The feasibility of creating and manipulating coherent quantum states on surfaces of functionalized semiconductor nanostructures is computationally investigated. Quantum dynamics simulations of electron-hole transfer between catechol molecules adsorbed on TiO$_2$-anatase nanostructures under cryogenic and vacuum conditions indicate that laser induced coherent excitations can be prepared and manipulated to exhibit controllable spatial Rabi oscillations. The presented computational methods and results are particularly relevant to explore the basic model components of quantum-information electro-optic devices based on inexpensive and readily available semiconductor materials.

Keywords: coherent control, wave-packet dynamics, adsorbates, semiconductors

Communicated by: to be filled by the Editorial

1 Introduction

The optical and mechanical manipulation of individual molecules has become a routine practice in many laboratories, holding great promise for several areas of science and technology [1–5]. Molecules exhibit an enormous diversity of structures and electronic properties. Once combined with solid state systems, they fit at the nanometer scale yielding their intrinsic properties to the functionalized host substrate material. Single molecules have already become the active part of nanoelectronic circuits [3] and have also been used for quantum-information processing [4]. In addition to nuclear spin states [4, 6], molecular vibrational modes have also been proposed for quantum bit (qubit) implementations [5]. However, the scalability of quantum information processing systems based on individual molecules in gases or liquids is expected to be hindered by molecular diffusion, leading to leakage and decoherence. In order to overcome these limitations, several solid-state qubit implementations are currently being pursued, including designs based on polarization states of electrons confined to quantum dots [7] and nuclear spin states of atomic impurities in semiconductors matrices [6]. Furthermore, quantum logic gates based on coupled quantum dots, treated as artificial molecules, have already been explored [8].

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In this work we investigate the feasibility of creating and coherently manipulating quantum states in functionalized semiconductor nanostructures, in an effort to explore realistic models of molecular qubits based on existing semiconductor materials. \textit{Ab-initio} molecular dynamics simulations are combined with semi-empirical electronic structure calculations in order to investigate electron-hole states, created by photoexcitation of catechol molecules functionalizing TiO$_2$-anatase nanostructures (see Fig. 1). TiO$_2$-anatase, functionalized with catechol molecules, constitutes an inexpensive material that has already been characterized both theoretically [9, 13] and experimentally [10–12], serving as a simple prototype model system upon which more complex molecular structures can be attached for specific phototransduction applications [14].

![Fig. 1](image)

**Fig. 1.** a) TiO$_2$-anatase functionalized with catechol molecules and density isosurface representing a nonstationary hole state delocalized on the molecular adsorbates. b) Energy diagram and photo-injection process depicted by arrows, including the splitting of closely lying HOMO energy levels. The inset shows the evolution of the time-dependent electronic population during electron injection from a photo-excited catechol adsorbate [13,36].

Functionalization results from the adsorption of molecules onto the semiconductor surface. As a result, surface complexes are formed and electronic states are introduced in the semiconductor band gap, sensitizing the host material for photo-absorption at frequencies characteristic of the molecular adsorbates (see energy level diagram in Fig. 1b). Photoexcitation of surface complexes often leads to femtosecond interfacial electron injection (see inset in Fig. 1b and arrows in the energy diagram) when there is suitable energy match between the photoexcited
molecular state and the electronic states of the conduction band in the semiconductor material [13,15–17,36]. Such a photoexcitation and relaxation process has already raised significant interest for a broad range of technological developments [18]. Here, we investigate an unexplored aspect of this fundamental process involving the dynamics of hole states left within the semiconductor band gap after photoinduced electron injection [36]. The distinctive characteristic of the interband states hosting electron-holes is that they remain off-resonance relative to the semiconductor valence and conduction bands. It is therefore of interest to explore the nontrivial question as to whether such an off-resonance condition can preserve quantum coherences, despite the partial intrinsic decoherence induced by thermal ionic motion, allowing for coherent optical manipulation of hole states with available femtosecond-pulse technology. This paper investigates such a problem in terms of quantum dynamics computations based on an approximate mixed quantum-classical approach (i.e., Ehrenfest mean-field nuclear dynamics) where the electrons are treated quantum mechanically and the nuclei evolve classically. The appreciation of conditions under which quantum coherences may be described according to mixed quantum-classical methodologies has been the subject of intense research [19–22], including the analysis of decoherence in similar composite models [23,24]. The applicability of mixed quantum-classical dynamics is found to be valid so long as the quantum subsystem (electronic dynamics) decoheres slowly and the remainder (nuclear dynamics), often coupled to a thermal bath, decoheres quickly [25].

