Approximate 2-degree-of-freedom digital control for a PFC boost converter

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Abstract: A PFC boost converter is a non-linear system whose small perturbation model is changed at each operating point depending on a duty ratio. The robust controller for the PFC boost converter is needed to suppress the output voltage change at load sudden change while attaining high power factor and low harmonic. In this paper, the combining method and design methods of two approximate 2-degree-of-freedom (A2DOF) controllers which simplify the overall controller and attain good robustness are proposed. The proposed controller is actually implemented on a Micro-processor and is connected to the PFC boost converter. Experimental studies demonstrate that the proposed robust controller is effective to suppress the output voltage change to improve power factor and to decrease harmonic.

Keywords: power factor correction, boost converter, approximate 2DOF, robust control, micro-processor

Classification: Electronic instrumentation and control

References

1 Introduction

Recent years, improving of power factor and reducing harmonic of power supply using non-linear electrical instrument are needed. Usually a current conduction mode boost converter is used for active power factor correction (PFC) in electrical instrument [1]. In a PFC boost converter, if a duty ratio, a load resistance and an input voltage are changed, the dynamic characteristics are varied greatly, that is, the PFC converter has non-linear characteristic. In many applications of the PFC converters, loads cannot be specified in advance, i.e., their amplitudes are suddenly changed from the zero to the maximum rating. These are prime reasons of difficulty for control of the PFC boost converter.

PID or root locus method etc. [1, 2, 3] have been considered in application of digital control for robustness. However, it is difficult to retain sufficient robustness and high power factor of the PFC converters by these techniques. A robust control method using an approximate 2-degree-of-freedom (A2DOF) for improving start-up characteristics and load sudden change characteristics of power converters has been proposed [4]. However, it was applied to buck DC-DC converters. The PFC boost converter needs a current controller and a voltage controller.

In this paper, the combining method and design method of two A2DOF controllers which simplify the overall controller and attain good robustness are proposed. This controller is actually implemented on a Micro-processor and is connected to the PFC converter. Experimental studies demonstrate that the digital controller designed by the proposed method satisfies the given specifications and is useful practically.

2 PFC boost converter

The PFC boost converter is shown in Fig. 1. In Fig. 1, the input AC voltage $v_{in}$ is 100 [VAC], the boost inductance $L_0$ is 150 [μH], the output capacitor $C_0$ is 940 [μF], the output load resistance $R_L$ changes from 300 [Ω] to 5 [kΩ] and the output voltage $v_o$ changes from 240 [VDC] to 385 [VDC]. And, $i_{in}$ is
a input AC current, \( i_L \) is an inductor current and \( v_{ac} \) is an absolute value of input AC voltage. Here, the inductor current \( i_L \) is controlled to follow the rectified input voltage \( v_{ac} \) for improving power factor, reducing harmonic and stablizing output voltage. Using a state-space averaging method, the static characteristics and the state equation of the controlled object in some neighborhood of some operating point become as follows [5]:

\[
V_s = \frac{(1 - \mu_s)V_i}{R_0 + (1 - \mu_s)^2}, \quad I_s = \frac{1}{R_L} \frac{V_s}{1 - \mu_s}
\]

\[
\dot{x}(t) = A_c x(t) + B_c u(t), \quad y(t) = C_c x(t)
\]

where

\[
A_c = \begin{bmatrix}
\frac{R_0}{L_0} & -\frac{1 - \mu_s}{L_0} & 1 \\
1 - \mu_s & -\frac{1}{L_0} & 0 \\
C_0 & -\frac{1}{R_L C_0} & 1
\end{bmatrix}, \quad B_c = \begin{bmatrix}
\frac{V_s}{L_0} \\
\frac{I_s}{L_0} \\
\frac{I_0}{C_0}
\end{bmatrix}, \quad C_c = \begin{bmatrix}
1 & 0 & 0
\end{bmatrix}
\]

\[
x(t) = \begin{bmatrix}
\Delta i_L(t) \\
\Delta v_o(t)
\end{bmatrix}, \quad y(t) = \begin{bmatrix}
y_i(t) \\
y_v(t)
\end{bmatrix} = \begin{bmatrix}
\Delta i_L(t) \\
\Delta v_o(t)
\end{bmatrix}
\]

\[
u(t) = \Delta \mu(t) = \mu - \mu_s
\]

