Symmetry Breaking in Six Dimensional Flux Compactification Scenarios

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Motivated by the electroweak hierarchy problem, we consider theories with two extra dimensions in which the four-dimensional scalar fields are components of gauge boson in full space, namely the Gauge-Higgs unification framework. We briefly explain the basics features of "flux compactification", i.e. compactification in presence of a background (magnetic) flux. In particular we recall how chirality and symmetry breaking can be obtained in this context. More in details, we find and catalogue all possible degenerate zero-energy stable configurations in the case of trivial or non-trivial 't Hooft flux, for a SU(N) gauge theory on a torus. We describe the residual symmetries of each vacua and the four-dimensional effective spectrum in terms of continuous and discrete parameters, respectively.

1 Introduction

All available data agree in indicating that the mass of the Higgs boson is of the order of the Electroweak scale, $v \sim \mathcal{O}(200)$ GeV. Such a mass is unnaturally light if there is new physics beyond the Standard Model (SM) to which the Higgs boson is sensitive. In fact the Higgs mass parameter is not protected by any symmetry and thus gets corrections which are quadratically dependent on possible higher scales, like the unification scale M_{GUT} or, ultimately, the Planck scale M_{Pl}.

Three different mechanisms have been devised in order to eliminate the quadratic sensitivity of the Higgs mass to the cutoff scale:

- *Supersymmetry*: bosonic and fermionic contributions to the quadratic divergences cancel each other in such a way that the Higgs mass remains affected only by a logarithmic sensitivity to the cutoff scale;

- *Technicolor* and *Little Higgs*: the Higgs is a Goldstone boson of a global custodial symmetry that it is only softly (spontaneously) broken;

- *Gauge-Higgs Unification*: the Higgs is a component of a higher dimensional gauge multiplet. The lightness of its mass is protected by the gauge symmetry itself.

Independently of the precise nature assumed for the Higgs field, all these proposals require, in one way or another, the appearance of new physics at about the TeV scale. While the first two approaches are being intensely studied, in practice they tend to be afflicted by rather severe fine-tuning requirements when confronted with present experimental data. Here, instead, we concentrate on the last and less explored possibility: Gauge-Higgs unification [1]. The idea is that a single higher dimensional gauge field gives rise to all the four-dimensional (4D) bosonic degrees of freedom: the gauge bosons, from the ordinary space-time components and the scalar

bosons (and the Higgs fields among them) from the extra ones. The essential point concerning the solution of the hierarchy problem is that, although the higher dimensional gauge symmetry is globally broken by the compactification procedure, however it always remains locally unbroken. Any local (sensitive to the UV physics) mass term for the scalars is then forbidden by the gauge symmetry and the Higgs mass would then only have a non-local and UV finite origin.

This idea has been widely investigated in the context of five- and six-dimensional orbifold compactification[2]. From the field theory point of view, a different and less explored possibility is to recover the idea of Gauge-Higgs unification in the context of *flux compactification*: compactification of the extra space-like dimensions on a manifold in which there exist a (gauge) background with a non-trivial field strength, compatible with Scherk-Schwarz periodicity conditions[3]. We'll review in the following the basic idea of five– and six-dimensional SS compactifications.

2 Scherk-Schwarz mechanism in five- and six-dimensional compactifications

Let's consider a $U(N)$ gauge theory on a $(4 + d)$-dimensional space-time [a] where the extra dimensions are compactified on an orthogonal d-dimensional torus $\mathcal{T}^d$. To completely define a field theory on a torus one has to specify the periodicity conditions: that is, to describe how the fields transform under the fundamental shifts $y \to y + l_a$, with l_a being the lengths of the non-contractile cycles of the torus. Let's denote with T_a the embeddings of these shifts in the fundamental representation of $U(N)$. If we want to preserve four-dimensional Poincaré invariance, the twists T_a must depend only on the extra-dimensional coordinates. The most general periodicity conditions for the gauge field A_M and for a generic field Φ in the fundamental representation of $U(N)$, read respectively:

$$\mathbf{A_M}(x, y + l_a) = T_a(y) \left[\mathbf{A_M}(x, y) + \frac{i}{g}\partial_M \right] T_a^\dagger(y) , \tag{1}$$

$$\Phi(x, y + l_a) = T_a(y)\, \Phi(x, y) . \tag{2}$$

These equations are derived from the fact that while individual (gauge or matter) fields may not be single-valued on the torus, any physical scalar quantity, like the Lagrangian, must be. The periodicity conditions in Eqs. (1,2) are usually referred as Scherk-Schwarz (SS) boundary conditions [3]. Let's describe in more details the five- and six- dimensional compactification procedure in presence of general SS boundary conditions.

