Network Cosmology

Dmitri Krioukov
CAIDA/UCSD
M. Kitsak, R. Sinkovits,
D. Rideout, D. Meyer, M. Boguñá
NetSci, Chicago, June 2012

Introduction

- Universal laws of network dynamics
- I will not tell you what they are exactly
- But they are unexpectedly similar to the laws of gravity in general relativity
- Complex networks and quantum networks, representing the spacetime of our universe at the Planck scale, exhibit:
  - Similar structure
  - Asymptotically identical dynamics of growth

Dynamics of complex networks

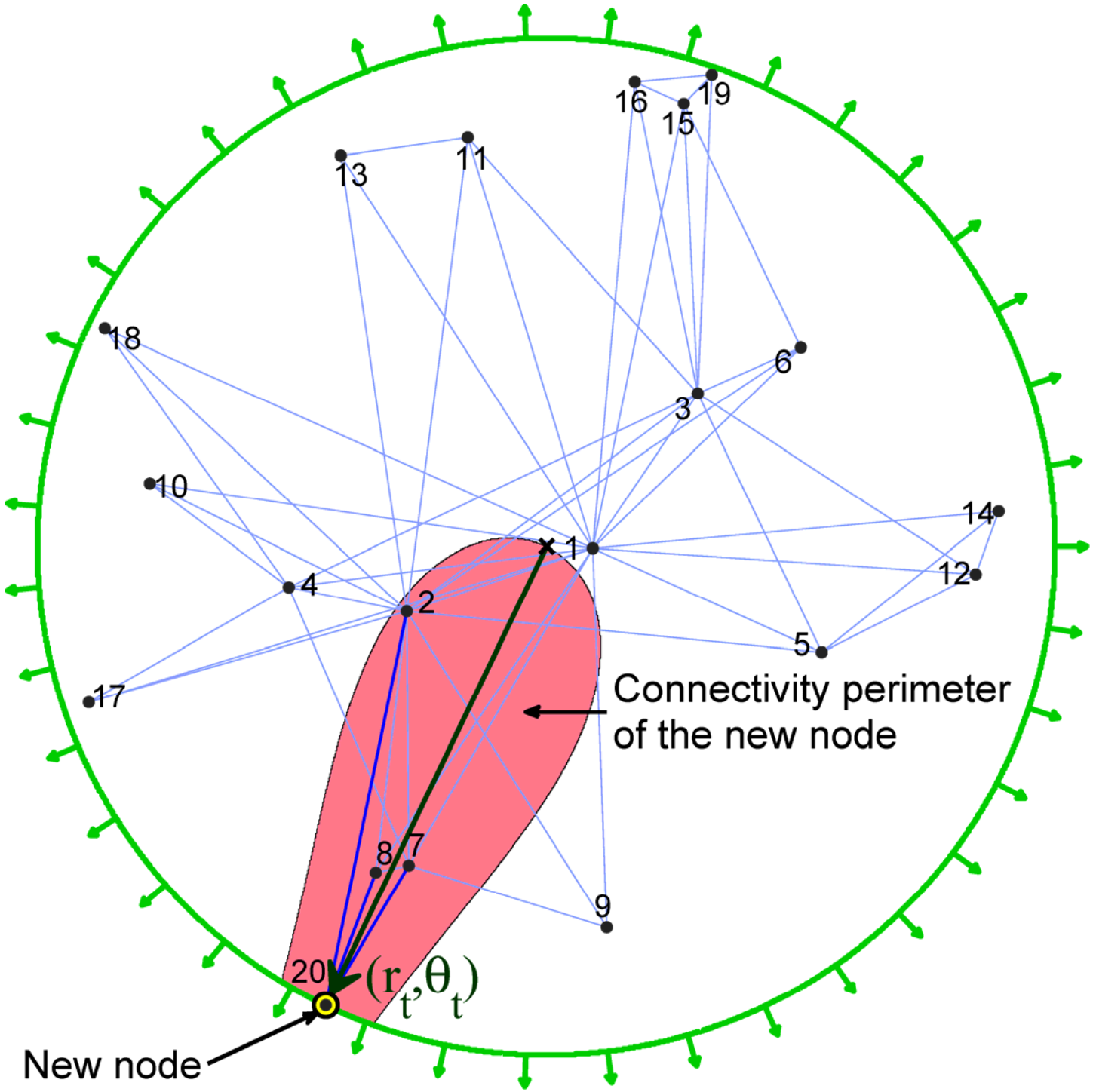
- Popularity is attractive, but so is similarity
- Growth model with trade-off optimization between popularity and similarity
  - Popularity ~ node birth time
  - Similarity ~ distances on a sphere
- Growing random geometric graphs in hyperbolic spaces
- ***Directly*** validated against the growth history of real networks (Internet, social trust, metabolic)

Random geometric graphs in hyperbolic spaces

- Take a compact region in a ***hyperbolic*** space, e.g., a circle of radius R
- Sprinkle N nodes into it via the Poisson point process ($R \sim \ln N$)
- Connect each pair of nodes if the distance between them is $x < R$
- Result: power-law graphs (γ=3) with strongest clustering

