \omega_i \) by evaluating the impact on the system transmission power from \( P_{s_i} \). The optimal choice of \( \omega_i \) for each CU depends on the interference structure of the system, thus requires global channel information. In fact, determining optimal \( \omega \) is of the same difficulty with getting the global optimum of the original optimization problem (6) and (7), which is very hard. Hence we adopt a heuristic way to adapt \( \omega_i \). Initially, for each CU \( i \), we set \( \omega_i(0) \) as the initial value. Then during each iteration, \( \omega_i \) can be updated as

\[ \omega_i(t+1) = \omega_i(t) + \alpha(t) |P_{s_i}^*(t) - P_{s_i}^*(t-1)|, \]  (17)

where \( \alpha(t) > 0 \) is a stepsize. We set \( \alpha(t) = \alpha_0 / \sqrt{t - t_0} \), for some constant \( \alpha_0 > 0 \) and positive integer \( t_0 \), and if \( t \leq t_0 \), \( \alpha(t) = 0 \). The intuition behind (17) is described as follows:

During the iterations, the larger the variance of \( P_{s_i} \) is, the larger impact of it on the total system transmission power, hence \( \omega_i \) is increased proportionally to the variance of \( P_{s_i} \) between the current time slot and the last time slot.

In the proof of proposition 1, the choice of \( \omega_i^* \) is in a most conservative way. Actually, \( \omega_i^* \) depends on the proportion of the channel gain divided by the interference level on the SN-DN link to that on the RN-DN link. The proportion is generally not large and can even be less than one, because most of the time the channel and interference condition on the RN-DN link is better in real systems due to two reasons: 1) More reliable radio technique is used in these links, and RNs are generally closer to the DNs/BSs; 2) RNs are not fully utilized as will be shown in the numerical results, hence the interference level on the RN-DN links is relatively low.

\section*{IV. NUMERICAL RESULTS AND DISCUSSIONS}

In the simulations, wireless links are uniformly distributed over a square field with dimension \( D \times D \). The path gain \( G_{i,j} \) between any two nodes (SN, RN or DN) is modeled as

\[ G_{i,j} = \frac{1}{d_{i,j}^\gamma}, \]

where \( d_{i,j} \) is the distance between node \( i \) and node \( j \). Other simulation parameters are listed in Table I. In each run, we first randomly generate \( N \) SNs and their corresponding DNs in this area, and the distance between any SN and its DN is upper bounded by a maximum distance \( D_M \). We then generate \( M \times N \) candidate RNs, denoted as set \( C_i \), and the selection of the RN for each SN follows a simple "the best worst" [7] criteria: each SN selects \( r_i = \arg \max_{j \in C_i} \min(G_{i,j}, G_{j,d}), \) This criteria aims to balance the two hops and can easily be implemented. During the simulations, if the power control problem is not feasible, then there is at least one CU whose combined SINR requirement is not satisfied, the one with the lowest SINR is shutting off and the algorithm is restarted. This is repeated until the algorithm converges and the SINR requirement of all CUs are satisfied. While more complicated RN selection and feasible set selection schemes can improve the performance, this is at a price of increasing implementation overhead and processing time, and is beyond the scope of this paper.

We first demonstrate the convergence of the proposed scheme. In this set of simulations, \( D = 1000, D_M = 50 \) and \( M = 6 \). We show the cumulative distribution function (CDF) of the iteration time needed for the algorithm under different conditions in Fig. 3. When the error tolerance is tight as \( \epsilon = 1 \times 10^{-6}; \) With large number of CUs \( N = 30 \), the average number of iterations needed is less than 30 for AF and 40 for DF; With \( N = 10 \), the average number of iterations needed is less than 10, which is very small. Generally, the