Here the equivalent \( R_0 \) resistance of inductor is 1.8 [Ω]. \( \mu \) is a duty ratio, \( \mu_s \) is a value at each operating point and \( \Delta \mu \) is a small perturbation from each operating point. From Eqs. (1) and (2), the parameters in \( A_c \) and \( B_c \) depend on the duty ratio \( \mu_s \). Therefore, the converter response will be changed depending on the operating point and other parameter variation. The load change and the duty ratio change of the controlled object are parameter changes in Eq. (2). Such parameter changes can be transformed to the equivalent disturbances \( q_u, q_y, y_i, y_v \) as shown in Fig. 2. Therefore, what is necessary is just to constitute the control system which pulse transfer functions from the equivalent disturbances to the output \( y \) become as small as possible in their amplitudes to suppress the influence of these parameter change. The controller which satisfies the following specifications will be designed.

1. The input voltage \( v_{in} \) is 100 [V AC], and the output voltage \( v_o \) changes from 240 [V DC] to 385 [V DC].
2. The step responses of the output voltage are almost the same at resistive loads where \( 300 \leq R_L < 5k [\Omega] \), and over-shoots in these step responses are less than 10 [%].
3. The dynamic load response is smaller than 5% (19.3 [V DC]) against change of load between 30~500 [W].
4. The power factor is over 0.99 at full load, and the harmonic is less than the standard prescribed in IEC/EN61000-3-2.
5. The control bandwidth of current control system is about 10 [kHz] and the control bandwidth of voltage controller is about 1 [Hz] for satisfying Spec. 4.
Under these specifications, the following three operating points for deciding one controller was selected from the static characteristics as follows:

**Point 1**: The output voltage is 385 [VDC]. The resistive load is 5 [kΩ]

**Point 2**: The output voltage is 385 [VDC]. The resistive load is 300 [Ω]

**Point 3**: The output voltage is 240 [VDC], The resistive load is 300 [Ω]

The gain and phase characteristics of the PFC boost converter in the neighborhood of each operating point are different. The A2DOF controller is designed to one operating point selected from these.

### 3 Digital robust current controller

The continuous system of Eq. (2) is transformed into the discrete system. Here, in order to compensate the delay time \( L_d \approx T_2 \) caused by the AD conversion time and the micro-processor operation time etc., one delay (state \( \xi_1 \)) is introduced to the input of the controlled object. The state-space equation of the new controlled object with one delay is described as follows:

\[
\begin{align*}
 x_d(k+1) &= A_dx_d(k) + B_du(k) \\
 y(k) &= C_dx_d(k)
\end{align*}
\]

\( x_d(k) = \begin{bmatrix} x(k) \\ \xi_1(k) \end{bmatrix}, \quad A_d = \begin{bmatrix} e^{A_cT} \\ 0 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ e^{A_cT}B_cT \end{bmatrix}, \quad C_d = C_c \\
A_{dt} = \begin{bmatrix} A_d & B_d \\ 0 & 0 \end{bmatrix}, \quad B_{dt} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_{dt} = \begin{bmatrix} C_d & 0 \end{bmatrix}
\]

To the system of Eq. (3), the model matching control system is constituted using the state feedback. The transfer function from the reference input \( r_i' \) to the output \( y_i \) is specified as follows:

\[
W_{r_i'y_i}(z) = \frac{(1 + H_1)(1 + H_2)(1 + H_3)(z - n_{1i})(z - n_{2i})}{(z + H_1)(z + H_2)(z + H_3)(1 - n_{1i})(1 - n_{2i})}
\]

(4)

Here \( H_i, i=1,2,3 \) are the specified arbitrary parameters, \( n_{1i} \) and \( n_{2i} \) are the zeros of the discrete-time controlled object. In Fig. 2(a), \( q_u \) and \( q_y \) are the equivalent disturbances with which the parameter changes of the controlled object are replaced. The transfer function between \( Q_i = [q_u, q_y] \) and \( y_i \) is defined as \( W_{Q_i'y_i}(z) \). It shall be specified that the relation of \( H_1 \) and \( H_3 \) become \( |H_1| >> |H_3| \) and \( H_2 \approx n_{1i} \). Then \( W_{r_i'y_i} \) can be approximated as

\[
W_{r_i'y_i}(z) \approx W_{m_i}(z) = \frac{1 + H_1}{z + H_1}
\]