2 Methods

2.1 Electronic Structure Methods

We consider simulations of quantum dynamics in a model system composed of catechol molecules adsorbed on the [101] surface of TiO$_2$-anatase nanostructures under cryogenic and vacuum conditions (see Figure 1a). The dimensions of the functionalized nanoparticle are 3.1 nm × 1.5 nm × 3.1 nm, along the [-101], [010] and [101] directions, respectively, with adjacent adsorbates 1.0 nm apart from each other. For future reference, the three molecular adsorbates Ω are denoted Left (L), Central (C) and Right (R) adsorbates. The optimized geometry is obtained, according with our previous studies, by geometry optimization with respect to the coordinates of the anchored catechol molecules and the TiO$_2$ surface [13,36]. Geometry relaxation is performed by using the Vienna Ab-initio Simulation Package (VASP/VAMP) [26,27], which implements the density functional theory (DFT) in a plane wave basis set, with the Perdew-Wang [28] generalized gradient approximation (GGA) and ultrasoft Vanderbilt pseudopotentials [29]. An ensemble of thermal configurations and nuclear trajectories is obtained by performing ab initio molecular dynamics simulations.

The electronic structure of the 3 nm particles is described according to a tight-binding model Hamiltonian gained from the extended Hückel (EH) approach [30,31]. Advantages of this method are that it requires a relatively small number of transferable parameters and is capable of providing accurate results for the energy bands of elemental materials (including transition metals) as well as compound bulk materials in various phases [31]. In addition, the EH method is applicable to large extended systems and provides valuable insight on the role of chemical bonding [32]. The EH method is therefore most suitable to develop a clear chemical picture of the underlying relaxation dynamics at the semiquantitative level, including
fundamental insight on the role that symmetry plays in the localization of holes on catechol molecular orbitals (MOs) (i.e., states with negligible overlap with d orbitals of nearby Ti$^{4+}$ ions in the TiO$_2$ host substrate).

The EH Hamiltonian is computed in the basis of Slater-type orbitals $\chi$ for the radial part of atomic orbital (AO) wavefunctions [13], including the 4s, 4p and 3d atomic orbitals of Ti$^{4+}$ ions, the 2s and 2p atomic orbitals of O$^{2-}$ ions, the 2s and 2p atomic orbitals of C atoms, and the 1s atomic orbitals of H atoms. The AOs $\{|\chi_i(t)\rangle\}$ form a mobile (nonorthogonal) basis set due to nuclear motion, with $S_{ij}(t) = \langle \chi_i(t) | \chi_j(t) \rangle$ the corresponding time-dependent overlap matrix elements. The overlap matrix is computed using the periodic boundary conditions along the [010] direction.

Diagonalization of the EH Hamiltonian predicts a 3.3 eV band gap for the 3.0 nm model system in its relaxed configuration. This result is consistent with the experimental value of 3.2 eV for the band gap of bulk TiO$_2$-anatase, 3.4 eV for 2.4 nm particles [33], and 3.7 eV band gap for the 1.2 nm model system [13], since the smaller the nanoparticle the larger is the band gap.

2.2 Quantum Dynamics Methods

We confine ourselves to an approximate mixed quantum-classical method in which the electrons are treated quantum mechanically and the nuclei classically. The nuclei evolve on an effective mean-field Born-Oppenheimer potential energy surface (PES), $V_{eff}$, according to classical trajectories $R^\xi = R^\xi(t)$ with initial conditions specified by the index $\xi$. The time-dependent electronic wave function is propagated for each nuclear trajectory $R^\xi(t)$, according to a numerically exact integration of the Time-Dependent Schrödinger Equation (TDSE),

$$\left\{ \frac{\partial}{\partial t} + \frac{i}{\hbar} H(t) \right\} |\Psi^\xi(t)\rangle = 0. \tag{1}$$

Here, $H(t)$ is the electronic EH Hamiltonian which depends on $t$ through $R^\xi(t)$. The actual calculation of $V_{eff}$, or equivalently of the set of trajectories $R^\xi(t)$, is a difficult problem [34]. However, in the present application both the ground and excited electronic state PESs involve bound nuclear motion of similar frequencies. Therefore, $V_{eff}$ is nearly parallel to the ground state PES. Nuclear trajectories $R^\xi(t)$ are therefore approximated according to ab initio-DFT Molecular Dynamics simulations.