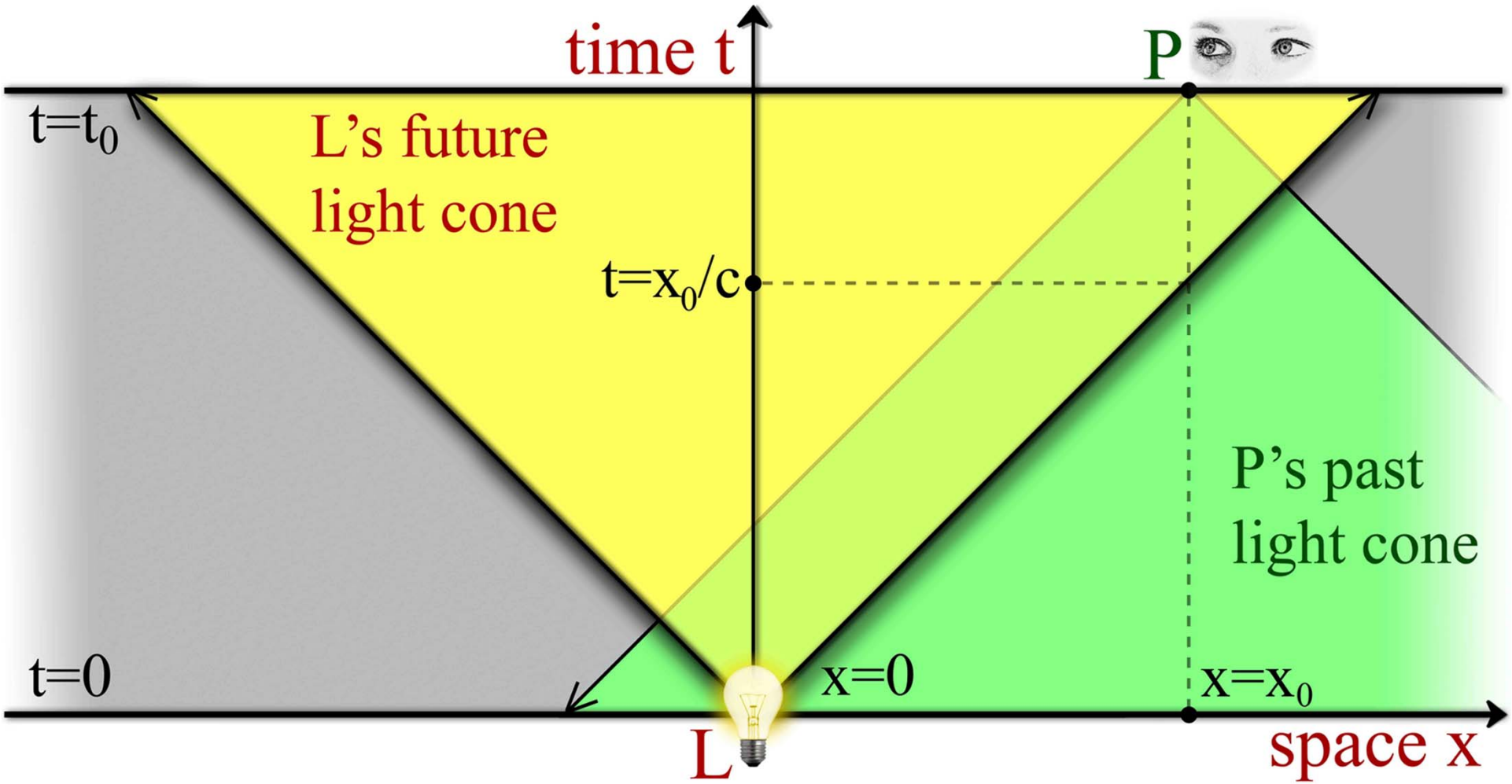
Growing geometric graphs in hyperbolic spaces

- Nodes join the network one by one $t = 1, 2, 3, \ldots$
- New node t appears at a random location on the edge of an expanding hyperbolic disc ($R \sim \ln t$)
- New node t connects to all existing nodes at hyperbolic distance $x < R$
- Result: power-law graphs ($\gamma=2$) with strongest clustering


16
19
15
13
11
18
6
3
10
14
1
12
4
2
5
17
Connectivity perimeter
of the new node
8
7
9
20
(r_t, θ_t)
New node