2.1 Scherk-Schwarz mechanism in five-dimensions

In the case of a five-dimensional theory compactified on a circle S^1 one has to define a single twist matrix $T(y)$. No restrictions have to be imposed on T except that it belongs to the $U(N)$ gauge group [b]. The $U(N)$ twist matrix can be, locally, decomposed as the product of an element $e^{iv(y)} \in U(1)$ and an element $\mathcal{V}(y) \in SU(N)$ as follows:

$$T(y) = e^{iv(y)}\, \mathcal{V}(y) . \tag{3}$$

It is always possible to choose a gauge, called the *symmetric* gauge[5], in which the $SU(N)$ vacuum configurations are trivial and the twist matrix is constant $\mathcal{V}^{sym} = V$ and can be parametrized as:

$$V = e^{2\pi i(\alpha \cdot H)} \qquad , \qquad \alpha \cdot H \equiv \sum_{j=1}^{N-1} \alpha^j H_j , \tag{4}$$

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[a] Throughout the paper, with x and y we will denote the four-ordinary and d-extra coordinates, respectively. Latin upper case indices M, N will run over all the extra-dimensional space, whereas greek and latin lower case indices μ, ν and a, b will run over the four ordinary and the extra-dimensions, respectively.

[b] The case of external automorphisms is not considered here. See for example [4]

where H_j are the $(N-1)$ generators of the Cartan subalgebra of $SU(N)$ and α_j are $(N-1)$ real continuous parameters $0 \leq \alpha_j < 1$. These parameters are non-integrable phases, which arise only in a topologically non-trivial space and cannot be gauged-away. When all the α_j are vanishing the periodicity conditions are trivial and consequently the initial symmetry is unbroken. If, instead, some of the α_j are non-vanishing, then symmetry breaking can occur. This mechanism is known as the *Scherk-Schwarz mechanism*[3].

In order to give an explicit expression for the gauge masses, one introduces the Cartan-Weyl basis for the $SU(N)$ generators. In addition to the Cartan subalgebra generators, H_j, one defines $N(N-1)$ non diagonal generators, E_r such that the following commutation relations are satisfied:

$$[H_j, H_k] = 0 \quad , \quad [H_j, E_r] = q_r^j E_r \, . \tag{5}$$

In this basis, the twist V acts in a diagonal way, that is

$$V H_j V^\dagger = H_j \quad , \quad V E_r V^\dagger = e^{2\pi i \, (\alpha \cdot q_r)} \, E_r \, , \tag{6}$$

and the four-dimensional mass spectrum reads simply:

$$m_{(k)}^2 = \frac{4\pi^2}{l^2} \, (n + \alpha \cdot q_k)^2 \quad , \quad n \in \mathbb{Z} \, . \tag{7}$$

For field components associated to a generator belonging to the Cartan subalgebra, H_j, one has $q_j = (0, ..., 0)$ and the spectrum reduce to the ordinary Kaluza-Klein (KK) one. For field components associated to the non-diagonal generators, E_r, one has, instead, $q_r \neq (0, ..., 0)$ and the mass spectrum is consequently shifted by a factor proportional to the non-integrable phases α^j. When all the $\alpha^j \neq 0$, then only the gauge field components associated to the generators of the Cartan subalgebra are massless. Therefore, the symmetry breaking induced by the twists, V, does not lower the rank of $SU(N)$. The maximal symmetry breaking pattern that can be achieved for an $U(N)$ symmetry group is given by:

$$U(N) \sim U(1) \times SU(N) \rightarrow U(1) \times U(1)^{N-1} = U(1)^N. \tag{8}$$