Results reported in Sec. 3 are obtained, according to the resulting propagation scheme, by sampling initial conditions $\xi$ for nuclear motion, integrating the TDSE over the corresponding nuclear trajectories and averaging expectation values over the resulting time-evolved wavefunctions. Converged results are typically obtained with less than 50 initial conditions representing the system thermalized under conditions of 100 K and constant volume. However, results are reported for averages of 100 initial conditions.

The TDSE, introduced by Eq. (1), is numerically integrated by expanding the time-dependent hole wavefunction

$$|\Psi^\xi(t)\rangle = \sum_q B_q(t) |\phi_q(t)\rangle, \tag{2}$$

in the basis of the instantaneous MOs

$$|\phi_q(t)\rangle = \sum_i C_{i,q}(t) |\chi_i(t)\rangle, \tag{3}$$
i.e., the instantaneous eigenstates of the generalized eigenvalue problem

\[ \mathbf{H}(t) \mathbf{C}(t) = \mathbf{S}(t) \mathbf{C}(t) \mathbf{E}(t), \tag{4} \]

with eigenvalues \( E_q(t) \).

The propagation scheme is based on the recursive application of the following short-time approximation:

\[ |\Psi^\xi(t + \tau/2)\rangle \approx \sum_q B_q(t) e^{-\frac{i}{\hbar} E_q(t) \tau/2} |\phi_q(t)\rangle, \tag{5} \]

where the evolution of the expansion coefficients

\[ B_q(t + \tau) = \sum_p B_p(t) e^{-\frac{i}{\hbar} \left[ E_p(t) + E_q(t + \tau) \right] \tau/2} \langle \phi_q(t + \tau) | \phi_p(t) \rangle \]

is approximated as follows,

\[ B_q(t + \tau) \approx B_q(t) e^{-\frac{i}{\hbar} \left[ E_q(t) + E_q(t + \tau) \right] \tau/2}, \tag{6} \]

in the limit of sufficiently thin time-slices \( \tau \).

The initial hole state, after electron-hole pair separation, is the Highest Occupied Molecular Orbital (HOMO) of the photoexcited surface-complex \( C \). The subsequent relaxation dynamics is quantitatively described in terms of the time-dependent hole populations \( P_\Omega(t) \) of the molecular adsorbates \( \Omega = (L, C, R) \).

The time-dependent hole population \( P_\Omega(t) \) is computed as follows,

\[ P_\Omega(t) = \text{Tr}\{ \hat{\rho}(t) \hat{\Pi}_\Omega \}, \tag{7} \]

where \( \hat{\Pi}_\Omega \) is the projection operator onto atomic orbitals of adsorbate \( \Omega \),

\[ \hat{\Pi}_\Omega = \sum_{k,j \in \Omega} |\chi_j\rangle S_{jk}^{-1} \langle \chi_k|, \tag{8} \]

and \( \hat{\rho}(t) \) is the reduced density operator associated with the electronic degrees of freedom,

\[ \hat{\rho}(t) = \sum_{\xi} p_\xi |\Psi^\xi(t)\rangle \langle \Psi^\xi(t)|, \tag{9} \]

where \( p_\xi \) is the probability of sampling the nuclear initial condition specified by index \( \xi \).