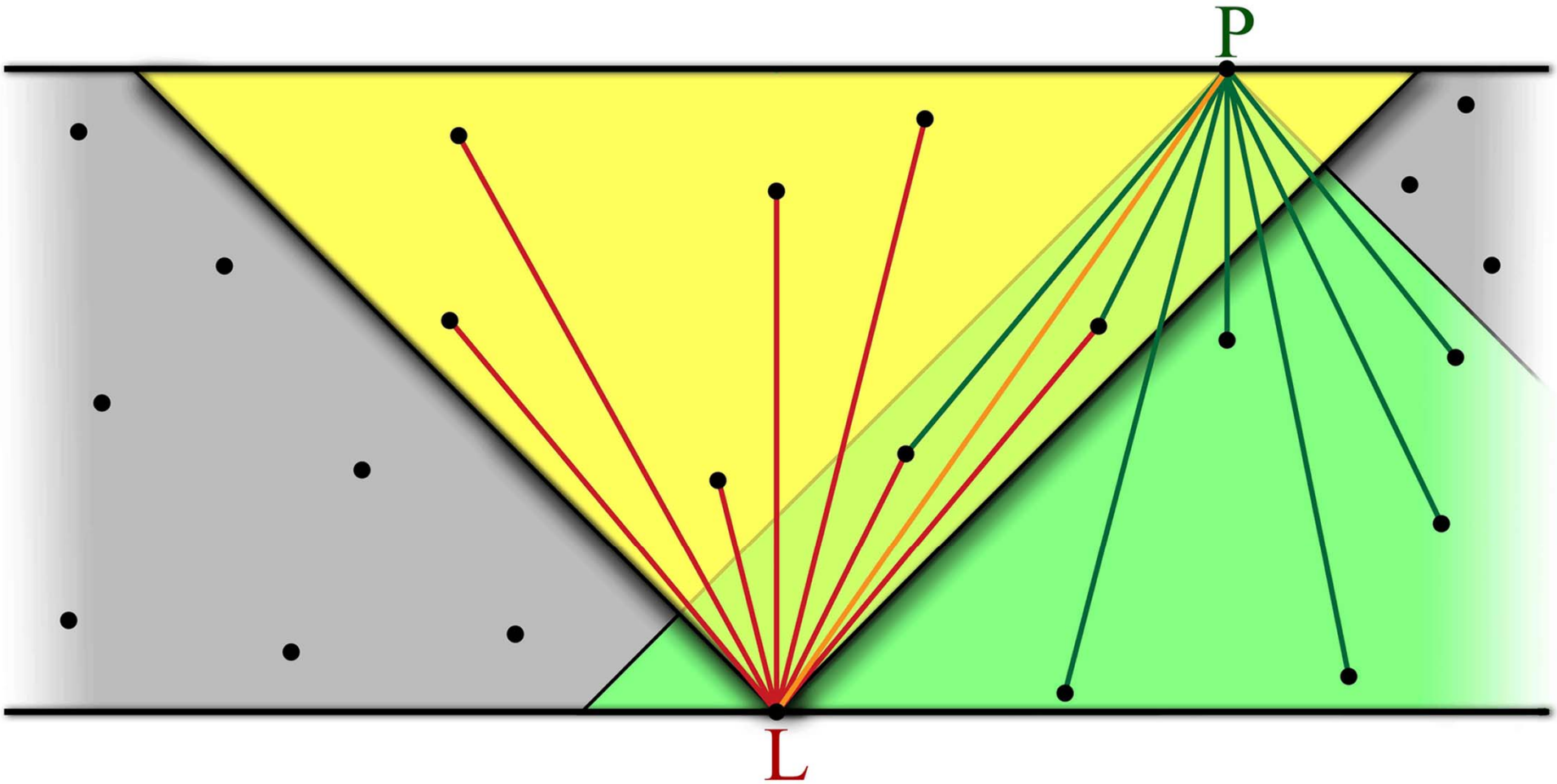
Relativistic spacetime

- Lorentzian manifold
- Distances can be
  - Negative (timelike)
  - Zero (lightcone)
  - Positive (spacelike)
- Causal structure
  - Defines the manifold up to a conformal factor


time t
P
t=t0
L's future
light cone
t=x0/c
P's past
light cone
t=0
x=0
x=x0
L
space x

Causal sets: Random Lorentzian graphs

- Take a compact region in a ***Lorentzian*** manifold
- Sprinkle N nodes into it via the Poisson point process
- Connect each pair of nodes if the distance between them is timelike ($x < 0$)
- Approach to quantum gravity (nodes are Planck-scale "atoms" of spacetime)