<table>
<thead>
<tr>
<th>Parameter</th>
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<tbody>
<tr>
<td>Background Noise ( \eta )</td>
<td>( 1 \times 10^{-6} )</td>
<td>SINR Requirement ( \gamma_i^s )</td>
<td>5</td>
</tr>
<tr>
<td>Maximum Power ( P_{\text{max}} )</td>
<td>1</td>
<td>Step Constant ( \alpha_0 )</td>
<td>5</td>
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<tr>
<td>Time shift ( t_0 )</td>
<td>1</td>
<td>Initial ( \omega_i(0) )</td>
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| TABLE I | SIMULATION PARAMETERS |

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In other words, relaying can keep the power consumption of each CU even when the number of simultaneous transmissions is growing. We also compare our proposed algorithm with the successive convex approximation GP (SC-GP) algorithm adopted in [7] and [9], which is shown to be able to reach the global optimum with high probability. From Fig. 4, we can see that the proposed scheme only consumes about 3% to 7% more power than SC-GP does for AF relaying.4 The DF relaying considered is different than the one in [7] because of the second constraint we use in (6), which means that in our algorithm, relaying is not compulsive if the SN-RN link is bad. However, problem (6) can not be solved by SC-GP, so only the DF example given by SC-GP is provided. It is also shown that the DF relaying is better than AF relaying5, because if relaying is beneficial, from (4) and (5), $\gamma_i^{AF} \leq \gamma_i^{DF}$ holds, thus DF relaying requires less power.

We then study the outage performance of relaying in Fig. 5. In this set of simulations, $D = 1000$, $D_m = 70$, and the number of candidate RNs per SN-DN pair is $M = 1, 6$ and 12 respectively. As the number of simultaneous transmissions increases, both AF and DF relaying outperforms no-relaying significantly. Even with $M = 1$, the performance gain is noticeable. It addition, the performance of DF and AF relaying is very close, and only when $M = 1$, DF relaying outperforms DF relaying slightly because the probability that the receiving SINR at the RNs can not satisfy $\gamma_i^{DF}$ is high. On the other hand, this disadvantage can be compensated by the lower interference level in DF relaying as a result of lower transmission power shown in Fig. 4. Besides what we have delivered in Fig. 5, in the simulations, we have also observed that, although each SN-DN pair is assigned with an RN, the usage (the percentage of RNs that with non-zero transmission power) is not 100%. For example, when $N = 70$, $M = 6$, the usage of RN is only 38.2% for DF, and 44.5% for AF. This indicates that with more advanced RN selection scheme, the performance can still be guaranteed even when the number of candidate RNs per SN-DN pair is fixed as 30. We change $N$ from 2 to 8, hence it is a crowded scenario with severe interference, which leads to high outage. It is shown that even under this circumstance, the transmission power with relaying is significantly reduced comparing to the no-relaying case, and from the flat slope of the curves, relaying shows robustness to the increasing of the interference level. In other words, relaying can keep the power consumption of

\[\alpha(t) = \alpha_0, \quad \text{as a constant, but it will degrade the performance of power control. Since this situation rarely happens in practical systems (like uplink in cellular systems, the RNs are generally close to the BSs and undergoes low interference level), we still use the step size described in Section III-C.}\]

Fig. 4 shows the benefit of using relaying in terms of saving transmission power. In this set of simulations, $D = 300$, $D_M = 70$, the number of candidate RNs is fixed as 30. We change $N$ from 2 to 8, hence it is a crowded scenario with severe interference, which leads to high outage. It is shown that even under this circumstance, the transmission power with relaying is significantly reduced comparing to the no-relaying case, and from the flat slope of the curves, relaying shows robustness to the increasing of the interference level. In other words, relaying can keep the power consumption of

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3This is due to the strict convexity of the local “virtual power” optimization problem for AF relaying as described in the Appendix.

4Because the SC-GP requires a feasible initial power allocation, we use the results obtained by our proposed algorithm as the initial value for SC-GP, leading to the same outage probability.

5In [7], DF performances rather bad because relaying is compulsive even if it is not beneficial, and the RN must decode the signal from the SN.
of candidate RN is limited. Thus the design of RN selection scheme in interference-limited environments is valuable and yet to be further investigated.