3 Results

3.1 Rabi Oscillations

Figure 2 (left panel) shows the time-dependent populations \( P_\Omega(t) \) of each adsorbate as a function of time (thick lines). Thin lines show the contributions of a single representative trajectory \( R^\xi \) to the total ensemble averages \( P_\Omega(t) \). Note that almost 90% of the hole population remains localized, throughout the entire relaxation time, on those atomic orbitals belonging to the adsorbate molecules. The remaining 10% of the hole population remains next to the molecular adsorbates, localized in the d orbitals of Ti\(^{4+}\) ions that have partial overlap with the molecular adsorbates [36]. These results are consistent with the fact that,
under vacuum conditions, the recombination of injected electrons back to the molecule (back-transfer) occurs in the nanosecond time-scale [35].

Figure 2 (left panel) indicates that the hole is transferred between adjacent catechol molecules, remaining localized in the mono-layer of adsorbate molecules rather than undergoing injection into the semiconductor host substrate. The underlying hole relaxation dynamics is therefore significantly different from the relaxation of the photoexcited electron which, according to the results reported in Ref. [13] as well as in the inset of Fig. 1, is usually completely injected into the semiconductor material within a few femtoseconds.

Note that hole transfer involves adsorbates with negligible overlap of AOs, since catechol molecules are anchored to the surface more than 1 nm apart from each other. However, near-resonant electronic states, localized in adjacent adsorbate molecules, are indirectly coupled by the common host-substrate giving rise the observed relaxation. Considering that there is no significant hole into the semiconductor structure, these results are consistent with a super-exchange hole transfer mechanism mediated by the semiconductor host substrate.

An approximate description of hole-tunneling between adsorbate molecules $\Omega$ and $\Omega'$, coupled by the host substrate, can be given in terms of the Rabi formulae:

$$ P_\Omega(t) = \frac{\gamma_{\Omega\Omega'}^2 / \hbar^2}{\Gamma_{\Omega\Omega'}} \sin^2 \left( \Gamma_{\Omega\Omega'} \cdot t \right) . \tag{10} $$

and $P_{\Omega'}(t) = 1 - P_\Omega(t)$. The calculated hole-tunneling period, obtained from Fig. 2, is $T \approx 42$ ps and has an exponential dependence with the separation between molecular adsorbates. The parameter $\Gamma_{\Omega\Omega'} \left[ (\gamma_{\Omega\Omega'} / \hbar)^2 + (\omega_{\Omega\Omega'} / 2)^2 \right]^{1/2}$, introduced by Eq. (10), is the Rabi frequency, $\gamma_{\Omega\Omega'}$ is the effective quantum coupling between resonant states and $\omega_{\Omega\Omega'}$ is the corresponding Bohr frequency. The parameters computed from Fig. 2 are $\omega_{LC} \approx 4.5 \omega_{RC}$ and $\gamma_{RC} \approx 1.12 \gamma_{LC}$. It is important to mention that the asymmetric nature of the underlying relaxation dynamics (as evidenced by the asymmetric population of the $R$ and $L$
adsorbates) is primarily due to the intrinsic asymmetry of the local substrate environment that interact with each adsorbate. Such small differences determine asymmetric couplings between molecular adsorbates and a preferential direction for the hole motion.

Figure 2 (right panel) shows a detailed analysis of the early time relaxation dynamics for a representative nuclear trajectory, including the quantitative description of the adsorbate time-dependent hole populations, the evolution of the adsorbate-semiconductor distance, and the evolution of energy differences between near-resonant electronic states of surface complexes localized in the semiconductor band-gap. The analysis of these results indicates that the time-dependent populations \( P(t) \) (top right panel) are correlated with the motion of the molecular adsorbates (middle right panel), since the adsorbate-semiconductor separation modulates the time-dependent energy differences \( \varepsilon_{RC} \) and \( \varepsilon_{LC} \) between the initially populated state \( C \) and the near-resonant electronic states in the \( R \) and \( L \) adsorbates, respectively (bottom right panel). The most significant population exchange, during this first picosecond of dynamics, is observed when the adsorbate-semiconductor separation reaches a maximum value, at approximately \( t = 0.41 \) ps (see Fig. 2, middle right panel). At that point, states \( L \) and \( C \) remain near-resonant for about 100 fs (see Fig. 2, top right panel) and exchange population without significantly populating any other state (e.g., states responsible for coupling states \( L \) and \( C \)).