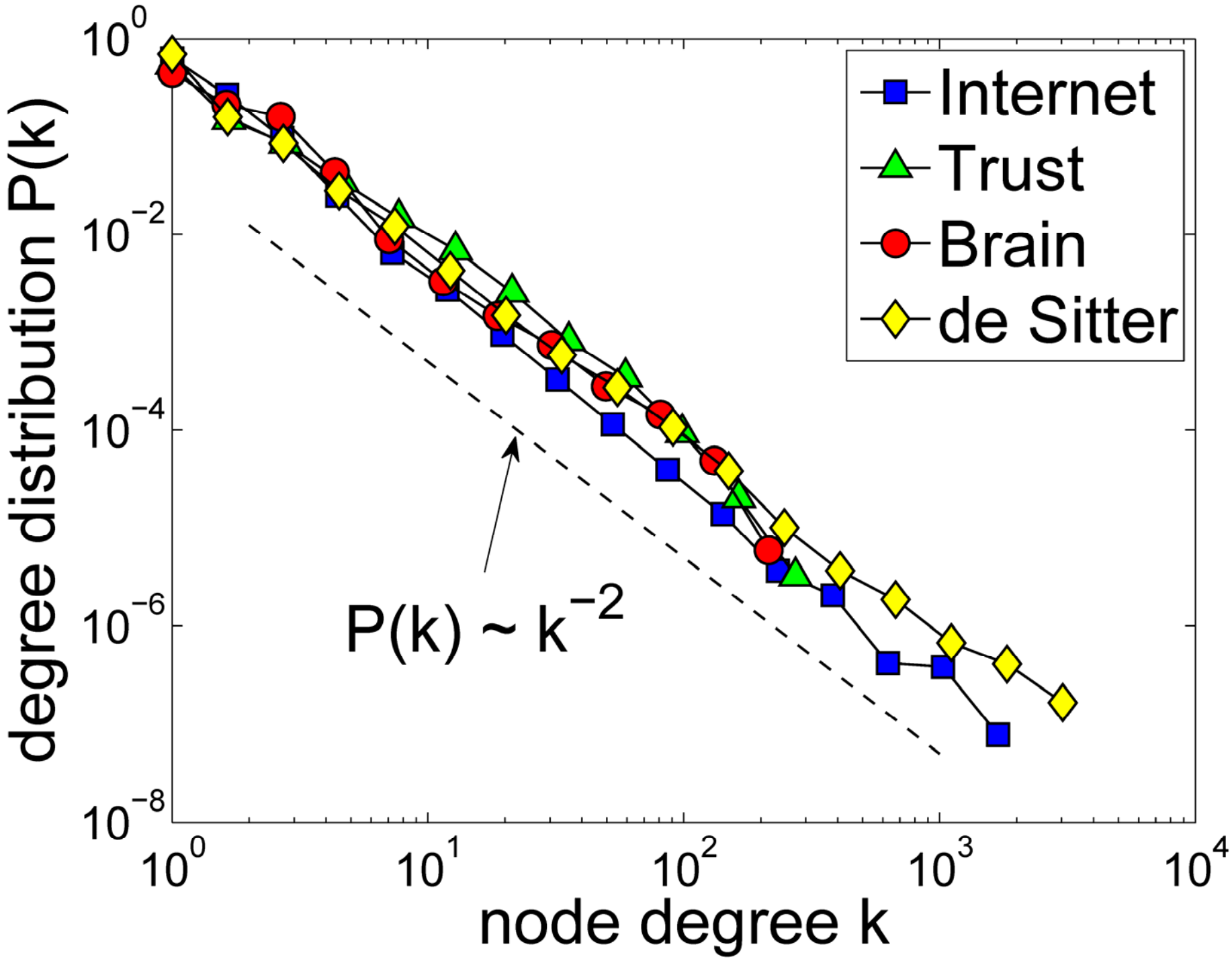

P
L

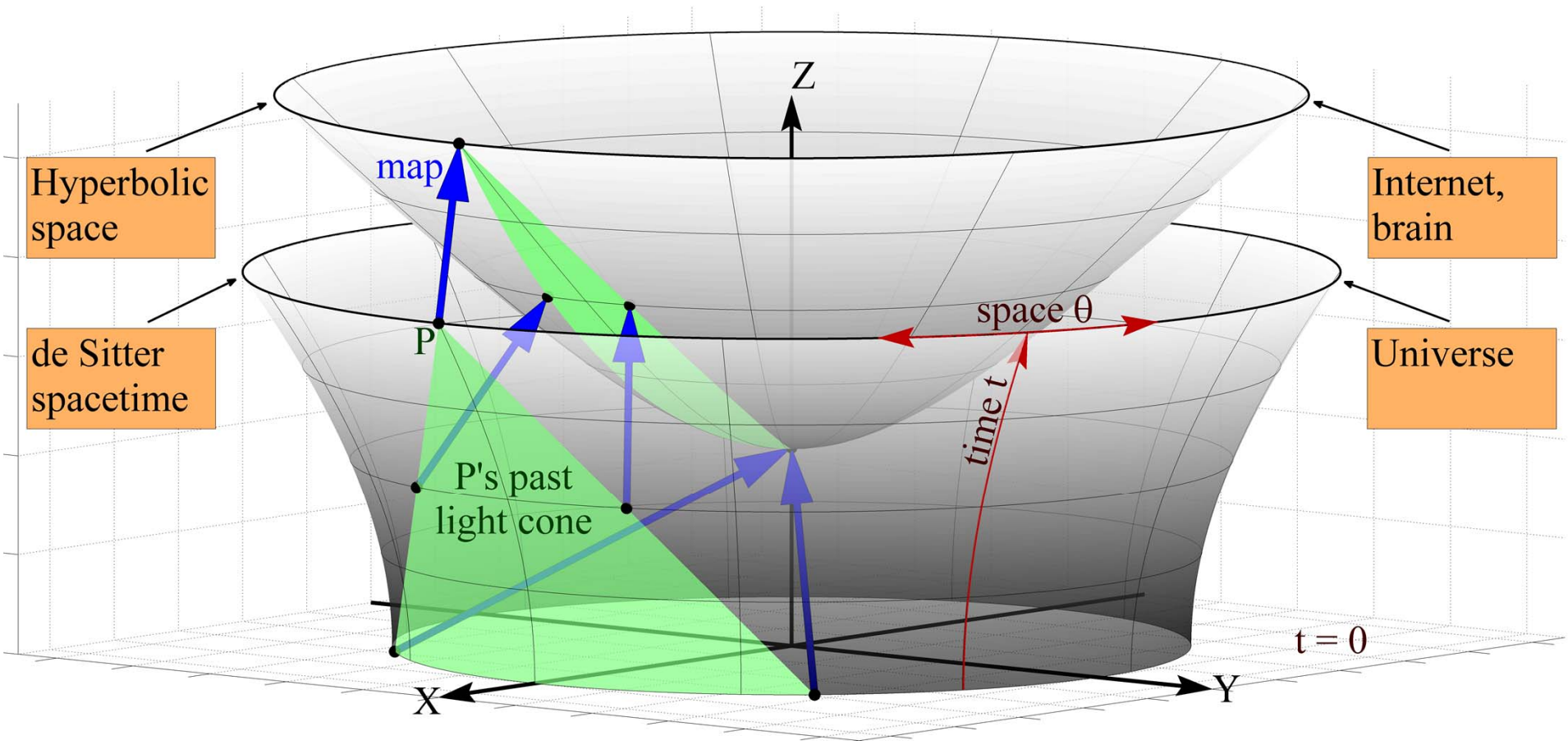
Universe