(5)

The system added the inverse system \( W_{m_i}^{-1}(z) \) and the filter \( K_i(z) \) to the system in Eq. (4) is constituted as shown in a current controller part of Fig. 2(b). In Fig. 2(b), the filter \( K_i(z) \) is as follows:

\[
K_i(z) = \frac{k_{z_i}}{z - 1 + k_{z_i}}
\]
The transfer functions of the system of Fig. 2 (b) from $r_i$ to $y_i$ and $Q_i$ to $y_i$ except for the voltage control system are given by the following equations.

\[
y_i = \frac{1 + H_1}{z + H_1} \frac{z - 1 + k_{z_i} W_{s_i}(z)}{W_{s_i}(z)} W_{s_i}(z) r_i
\]

\[
y_i = \frac{1 + H_1}{z + H_1} \frac{z - 1 + k_{z_i} W_{s_i}(z)}{W_{s_i}(z)} W_{Q_i y_i}(z) Q_i
\]

where

\[
W_{s_i}(z) = \frac{(1 + H_3)(z - n_{2_i})}{(z + H_3)(1 - n_{2_i})}
\]

Here, if $W_{s_i}(z) \approx 1$ in the frequency range which satisfies the relation of $|H_3| \ll |H_1|$ and $|H_3| \ll |1 - k_{z_i}|$, then the Eqs. (7), (8) become as follows:

\[
y_i \approx \frac{1 + H_1}{z + H_1} r_i
\]

\[
y_i \approx \frac{z - 1}{z + H_1} W_{Q_i y_i}(z) Q_i
\]

From Eqs. (9), (10), it turns out that the characteristics from $r$ to $y_i$ can be specified with $H_1$ and the characteristics from $q_u$ and $q_y$ to $y_i$ can be specified with $k_{z_i}$ independently. That is the system in Fig. 2 (b) is the approximate 2DOF system, and its sensitivity against the disturbances becomes lower with the increase of $k_{z_i}$.

### 4 Digital robust voltage controller

Add the multiplier in front of the reference input $r_i$ of the current control system. Let the inputs of the multiplier be $v_{ac}$ and the new input $u_v$. This addition is for making the inductor current $i_L$ follow the AC voltage $v_{ac}$.

The system of the current control system part of Fig. 2 (b) becomes the controlled object for the voltage controller. Derive the simple controlled object from this for designing the voltage controller. Let $u_v$ as $u_v = G_v r'_v$. $G_v$ is a gain for setting the DC gain of the transfer function from $r'_v$ to $y_v$ to 1. Then the transfer function of this controlled object is as follows:

\[
W_{r'_v y_v}(z) = \frac{(1 + H_2)(1 + H_1)(1 + p_{1_v})(1 + p_{2_v})(z - n_{1_v})(z - n_{2_v})}{(z + H_2)(z + H_1)(z + p_{1_v})(z + p_{2_v})(1 - n_{1_v})(1 - n_{2_v})}
\]

(11)

Here $n_{1_v}$, $n_{2_v}$ are zeros of the transfer function from $r'_v$ to $y_v$. In the system of Fig. 2 (b) except for the parts of $K_v(z)$ and $W_{m_v}^{-1}(z)$, $|H_2| > > (|H_1|, |p_{1_v}|, |p_{2_v}|)$, the controlled object $W_{r'_v y_v}$ for the voltage controller is approximated as:

\[
W_{r'_v y_v}(z) \approx W_{m_v}(z) = \frac{1 + H_2}{z + H_2}
\]

(12)