The extent to which these results are significant is associated with the survival of electronic coherences, responsible for the Rabi oscillations shown in Fig. 2. To this end, we examine the diagonal and off-diagonal elements of the reduced density matrix \( \rho(t) \), introduced by Eq. (9), in the basis of catechol MOs. The analysis, presented in Fig. 3, is focused on the calculation of matrix elements of \( \rho(t) \) associated with the HOMOs of the isolated catechol molecules \( L,C, \) and \( R \), which can be represented in occupancy notation as register states \( |100\rangle, |010\rangle, \) and \( |001\rangle \), respectively. Note that in this notation, \( \hat{\rho}(0) = |010\rangle\langle 010| \). The

![Fig. 3](image-url)  
Fig. 3 Left panel: Comparison of the trace of the square of the reduced density matrix (gray curve) and the time evolution of the hole population on the surface complexes \( L,C,R \) (black curves). Ensemble averages were converged by sampling over 100 different initial conditions \( \xi \). Right panel: Real (upper panel) and imaginary (lower panel) parts of non-zero off-diagonal matrix elements of reduced density matrix for register states \( |100\rangle, |010\rangle, \) and \( |001\rangle \) demonstrating coherent super-exchange hole tunneling dynamics for the first 50 ps of dynamics.

The black curves in Fig. 3 (left panel) show that the diagonal elements of the reduced density
matrix for states $|100\rangle, |010\rangle$ and $|001\rangle$ correspond closely to the total adsorbate populations $P_L(t), P_C(t)$ and $P_R(t)$ reported in Fig. 2, indicating that most of the hole population remains localized in these adsorbate states throughout the simulation time. Furthermore, the non-zero off-diagonal elements, shown in Fig. 3 (right panel), demonstrate that the hole relaxation dynamics remains remarkably coherent for the entire simulation time despite thermal nuclear motion. (Note: For clarity, results are presented for the first 50 ps of dynamics. It is, however, important to mention that our calculations indicate that both Rabi oscillations and quantum coherences are preserved for at least hundreds of picoseconds.)

The observed coherent localization in the space of electronic states is mainly due to the finite size of the nanostructure, where the register states are coupled by the common host substrate but remain off-resonant relative to valence and conduction bands (manifolds) of electronic states [13]. In contrast, the analogous relaxation dynamics on extended systems is expected to delocalize the hole excitation on the manifold of near-resonant electronic states introduced by the adsorbate molecules. Hole excitations in disordered extended systems with low surface coverage, however, are expected to have an intermediate behavior and exhibit localized coherent relaxation within small disjoint islands of adsorbate molecules.

A quantitative measure of intrinsic decoherence (see Fig. 3, left panel) is obtained by computing the trace of the square of the reduced density matrix $\text{Tr}[^{2}\rho(t)]$ [37–39]. We refer to the decoherence induced by the nuclear motion on the quantum subsystem as intrinsic decoherence to make a distinction from cases where decoherence is caused by the direct coupling of the quantum subsystem with an external bath. Such a quantity measures the decay of purity as the initial state becomes a statistical mixture of electronic states due to decoherence induced by thermal nuclear motion. The gray curve in Fig. 3 (left panel) shows that $\text{Tr}[^{2}\rho(t)]$ decays about 15% at very early times due to partial mixing in the initially photoexcited surface complex $C$. However, throughout the rest of the propagation time, $\text{Tr}[^{2}\rho(t)]$ remains approximately constant, decaying at a much lower rate while the hole tunnels between adjacent molecular adsorbates. These results thus indicate that the underlying hole relaxation dynamics remains highly coherent on the surface of the nanoparticle in spite of thermal nuclear motion.

### 3.2 Model System

The analysis of spatial Rabi oscillations, presented in Sec. 3.1, indicates that coherent superexchange hole transfer between adsorbate molecules attached to TiO$_2$ semiconductor nanostructures, results from multiple scattering events where near-resonant states localized in adsorbate molecules become indirectly coupled by intermediate states in the semiconductor host substrate. Coupling events last for about 100 fs and occur once or twice every ps as thermal fluctuations modulate the electronic couplings and the resonance conditions (see Fig. 2, right panel). It is, therefore, natural to expect that similar coherent Rabi oscillations should be observable in other systems, whenever donor and acceptor states become indirectly coupled by mediating states, so long as the life-time of the mediating state is longer than the duration of a single scattering event.