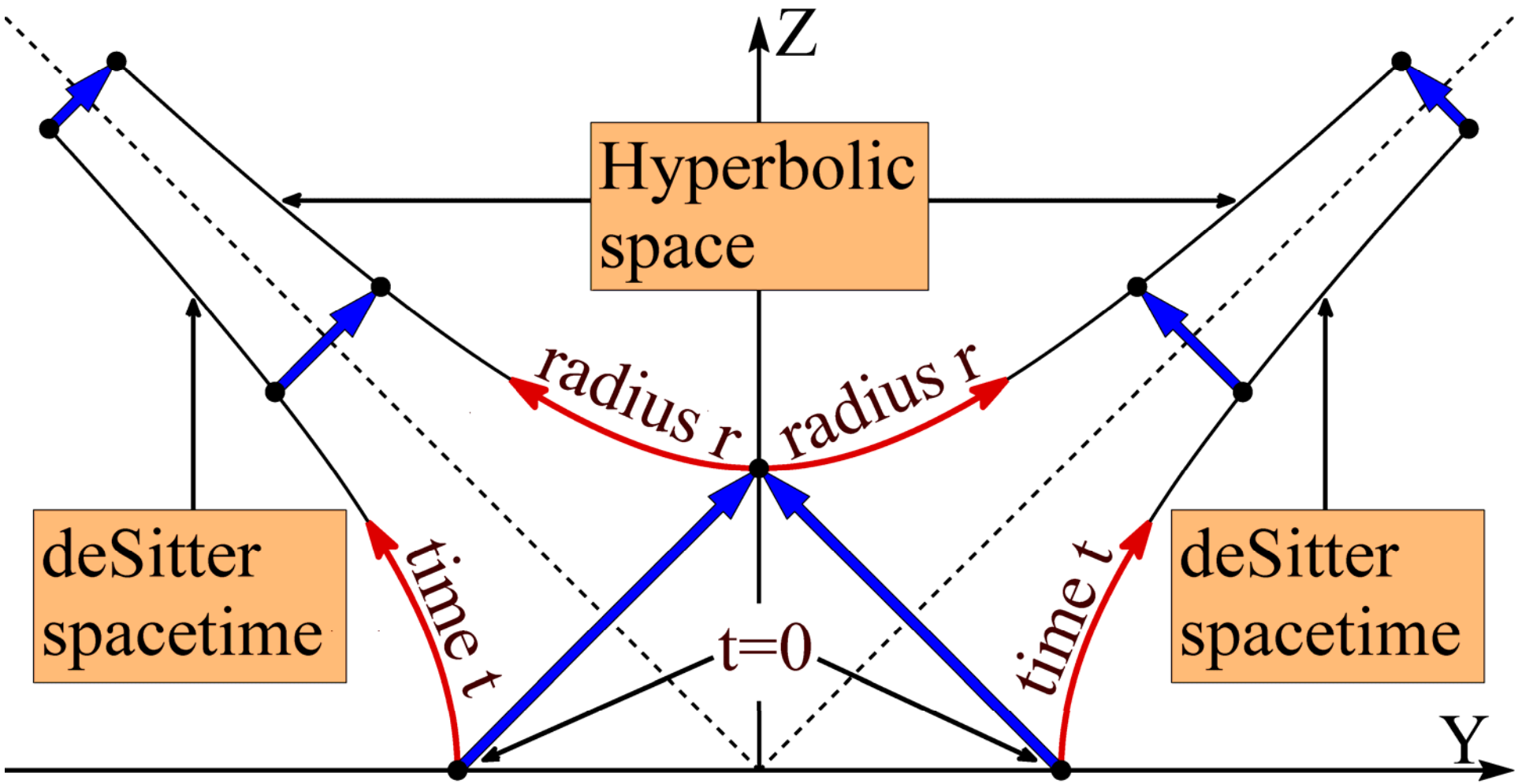
- Expands
- Accelerates (Nobel prize)
- Is asymptotically de Sitter spacetime