The inverse system $W_{m_v}^{-1}(z)$ and the filter $K_v(z) = k_{z_v}/(z - 1 + k_{z_v})$ are added to the current controller control part of shown in Fig. 2 (b). In the current control system of Fig. 2 (b), the transfer function between $Q_v = [q_u \ q_y]^T$ to $y_v$ is defined as $W_{Q_v y_v}(z)$. Then the transfer functions from $r_v$
to \( y_v \) and the equivalent disturbance \( Q_v \) to \( y_v \) of the system in Fig. 2 (b) are given by

\[
\begin{align*}
\text{to } y_v & = \frac{1 + H_2}{z + H_2} \left( z - 1 + k_{zv} \right) W_{s_v}(z) r_v \\
\text{to } y_v & = \frac{z - 1}{z - 1 + k_{zv}} \left( z - 1 + k_{zv} \right) W_{Q_v y_v}(z) Q_v
\end{align*}
\]

(13)

(14)

where

\[
W_{s_v}(z) = \frac{(1 + H_1) \left( 1 + p_{1v} \right) \left( 1 + p_{2v} \right) (z - n_{1v}) (z - n_{2v})}{(z + H_1) \left( 1 + p_{1v} \right) \left( 1 + p_{2v} \right) (z - n_{1v}) (1 - n_{2v})}
\]

Here, if \( W_{s_v}(z) \approx 1 \) in the frequency range which satisfies the relation of
Fig. 3. Experimental results.

\[ |H_1| \ll |H_2| \quad \text{and} \quad |H_1| \ll |-1 + k_{zv}|, \quad \text{then Eqs. (13), (14) becomes as follows:} \]

\[
y_v \approx \frac{1 + H_2}{z + H_2} r_v \quad (15)
\]

\[
y_v \approx \frac{z - 1}{z - 1 + k_{zv}} W_{Qv} y_v(z) Q_v \quad (16)
\]

That is, the voltage control system in Fig. 2 (b) is the A2DOF system. Transforming the controller in Fig. 2 (b) equivalently, the A2DOF digital PFC control system will be obtained as shown in Fig. 2 (c). All specifications will be satisfied with only one controller of Fig. 2 (c).

5 Experimental results

The controller is designed to Eq. (2) in the neighborhood of the operating point 2. In this experiment, the micro-processor SH7216 by Renesas Electronics Corp. is used. The experimental results of 20 [V\text{DC}] step responses at
each operating point using the proposed controller and the PI controller are shown in Fig. 3 (a) and (b), respectively.

Fig. 3 (a) shows that even if $R_L$ and $V_s$ are changed by change of the operating point, the step responses are not changed. It turns out that the reference characteristics are maintained without being affected by the parameter variations, i.e., the voltage control system is the A2DOF system and the disturbance gain characteristics are small enough.

Fig. 3 (b) shows that the step responses are changed by change of the operating points and have over-shoots. It turns out that since the PI control system is the 1DOF, Spec. (2) is not satisfied.

The experimental result of the steady state at operating point 2 and point 3 are shown in Fig. 3 (c), (d), (e), (f). In Fig. 3 (c) and (d), the input current waveform and phase are almost same as the input voltage waveform and power factor of converter at full load is 0.994. It is checked that harmonic satisfies the Spec. (4). In Fig. 3 (e), the input current waveform is the almost same as the input voltage waveform even if the operating point is changed. That is, the proposed current control system is the A2DOF and the disturbance gain characteristics are small enough. However, in Fig. 3 (f), the input current waveform differs from the input voltage waveform considerably. That is, the PI current control system is not robust.

The experimental result of load sudden change using the proposed controller and the PI controller are shown in Fig. 3 (g), (h), respectively. In Fig. 3 (g), the output voltage variation in sudden load change is less than $10 \text{[V}_{\text{DC}}\text{]}(2.60 \%)$ using the proposed controller. Since the proposed control system can set up the gain characteristics of the transfer function from $Q_v$ to $y_v$ small regardless of the reference characteristics, the variation of the output voltage can be suppressed small. In Fig. 3 (h), the output voltage variation is over $20 \text{[V}_{\text{DC}}\text{]}(5.19 \%)$ using the PI controller and it cannot satisfy Spec. (3). It turns out that the proposed controller is effective practically.

6 Conclusion

In this paper, the concept of the digital controller which attains good robustness for the non-linear PFC boost converter was given. The proposed digital controller was implemented on the micro-processor. It was shown from experiments that the digital controller combined by the two A2DOF can suppress the variations of the step responses at different operating points and the output voltage variations at sudden load changes while attaining high power factor and low harmonic. This fact demonstrates the usefulness and practicality of our proposed method.