In order to explore this fundamental aspect of the observed quantum coherent Rabi oscillations, consider the simplest possible implementation of multiple STInulated Raman Adiabatic Passage (STIRAP) events [40–42] in the three-level system depicted in Fig. 4, described by
Fig. 4. Three-level excitation model scheme. States $|1\rangle$ and $|3\rangle$ are coupled by a train of Stokes and pump laser pulses, described by $H_{23}(t)$ and $H_{12}(t)$, respectively, slightly detuned relative to the intermediate state $|2\rangle$. The life-time of state $|2\rangle$ is longer than the duration of the pulses.

The following time-dependent model Hamiltonian,

$$H(t) = \begin{pmatrix} E_1 & H_{12}(t) & 0 \\ H_{12}(t) & E_2 & H_{23}(t) \\ 0 & H_{23}(t) & E_3 \end{pmatrix},$$

written in the basis of states $|1\rangle$, $|2\rangle$ and $|3\rangle$, with $E_1 = 0$ meV, $E_2 = 28.60$ meV and $E_3 = 5.72$ meV, respectively. The coupling strength between states is determined by a train of femtosecond pump and Stokes pulses described by $H_{12}(t)$ and $H_{23}(t)$ (vide infra). These pulses induce population exchange between states $|1\rangle$ and $|3\rangle$, according to standard three-state two-photon Raman processes, leaving $|2\rangle$ without any significant population. Note that here, in analogy to the adsorbate states in the semiconductor nanostructure, donor and acceptor states ($|1\rangle$ and $|3\rangle$) are only indirectly coupled by the mediating state $|2\rangle$, as modulated by $H_{12}(t)$ and $H_{23}(t)$. Starting with population in state $|1\rangle$, the counterintuitive order of pulses is implemented with the Stokes pulse $H_{23}(t)$ preceding the pump pulse $H_{12}(t)$ as indicated in Fig. 5 (right lower panel), first coupling the acceptor $|3\rangle$ and intermediate state $|2\rangle$, and then coupling the donor $|1\rangle$ and intermediate $|2\rangle$. Here, $H_{23}(t)$ and $H_{12}(t)$ are Gaussian pulses, depicted in Fig. 5 (right lower panel), with a pulse-width of 134 fs and a relative pulse delay of 138 fs. Such a coupling scheme transfers population between $|1\rangle$ and $|3\rangle$ without significantly populating state $|2\rangle$ (see Fig. 5, right upper panel). While the pulse delay and coupling strengths could be adjusted in order to achieve partial or complete population transfer, parameters have been chosen in order to mimic the time-dependent populations shown in Fig. 2. Under typical experimental conditions, there might be some diabatic loss of population to $|2\rangle$, but this should not significantly affect the overall transfer mechanism. Figure 5 (left panel) shows the evolution of time-dependent populations $P_{ij}(t)$ that result from extending such a coupling scheme periodically with a period of about 1.72 ps. Analogously to
the relaxation dynamics observed in functionalized semiconductor nanostructures (see Fig. 2, left panel), coherent Rabi oscillations are observed with periodicity of about 42 ps.

![Time-dependent populations](image)

**Fig. 5.** Left panels: Time-dependent populations $P(t)$ of states $|1\rangle$, $|2\rangle$ and $|3\rangle$ (green, blue and black lines, respectively) for the 3-level STIRAP system depicted in Fig. 4. The finite life-time of intermediate state $|2\rangle$ is $\tau_2 = 10$ ps (left top panel) and $\tau_2 = 1$ ps (left lower panel). Time-dependent populations show repeated partial transfer due to the train of pump and Stokes laser pulses, described by $H_{12}(t)$ and $H_{23}(t)$, inducing multiple STIRAP events throughout the whole relaxation time. Right top panel: Time-dependent populations $P(t)$ during the early time dynamics. Right lower panel: counterintuitive order of coupling pump and Stokes pulses, $H_{12}(t)$ (red) and $H_{23}(t)$ (black), respectively.