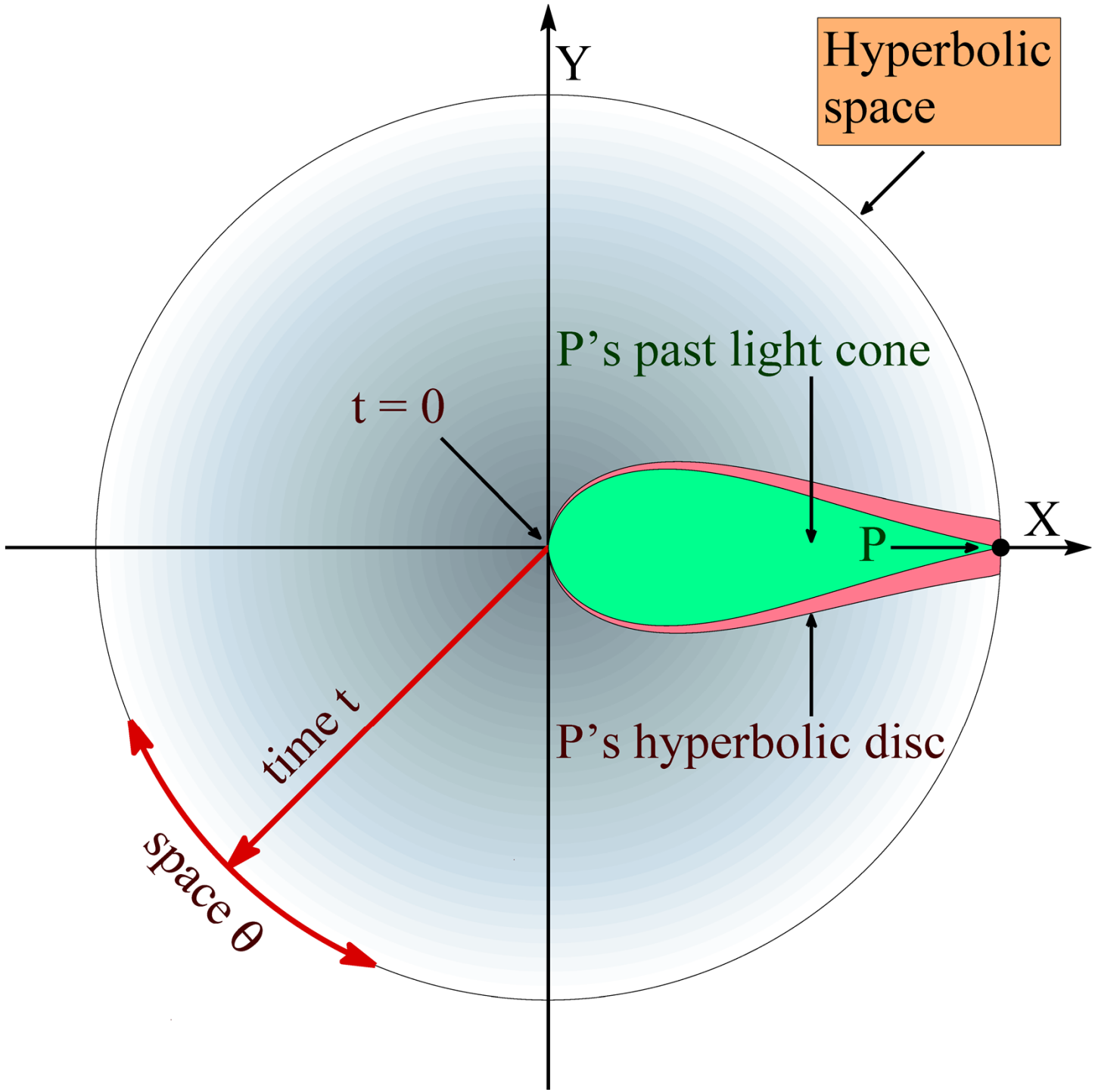
Main results:
Causal sets in de Sitter spacetime

- Are power-law graphs ($\gamma=2$) with strongest clustering
- Grow asymptotically identically to random geometric graphs in hyperbolic spaces (popularity × similarity optimization)

