ABSTRACT
We explore a full-duplex technique in wireless communication particularly for relay networks. We consider the relay to operate in full-duplex, which occurs when transmission and reception are conducted in the same channel. We investigate potential benefits of full-duplex technique in relay networks, which uses multiple antennas for transmission and reception combined with the Amplify-Forward (AF) scenario. We study the effects of multiple antennas in terms of relay capacity. We derive an ergodic capacity expression using the Tracy-Widom distribution. Using Singular Value Decomposition (SVD) and perfect Channel State Information (CSI), we investigate these three scenarios: First, we consider the relay to have an antenna larger than that of both source and destination. Second, we consider both relay and destination to have an antenna larger than that of the source. Third, we consider both relay and source to have antenna larger than that of the destination. We show the results that the capacity of the relay with a full-duplex technique is almost twice the capacity of an half-duplex. We show that increasing the number of destination antennas is not help much when one of the source antennas is small. Moreover, the capacity decreases due to a channel hardening effect, when the number of destination antennas is larger than that of the source.

Keywords: Ergodic capacity; Full-duplex relay; Multiple antennas; Self-interference

1. INTRODUCTION
Wireless networks with relay configurations have been investigated by many researchers, particularly for increasing the system performance and extending the cell coverage (Cover, et al., 1979; Kramer, et al., 2003; Lee, et al. 2010). A relay network with high spectral efficiency and throughput has been investigated in various research (Kang, et al., 2009; Riihonen, et al., 2011). To enhance the capacity of the relay network, a Multiple-Input, Multiple-Output (MIMO) system has been studied in other research, (Lee, et al., 2010; Wang, et al., 2005; Madsen, et al., 2005). A relay-assisted cooperative system can be divided into two main methods: amplify-and-forward (AF) and decode-and-forward (DF). In the relay networks, the AF method shows benefits of low computational complexity and simple implementation compared to the DF method, because the relay only processes linearly the received signal baseband (Lee, et al., 2010). Wang, et al., 2005 and Madsen, et al., 2005, have studied the MIMO AF relaying system in terms of capacity based on a Singular Value Decomposition (SVD) with perfect Channel State Information (CSI) at the relay network.

Permalink/DOI: http://dx.doi.org/10.14716/ijtech.v6i2.1003
Another interesting classification of the relaying scheme is the operation models: a half-duplex (HD) and a full-duplex (FD). A FD relaying model promises to provide more spectrum efficiency rather than an HD relaying model, (Riihonen, et al., 2011). However, the FD model has a drawback known as self-interference. The self-interference is due to signal leakage from the relay’s transmission to its own reception. Therefore, it is important that the self-interference is sufficiently mitigated. Interestingly, recent advance techniques in hardware design have demonstrated that the self-interference can be suppressed to a degree that possibly allows for FD transmission (Jain, et al., 2011). The self-interference suppression can be categorized as passive cancellation and active cancellation (Sabharwal, et al., 2014). Passive suppression isolates the transmit and receive antennas using techniques such as directional antennas, absorptive shielding, and cross-polarization (Everett, et al., 2014). Active suppression uses the knowledge of its own transmit signal to cancel the self-interference, which typically includes analog cancellation, digital cancellation and spatial cancellation.

In this paper we study the ergodic capacity of FD MIMO AF relay channel. Lee, et al. 2010; Tang, et al., 2007; Medina, et al., 2012 show the capacity of a MIMO AF relaying channel using the SVD technique when the Channel State Information (CSI) is available at the relay and destination nodes. However, they investigated the capacity only for the half-duplex model where self-interference does not exist. Moreover, we assume there is CSI at both destination and relay, where the capacity optimal is derived based on the SVD method (Tang, et al., 2007; Medina, et al., 2012). We know that calculation of the exact ergodic capacity is extremely difficult and complicated, since the expectation of the instantaneous capacity should be integrated over the channel distribution. We apply the Tracy-Widom law (Dieng, et al., 2011) that exploits the eigenvalue distribution of complex Wishart matrices. Then, we derive asymptotic closed-form expressions on the ergodic capacity of the FD MIMO AF relay channel for several antenna configurations. We considered three asymptotic analyses: First, we considered the relay to have antenna larger than that of both source and destination. Second, we considered both relay and destination to have antenna larger than that of the source. Third, we consider both relay and source to have antenna larger than that of the destination.