The comparison between Fig. 5 and Fig. 2 provides insight into the underlying hole-relaxation mechanism observed in semiconductor nanostructures. It is concluded that thermal fluctuations transiently couple the otherwise energetically isolated states of adjacent adsorbates to MOs delocalized in the nanostructure, mediating population transfer between spatially isolated adsorbate states according to multiple STIRAP-like events. The robustness of the observed Rabi oscillation is found to rely upon the life-time of the mediating state, which must be longer that the duration of a single coupling event.

### 3.3 Optical Coherent Control

This section investigates the feasibility of achieving coherent control of the underlying superexchange hole transfer relaxation responsible for the coherent Rabi oscillations discussed in Secs. 3.1 and 3.2. The specific coherent-control scheme implemented involves a sequence of ultrashort laser pulses that couple a populated state in an adsorbate molecule with an auxiliary state, inducing a sequence of $\pi$ phase-shifts in the time-evolved wave-packet component corresponding to the initially populated state of interest. Such a coherent control scenario is inspired in other successful approaches for modulating quantum relaxation dynamics based
on multiple pulses [43–47]. All of these approaches are particularly suited for applications
to quantum information processing since they preserve the coherent nature of the unitary evolution. In contrast, other approaches proposed to control decay of excited states are based on the frequent collapse \(i.e.,\) measurement of the time-dependent wavefunction \([48, 49]\) and therefore do not preserve the coherent nature of quantum dynamics.

In the present implementation, the time evolution operator that describes the transformation due to a single ultrashort \(2\pi\)-pulse is

\[
\hat{U}_{2\pi} = \mathbf{I} - 2\frac{|010\rangle\langle010|}{\langle010|010\rangle},
\]

where \(\mathbf{I}\) is the identity operator and \(|010\rangle\) is the initially populated state. Therefore, the transformed wavefunction after applying a \(2\pi\)-pulse to the time-evolved wavefunction \(|\Psi_t\rangle\) is \(|\Psi_t^{(2\pi)}\rangle = \hat{U}_{2\pi} |\Psi_t\rangle\). We proceed to investigate the effect of a sequence of ultrashort \(2\pi\) pulses on the relaxation of the time-dependent wave-packet, considering that the width of the \(2\pi\) pulses is much shorter than the interval \(\tau\) between pulses \((\tau = 550\) fs\).

Fig. 6. Upper-left panel: hole population \(P_{2\Omega}\) for the three adsorbates; arrows indicate the start and the end of the sequence of pulses. Other panels show the real (black) and imaginary (blue) parts of the off-diagonal density matrix elements, indicated by labels.

Figure 6 illustrates the perturbational effect of a sequence of ultrashort \(2-\pi\) pulses on the hole relaxation dynamics during the first 100 ps of dynamics. The \(2-\pi\) pulses are applied in the \(t = 15–60\) ps time window (indicated with arrows) at intervals of 550 fs, starting at \(t_i = 15\) ps when there is maximum entanglement between adsorbates \(C\) and \(R\) \(i.e.,\) when \(|\rho_{010,001}\rangle\) is maximum. Fig. 6 shows the resulting time-dependent hole populations \(P_{2\Omega}(t)\) (upper-left panel). The other panels show the matrix elements of the time-dependent reduced density matrix \(\rho^{3\times3}_{\Omega\Omega}(t)\), in terms of real (black) and imaginary (blue) components associated with the
pairs of states indicated by the labels. These are the most significant matrix elements since, according to Sec. 3.1, the subset of adsorbate states ($L, C, R = |100>, |100>, |100>$) localize most of the hole population throughout the relaxation dynamics.

The main effect of the optical pulses is to suspend Rabi oscillations, keeping constant the population of adsorbate $R$. Population locking results from the highly oscillatory evolution of the off-diagonal matrix elements $\rho_{010,001}$ and $\rho_{100,010}$, due to the perturbational effect of the pulses, affecting the interference with mediating intermediate states. However, since coherences are intrinsically preserved, the system is able to re-establish the coherent Rabi oscillations once the sequence of pulses is over at 60 ps.