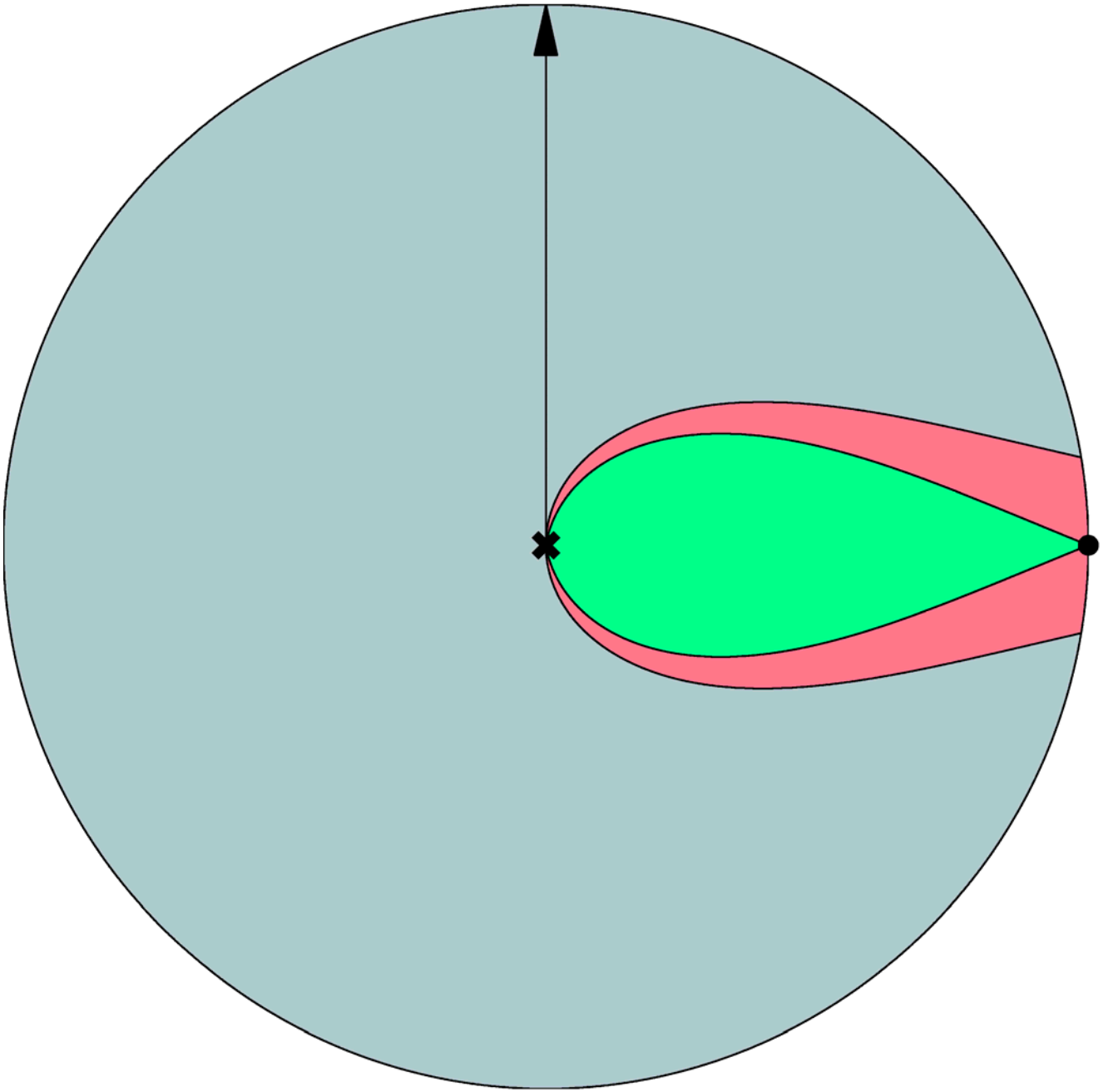

degree distribution P(k)
node degree k
Internet
Trust
Brain
de Sitter
P(k) ~ k^−2
10^0
10^−2
10^−4
10^−6
10^−8
10^1
10^2
10^3
10^4


Hyperbolic space
de Sitter spacetime
Internet, brain
Universe
Z
map
P
space θ
time t
P's past light cone
t = 0
X
Y

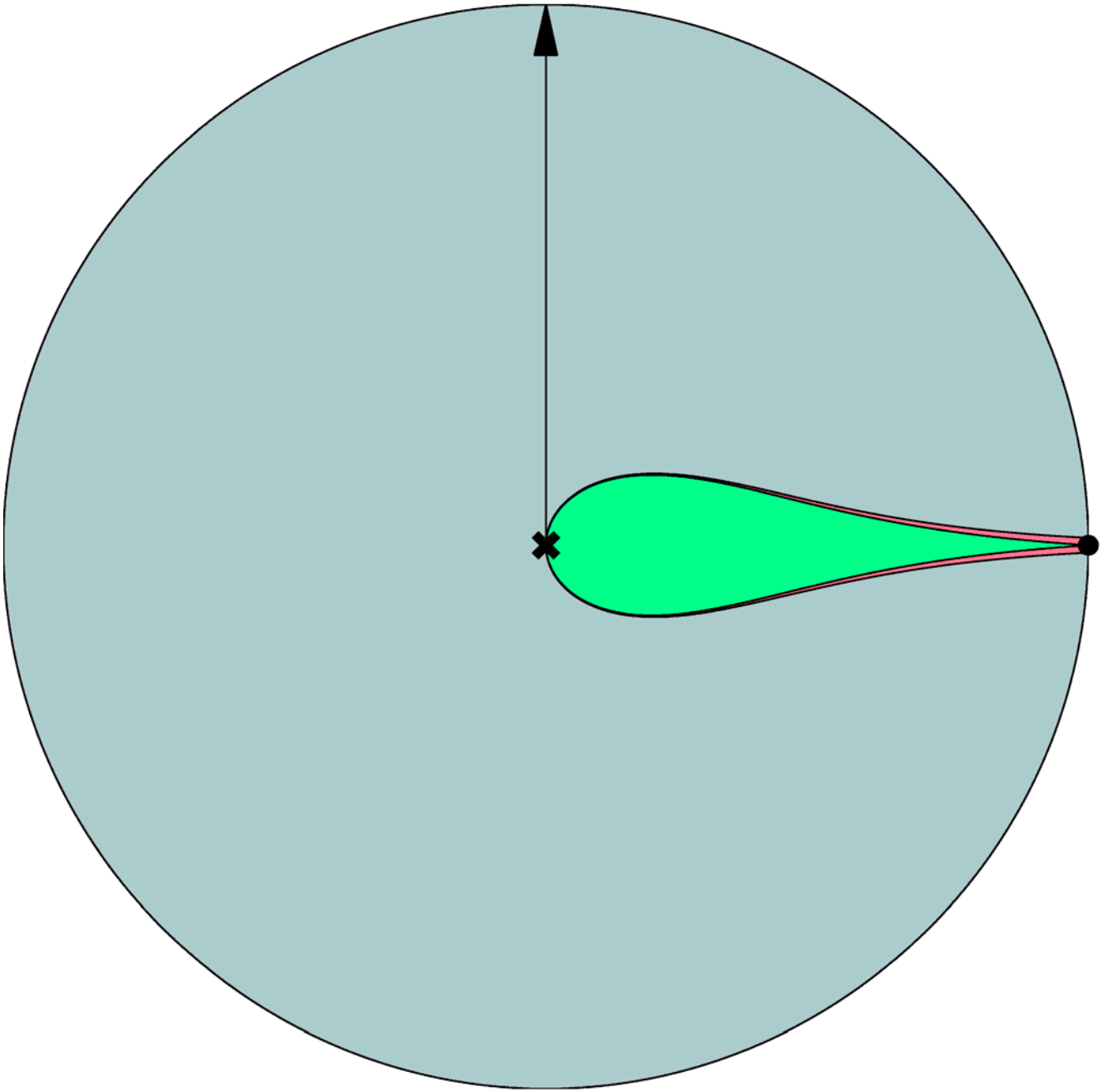

Z
Hyperbolic space
radius r
radius r
deSitter spacetime
time t
t=0
time t
deSitter spacetime
Y