V. Conclusion

We have proposed a distributed power control algorithm to minimize the total system transmission power subject to the SINR requirements in interference-limited environments. The proposed algorithm only requires locally measured SINR, and the computational overhead is low. Simulation results have shown that the proposed algorithm performs close to the optimal solution and the benefits of relaying under interference-limited conditions are demonstrated. The proposed power control algorithm can be utilized in multiple ways, such as evaluating the effectiveness of relaying in multi-cell environment where co-channel interference exists among cells. Moreover, as the future work, interference-aware RN selection scheme should be considered together with the power control algorithm to effectively exploit the spatial diversity that relaying provides.

Acknowledgment

The authors would like to express their sincere thanks to Hitachi R&D Headquarter for the continuous supports. Furthermore, the authors gratefully acknowledge the support from the R&D Center, China Mobile Communications Corporation.

References


Appendix

Proof: (Proof of Proposition 1) Because we assume that the power control problem is feasible, the violation to the maximum power constraint is not considered.

First, for DF relaying, from (11), assume that for each CU i, \(\omega_i > a_i/c_i\) always exists, then the solution will be \(P_{s_i}^{(t)} = \gamma_i^*/a_i, P_{r_i}^{(t)} = 0\), for \(a_i > b_i\) or \(P_{s_i}^{(t)} = \gamma_i^*/b_i, P_{r_i}^{(t)} = (b_i-a_i)^*/(b_i)c_i\), for \(a_i < b_i\). Now our proposed algorithm is equivalent to a two-stage power control problem: The first stage is on channel 1, each SN tries to achieve max\{\(\gamma_{sd, i}, \gamma_{sr, i}\)\} \(\geq \gamma_i^*\); The second stage is on channel 2, each RN tries to fulfill the remaining SINR gap max\{\(\gamma_i^* - \gamma_{sd, i}, 0\)\}, based on the result of the first stage. It can be easily shown that the update of \(P_{s_i}^{(t)}\) and \(P_{r_i}^{(t)}\) in each iteration can be categorized as a standard interference function [10] respectively, hence the convergence to a fixed point is guaranteed as proved in [10]. Moreover, if we let:

\[
\omega_i = \frac{G_i d_i (\sum_{j=1}^N G_{r_j,d_j} P_{max} + \eta)}{\eta G_{r_i,d_i}}, \tag{18}
\]

Since \(\omega_i > a_i/c_i\) always holds (from the definition in (8) and the maximum power constraints), for any \(\omega_i > \omega_i^*\), the algorithm will converge.

Second, for AF relaying, we construct "virtual powers" for each CU i: \(P_{s_i}'\) and \(P_{r_i}'\), and they do not have non-negative constraints, i.e., the virtual powers can take negative values. Actually, each CU first solves a local optimization problem in terms of the virtual powers:

\[
\begin{align*}
\text{minimize} & \quad \omega_i P_{s_i}' + P_{r_i}' \\
\text{subject to} & \quad P_{r_i}' = f(P_{s_i}') \\
\end{align*}, \tag{19}
\]

where \(f(x)\) is defined in (13). Solving this problem is actually solving the formula (14), and the solution is given by (15). Assume that the solution to (19) always exists, say, \(P_{s_i}'\) and \(P_{r_i}'\), then each CU sets the transmission power as \(P_{s_i}^{(t)} = \min\{P_{s_i}', \gamma_i^*/a_i\}\) and \(P_{r_i}^{(t)} = \max\{0, P_{r_i}'\}\). Because \(f(P_{s_i}')\) is convex and with the assumption that the solution to (19) always exists, by Debreu’s theorem [11], the proposed algorithm will converge to an equilibrium.

Now it remains to guarantee the existence of the solution to (19), which requires that \(\omega_i > \frac{a_i b_i}{c_i(a_i+b_i)}\) (from (15)). Similar to DF relaying, if we set \(\omega_i^*\) the same as (18), since \(\omega_i^* > \frac{a_i b_i}{c_i(a_i+b_i)}\) always holds (from the definition in (8) and the maximum power constraints), for any \(\omega_i > \omega_i^*\), the algorithm will converge to an equilibrium.