The rest of the paper is organized as follows. Section 2 describes the system model. In Section 3, we formulate the asymptotic ergodic capacity of three different scenarios. Section 4 presents some analytical results and discussion, and the conclusion is drawn in Section 5.

2. SYSTEM MODEL

For the system model, we consider the wireless relay networks with a full-duplex scenario. The relay is assumed to be operated in a full-duplex AF that uses a linear transformation on the received signal and retransmits the signal. Source, relay, and destination adopt multiple antennas which are equipped with $M, 2N, L$ antennas, respectively as shown in Figure 1. $2N$ means that relay has $N$ transmit antennas and $N$ receive antennas. Moreover, we assume that the direct link between source and destination can be ignored due to a large path loss.

![Figure 1 System model of FD MIMO AF relay networks](image-url)
Assuming that there is processing delay $\tau$ at the relay, we can write the transmitted signal from the relay at time $n$ as
\[ x_t[n] = G(H_1 x[n] + H_s x_r[n] + n_1[n]) \]  
and
\[ x_r[n] = Gx_t[n-\tau] \]
where $x_t[n]$ is the received signal coming from the previous transmitted signal, $x_r[n-\tau]$. Using (1) and (2) recursively, the overall relay output can be written as (Zheng, 2015)
\[ x_t[n] = G(H_1 x[n-\tau] + H_s x_r[n-\tau] + n_1[n-\tau]) \]
\[ = G \sum_{j=0}^{j} (H_s G)^j (H_1 x[n-j\tau-\tau] + n_1[n-j\tau]) \]
where $j$ is the index of delayed symbols. $G$ is the $N \times N$ weighting matrix. $H_1$ and $H_s$ are the $N \times M$ channel matrix of source and relay and the $N \times N$ residual self-interference channel matrix, respectively. $n_1$ denotes the $N \times 1$ Gaussian relay noise vector with zero mean and $E[n_1[n_1]] = \sigma_n^2 I_N$. In this scenario, we add the Zero Forcing (ZF) constraint such that the optimization of $G$ nulls out the residual self-interference from the relay output to relay input. From (3), the following condition is sufficient,
\[ GH=G = 0 \]
Consequently, (3) becomes
\[ x_t[n] = G(H_1 x[n-\tau] + n_1[n-\tau]) \]
We assume that the relay transmit signal covariance $E[x_t x_t^H]$, with the covariance matrix
\[ E[x_t x_t^H] = P_s G H_1 H_1^H G^H + \sigma_n^2 G G^H \]
The relay output power constraint is
\[ \text{tr} \left( G \left( \frac{P_s}{M} H_1 H_1^H + \sigma_n^2 I_N \right) G^H \right) \leq P_r. \]
The received signal at the destination can be written as
\[ y[n] = H_2 x_t[n] + n_2[n] \]
\[ = H_2(G(H_1 x[n-\tau] + n_1[n-\tau]) + n_2[n] \]
where $H_2$ and $n_2$ are the $L \times N$ channel matrix of source-destination and the $L \times 1$ Gaussian relay noise vector with zero mean and $E[n_2[n_2]] = \sigma_2^2 I_L$. We denote the average transmission power of the source and the relay by $P_s$ and $P_r$, respectively. We can write the received Signal to Noise Ratio (SNR) as
\[ \gamma_D = \frac{P_s (H_2 G H_1)(H_2 G H_1)^H}{\sigma_n^2 H_2 G G^H H_2^H + \sigma_2^2 I_N} \]
Moreover, the instantaneous capacity of full-duplex MIMO AF relay networks can be expressed as
\[ C = \log_2 \left[ I_L + \frac{P_s}{M} (H_2 G H_1)(H_2 G H_1)^H \times \left( \sigma_n^2 H_2 G G^H H_2^H + \sigma_2^2 I_N \right)^{-1} \right] \]
Using an SVD decomposition technique, we can form the matrix $\mathbf{G}$ as