Y
Hyperbolic space
P's past light cone
t = 0
X
P
time t
P's hyperbolic disc
space θ

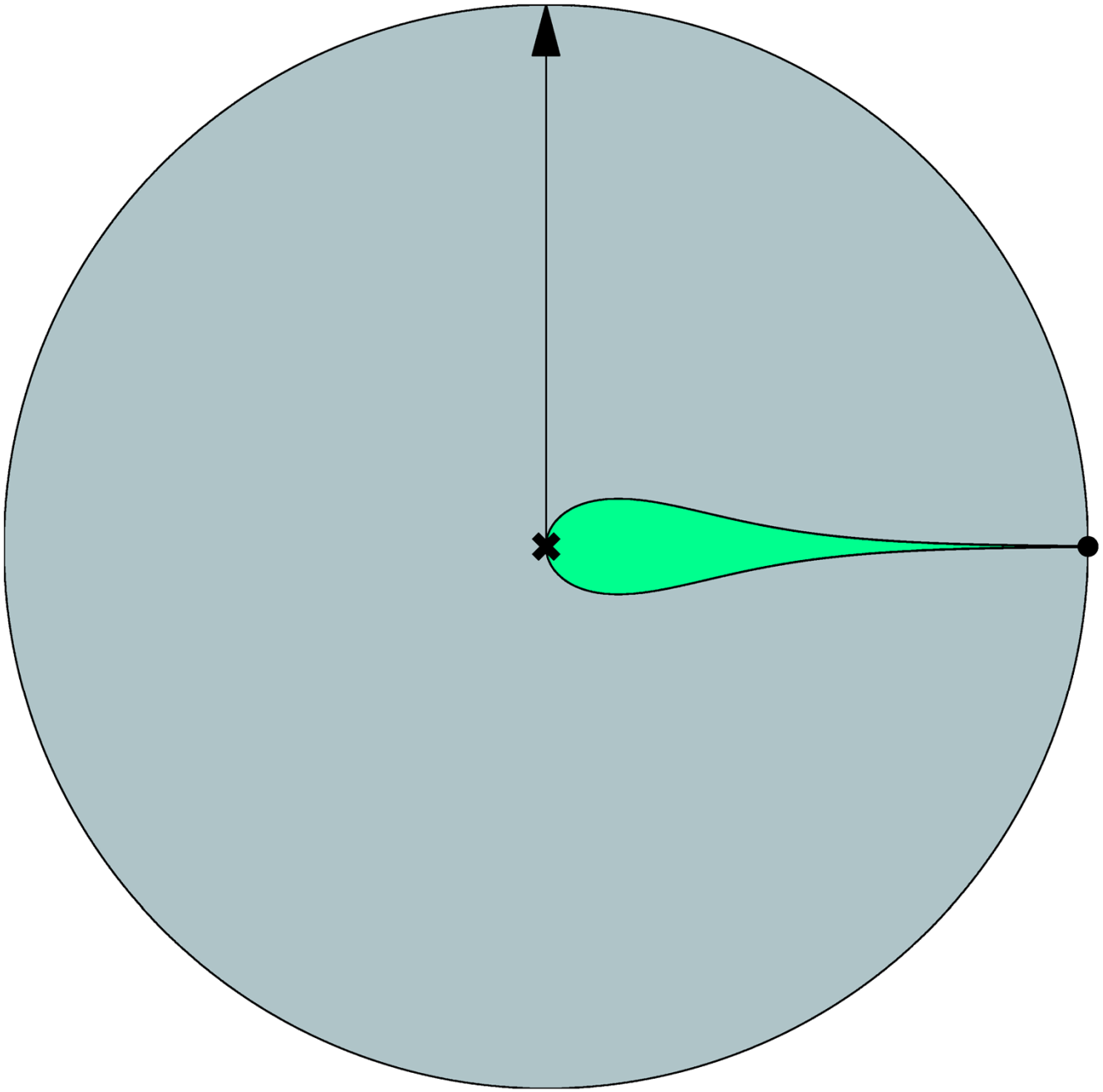
N = 40



N = 200



N = 2000



Conclusions

- The ***large-scale*** structure and dynamics of complex networks and de Setter causal sets are asymptotically identical
- De Sitter spacetime is a solution of Einstein's equations for an ***empty*** universe with positive vacuum energy (***dark energy***)
- Einstein's equations describe the ***large-scale*** dynamics of not only the universe but also of complex networks

Implications

- The universe is not empty: matter introduces inhomogeneities and anisotropies at smaller scales
- Distribution of nodes in hyperbolic spaces in real networks after mapping is not homogeneous either
- Einstein's equations may as well apply to complex networks at ***smaller scales*** describing gravity-like micro-dynamics of nodes and links
- Dark energy may be an illusion emerging from yet unknown optimization principles