![Fig. 7. Upper-left panel: hole population $P_0$ for the three adsorbates; arrows indicate the start and the end of the sequence of pulses. Other panels show the real (black) and imaginary (blue) parts of the off-diagonal density matrix elements, indicated by labels.](image)

It is important to note that the resulting dynamics, generated by a sequence of $2\pi$ pulses, strongly depends on the position along the Rabi cycle where the sequence of pulses starts acting on the system (i.e., the time when the train of $2\pi$ pulse is 'turned on'). In order to illustrate this aspect, we consider starting the sequence of pulses at a time when there is minimum entanglement between adsorbates $C$ and $R$ adsorbates (i.e., when $|\rho_{010,001}|$ is small). Fig. 7 shows that the resulting time-dependent hole populations $P_0(t)$ (upper-left panel) are significantly different from those shown in Fig. 6, resulting from a different start for sequence of pulses. Fig. 7 also shows more evidently that while the sequence of pulses is acting on the system the initially populated state, $C = |010>$, interferes with states in the semiconductor structure partially injecting hole population. Therefore, when the sequence of pulses is over, Rabi oscillations are re-established but with a reduced amplitude.
4 Summary and Conclusions

We have shown that mixed quantum-classical simulations, based on \textit{ab initio}-DFT Molecular Dynamics, can be efficiently implemented to simulate quantum electronic dynamics, revealing the feasibility of super-exchange hole tunneling dynamics in a model study of functionalized TiO$_2$-anatase nanostructure under cryogenic and vacuum conditions. It is found that quantum coherences, between hole states localized energetically deep in the semiconductor band gap, can persist for hundreds of picoseconds despite the partial intrinsic decoherence induced by thermal ionic motion. The observed coherent hole population transfer among adjacent adsorbates is mainly stabilized by the finite size of the nanostructure, where register states are coupled by the common host substrate but remain off-resonant relative to the valence and conduction bands (manifolds) of electronic states. Carrier-phonon, and likely carrier-carrier, scattering mechanisms which would otherwise lead to the commonly observed ultrafast decoherence in semiconductor spectroscopy \cite{50} are highly suppressed by the off-resonance and symmetry conditions of the electronic states in surface complexes. It is concluded that the predicted observation of Rabi oscillations, associated with adsorbate electronic populations, could provide a simple experimental probe of coherent quantum relaxation dynamics associated with super-exchange hole transfer. Considering the great technological interest in encoding and manipulating quantum states in inexpensive and readily available semiconductor materials \cite{51}, we anticipate significant experimental interest in examining the predicted relaxation dynamics reported in this paper.

We have investigated the feasibility of coherently controlling the underlying hole relaxation dynamics by multiple 2-$\pi$ pulses. The reported results suggest that functionalized TiO$_2$ semiconductor materials can be photo-excited to exhibit coherently controllable spatial Rabi oscillations, possibly offering a readily available platform to encode logical hole states within the subspace of adsorbate surface complexes. Such a subset of adsorbates states seem to offer symmetry and off-resonance conditions suitable to protect the evolution of quantum states against the decoherence effect of the lattice motion, naturally provide a passive error prevention scheme \cite{52}. The reported calculations associated with functionalized TiO$_2$-anatase nanostructures (and mimic 3-level multipleSTIRAP system) indicate that coherent wave-packet dynamics, resulting from multiple scattering events between states indirectly coupled by intermediate states, could be preserved for hundreds of picoseconds whenever the lifetime of the mediating intermediate state is longer than the duration of each scattering event. Finally, we conjecture that the ability to assemble molecules on nanostructure solid surface, together with the rich variety of properties characteristic of potential molecular adsorbates, make functionalized semiconductors a promising alternative for the construction of quantum information photo-optic devices.

Acknowledgments

L.G.C. Rego acknowledges the financial assistance from CAPES/Brazil, under the PRODOC program. V.S.B. acknowledges supercomputer time from the National Energy Research Scientific Computing (NERSC) Center and financial support from Research Corporation, Research Innovation Award # R10702, a Petroleum Research Fund Award from the American Chemical Society PRF # 37789-G6, a junior faculty award from the F. Warren Hellman Family, the National Science Foundation (NSF) Career Program Award CHE # 0345984, the NSF
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