$$
\mathbf{G} = \mathbf{V}_G \Sigma_G \mathbf{U}_G^H
$$

where $\Sigma_G$ is an $N \times N$ diagonal matrix as $\Sigma_G = \text{diag}(\sqrt{g_1}, \ldots, \sqrt{g_N})$, and the $N \times N$ unitary matrices $\mathbf{U}_G$ and $\mathbf{V}_G$ are obtained through the SVD of the channels. If we plug (11) into (10) and (7), we have the optimization problem as,

$$
\max_{\min(M, N, L)} \sum_{k=1}^{\min(M, N, L)} \log_2 \left( 1 + \frac{P_s \lambda_{1,k} \lambda_{2,k} g_k}{M \sigma_{n_1}^2 \lambda_{2,k} g_k + \sigma_{n_2}^2} \right)
$$

subject to

$$
\sum_{k=1}^{\min(M, N, L)} g_k \left( \frac{P_s}{M} \lambda_{1,k} + \sigma_{n_1}^2 \right) \leq P_r
$$

where $\mu$ and $\gamma$ are constant to satisfy the relay power constraint and the ratio of the noise variances $\sigma_{n_2}^2/\sigma_{n_1}^2$. Moreover, $[x]^+ = \max(0, x)$.

3. **ASYMPTOTIC ERGODIC CAPACITY FULL-DUPLEX MIMO**

Now, we consider the asymptotic analysis for ergodic capacity of full-duplex MIMO AF relay networks. From (12), we apply the capacity optimal relay matrix in (14) and take expectations over channel realizations, we express the ergodic capacity as,

$$
E[C] = \sum_{k=1}^{\min(M, N, L)} E[C_k]
$$

where the ergodic capacity of each stream $E[C_k]$ is defined as

$$
E[C_k] = E \left[ \log_2 \left( 1 + \frac{P_s \lambda_{1,k} \lambda_{2,k} g_k}{M \sigma_{n_1}^2 \lambda_{2,k} g_k + \sigma_{n_2}^2} \right) \right]
$$

The expectation of (16) should be taken over the eigenvalues of Wishart matrices $\lambda_{1,k}$ and $\lambda_{2,k}$ that have a joint distribution of $\lambda_{1,k}$ and $\lambda_{2,k}$ in (Telatar, 1999). However, it is difficult to obtain a closed-form solution. Therefore, we considered an asymptotic ergodic capacity based on antenna configurations. We proposed three scenarios: i) Large relay antennas, and fixed destination and source antennas, ii) Fixed source antennas, and large destination and relay antennas, and iii) Large source and relay antennas, and fixed destination antennas.

3.1. **Large Relay Antennas, and Fixed Destination and Source Antennas**

As $N$ goes to infinity, we can express that $\mathbf{H}_1^H \mathbf{H}_1 / N$ and $\mathbf{H}_2^H \mathbf{H}_2 / N$ converge to $\mathbf{I}_M$ and $\mathbf{I}_L$, respectively (Lee, et al., 2010). All the elements of $\mathbf{H}_1$ and $\mathbf{H}_2$ are i.i.d. Gaussian random variables and due to convergence, the asymptotic eigenvalues for fixed $M$ and $L$ can be written as

$$
\lambda_{1,k} = N \quad \text{for} \quad k = 1, 2, \ldots, M,
$$

(17)
\[ \lambda_{2,k} = N \quad \text{for} \quad k = 1, 2, \ldots, L. \quad (18) \]

From (13) and (17), we can easily obtain the optimal eigenvalues of matrix \( G \) as

\[ g_k = \frac{P_r}{\min(M, L) \left( \frac{P_s}{M} N + \sigma_{n_1}^2 \right)} \quad (19) \]

To get the asymptotic ergodic capacity, we can substitute (17), (18), and (19) to (15) as

\[ E[C]_1 = \min(M, L) \log_2 \left( 1 + \frac{P_s P_r N^2 \times \frac{1}{\sigma_{\lambda_2}^2 P_s M N + \sigma_{\lambda_1}^2 \min(M, L) (P_s N + \sigma_{n_1}^2 M)} \right) \quad (20) \]

3.2. Fixed Source Antennas, and Large Destination and Relay Antennas

\( \mathbf{H}_1^H \mathbf{H}_1 / N \) converges to \( \mathbf{I}_M \) and \( \lambda_{1,k} \) becomes \( N \), when \( N \) goes to infinity and \( M \) is fixed. Moreover, when \( L \) and \( N \) go to infinity, the eigenvalue of \( \mathbf{H}_1^H \mathbf{H}_2 \) converges to the Tracy-Widom (TW) distribution (Lee, 2010; Johansson, 2000; Johnstone, 2001; Soshnikov, 2002) as

\[ p \left( \frac{\lambda_{2,k} - a_{LN}}{b_{LN}} \right) \rightarrow \text{TW} \quad (21) \]

where

\[ a_{LN} = \left( \sqrt{L} + \sqrt{N} \right)^2 \quad (22) \]

\[ b_{LN} = \left( \sqrt{L} + \sqrt{N} \right) \left( \frac{1}{\sqrt{L}} + \frac{1}{\sqrt{N}} \right)^{1/2} \quad (23) \]

Suppose that TW distribution has mean and variance as \( \mu_k \) and \( \sigma_k^2 \), the mean and variance of \( \lambda_{2,k} \) can be written as (Lee, et al., 2010)

\[ E[\lambda_{2,k}] = b_{LN} \cdot \mu_k + a_{LN}, \quad (24) \]

\[ E[(\lambda_{2,k} - E[\lambda_{2,k}])^2] = b_{LN}^2 \cdot \sigma_k^2 \quad (25) \]

where \( \mu_k \) and \( \sigma_k^2 \) can be calculated as in (Hochwald, et al. 2004), and they have finite values for fixed \( k \). Table I presents the value of \( \mu_k \).

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( \mu_1 )</th>
<th>( \mu_2 )</th>
<th>( \mu_3 )</th>
<th>( \mu_4 )</th>
<th>( \mu_5 )</th>
<th>( \mu_6 )</th>
<th>( \mu_7 )</th>
<th>( \mu_8 )</th>
<th>( \mu_9 )</th>
<th>( \mu_{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.7711</td>
<td>-0.6754</td>
<td>-5.1713</td>
<td>-6.4745</td>
<td>-7.6572</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The ratio of eigenvalues $\lambda_{2,1}/\lambda_{2,k}$ converges to one for $k = 1,...,M$ as $N$ and $L$ go to infinity for fixed $M$. Therefore, $g_k$ for $k = 1,...,M$ satisfying power constraint of the relay is given by

$$g_k = \frac{P_r}{\min(M, L) \left( \frac{P_r}{M} N + \sigma_n^2 \right)} \quad (26)$$

Substituting $\lambda_{1,k} = N$, (26) and the mean of $\lambda_{2,k}$ in (24) into (15), we can write the capacity as

$$E[C]_2 = \sum_{k=1}^{M} \log_2 \left( 1 + P_s P_r N (b_{LN} \cdot \mu_k + a_{LN}) \times \frac{1}{\sigma_{\lambda_2}^2 M (P_s N + \sigma_{\lambda_2}^2 M) + \sigma_{\lambda_2}^2 P_r (b_{LN} \cdot \mu_k + a_{LN})} \right). \quad (27)$$

C. Large source and relay antennas, and fixed destination antennas

Then we considered a closed-form for asymptotic ergodic capacity when $N$ and $L$ go to infinity and that of source antennas is fixed. $H_2^H H_2/N$ converges to $I_L$, as $N$ goes to infinity and $L$ is fixed. Therefore, the eigenvalue of $H_2^H H_2, \lambda_{2,k}$, becomes $N$. The biggest eigenvalue of $H_1^H H_1$ meets the corresponding TW distribution as

$$p \left( \frac{\lambda_{1,k} - a_{MN}}{b_{MN}} \right) \rightarrow \text{TW} \quad (28)$$

From (28), the asymptotic mean and variance of $\lambda_{1,k}$ can be written as

$$E[\lambda_{1,k}] = b_{MN} \cdot \mu_k + a_{MN}, \quad (29)$$

$$E[(\lambda_{1,k} - E[\lambda_{1,k}])^2] = b_{MN}^2 \cdot \sigma_k^2 \quad (30)$$

where $a_{MN}$ and $b_{MN}$ can be computed using a similar method as (22) and (23), respectively. Since the $f_k$ depends on the ratios of the eigenvalues in (14), $f_k$ becomes a constant value, which is independent of $k$ (Lee, et al., 2010). Thus, we can write $f_k$ based on the power constraint on the relay as,

$$g_k = \frac{P_r}{P_s \sum_{i=1}^{L} \lambda_{1,i} + L \sigma_n^2} \quad (31)$$

Now, we can substitute $\lambda_{2,k} = N$, (29) and (31) into (15) as

$$E[C]_3 = \sum_{k=1}^{L} \log_2 \left( 1 + P_s P_r N (b_{MN} \cdot \mu_k + a_{MN}) \times \frac{1}{\sigma_{\lambda_1}^2 \sigma_{\lambda_2}^2 \mu_k + a_{MN} \cdot \sum_{l=1}^{L} (b_{MN} \cdot \mu_l + a_{MN})} \right). \quad (32)$$

4. RESULTS AND DISCUSSION

In this simulation, we assume that $P_s = P_r = P$ and $\sigma_n^2_1 = \sigma_n^2_2 = 1$ and the Signal to Noise Ratio (SNR) is defined as $P/\sigma_n^2$. In Figure 2, we plot the analytical result between the half-duplex model (Lee, et al., 2010) and the full-duplex model when the number of relay antennas is large, and the number of source and destination antennas is fixed. We can see that as $N$ increases, the capacity of the both models increases. As we expected, the capacity of the full-duplex is larger than the capacity of the half-duplex. However, we can see that the capacity of the full-duplex is less than twice the capacity of the half-duplex, because the full-duplex model utilizes half antennas for transmission and the other half antennas for reception. Moreover, the gap between the full-duplex model and the half-duplex model becomes larger as the SNR increases.
We plot the second scenario where the number of relay and destination antennas is large and the number of source antennas is fixed as shown in Figure 3. We set SNR = 20 dB. We can see that increasing the number of destination antennas is not very helpful to improve the capacity in both models. As the number of destination antennas increases, the mean of the eigenvalues $\lambda_{2,k}$ increases as shown in (24). In practical situations, increasing the number of destination antennas causes an increase in the received power at the destination node. Thus, as the received power increases, the capacity also increases, but in logarithmic scale.

In Figure 4, we plotted the analytical result with large source and relay antennas and fixed destination antennas. We set SNR = 20 dB. We can see that the capacity of both models decreases as the number of source antennas increases. When the number of source antennas $M$
is larger than that of destination antennas \( L \), the rank of \( \mathbf{H}_1 \) becomes greater than the rank of \( \mathbf{H}_2 \). Without CSI at source, we assume that the source transmits the signal per stream with equal power. However, the number of streams is limited by the rank of \( \mathbf{H}_2 \). In this connection, the transmit power per stream from the source antennas is decreased and the capacity is reduced. We also note that the number of source antennas should not exceed the number of destination antennas.

Figure 4 Comparison of capacity relay networks between half-duplex and full-duplex with large \( N \) and \( M \), and fixed \( L \)

5. CONCLUSION

We derived the ergodic capacity of full-duplex MIMO amply-forward relay channel using the Tracy-Widom distribution. We considered Singular Value Decomposition (SVD) based scheme with perfect Channel State Information (CSI). We derived the asymptotic ergodic capacity for full-duplex model for three scenarios: First, we considered the relay to have antenna larger than that of both source and destination. Second, we considered both relay and destination to have antenna larger than that of source. Third, we considered both relay and source to have antenna larger than that of destination. We show in the simulation results that the capacity of the full-duplex model cannot achieve twice the capacity of the half-duplex. This is because the capacity of full-duplex uses half antennas for transmission and the other half antennas for reception. We show that increasing the number of destination antennas does not help much for improving the capacity when the number of source antennas is fixed. Moreover, we show that adding the number of source antennas can decrease the capacity when the number of destination antennas is fixed.

6. REFERENCES