Scherk-Schwarz symmetry breaking mechanism can be used to break both global (flavour symmetries, supersymmetry) or local symmetries. In the case of gauge symmetry breaking the SS phase, α, can be interpret as the vev of the extra-dimensional component of the gauge fields, $\langle A_5 \rangle$. At classical level the scalar potential is flat and consequently the phases α^i are undetermined. Their values must be dynamically determined at the quantum level[10,5] minimizing the one-loop effective potential. If, at the minimum, any of the α_i is non-vanishing then the gauge symmetry is spontaneously broken. This dynamical and spontaneous symmetry breaking mechanism is conventionally known as the *Hosotani mechanism*. At the same time, the extra-dimensional component of the gauge field, A_5, is a scalar field that can be identified with the Higgs field and that acquires a finite mass term. The non-local nature of this symmetry breaking protects the theory from ultraviolet divergences and makes it a promising candidate mechanism to break the electroweak symmetry and to provide an Higgs field free from quadratic divergences.

2.2 *Scherk-Schwarz mechanism in six-dimensions*

In the case of a six-dimensional theory compactified on a torus T^2, one can introduce a different twist, $T_a(y)$, along each of the two independent cycles. The twists cannot be chosen arbitrarily but they have to satisfy the following $U(N)$ 't Hooft consistency condition[6,7]:

$$T_1(y + l_2) \, T_2(y) \;=\; T_2(y + l_1) \, T_1(y) \, . \tag{9}$$

This condition is obtained imposing that (for any fields included in the theory) the value of the field at the final point $(y_1 + l_1, y_2 + l_2)$, starting from the initial point (y_1, y_2) has to be independent on the followed paths.

The $U(N)$ twist matrices can be, locally, decomposed as the product of an element $e^{iv_a(y)} \in U(1)$ and an element $\mathcal{V}_a(y) \in SU(N)$ as follows:

$$T_a(y) = e^{iv_a(y)} \mathcal{V}_a(y). \tag{10}$$

Using this parametrization, the consistency conditions in Eq. (9) can be splitted into the $SU(N)$ and $U(1)$ part, respectively:

$$\mathcal{V}_1(y + l_2) \mathcal{V}_2(y) = e^{2\pi i \frac{m}{N}} \mathcal{V}_2(y + l_1) \mathcal{V}_1(y) \tag{11}$$

$$\Delta_2 v_1(y) - \Delta_1 v_2(y) = 2\pi \frac{m}{N}, \tag{12}$$

with $\Delta_a v_b(y) = v_b(y + l_a) - v_b(y)$. The $SU(N)$ consistency condition, Eq. (11), tells us that the twists, $\mathcal{V}_a$ must commute, on the fundamental plaquette, modulo a phase factor belonging to the center of $SU(N)$. The integer $m = 0, 1, .., (N-1)$ (modulo N) is a gauge invariant quantity called the non-abelian 't Hooft flux[6]. Furthermore, Eq. (12) tells us that it must coincide with the value of a quantized abelian magnetic flux living on the torus or, in other words, with the first Chern class of $U(N)$ on $\mathcal{T}^2$.

It is well known, that the presence of a stable magnetic background, associated with the abelian subgroup $U(1) \in U(N)$ and living only on the two extra dimensions, can induce chirality[8] in four-dimensions. A non-vanishing value of the 't Hooft flux m is indeed necessary for having four-dimensional chiral matter fields. A general description of fermions and chirality in the context of 6D $U(N)$ theories compactified on a two-dimensional torus can be found in [9].

From the other side, from Eqs. (11,12) it appears evident that the presence of the quantized abelian magnetic flux deeply affects the non-abelian subgroup $SU(N) \in U(N)$, giving rise to a non-trivial 't Hooft non-abelian flux. While the symmetry breaking pattern for a $SU(N)$ theory in presence of trivial non-abelian 't Hooft flux ($m = 0$) is well-known in the literature[10,5], the field theory and phenomenological analysis of the non-trivial ($m \neq 0$) 't Hooft flux has been explored only recently, in [11,12]. Here, it has been shown that exists a gauge, denominated, as the five dimensional case, the *symmetric gauge*, in which the $SU(N)$ twists can always be chosen as constant, i.e. $\mathcal{V}_a^{sym} = V_a$, with V_a constant matrices satisfying the $SU(N)$ 't Hooft consistency conditions:

$$V_1 V_2 = e^{2\pi i \frac{m}{N}} V_2 V_1. \tag{13}$$

In the symmetric gauge, the $SU(N)$ vacuum configurations are trivial and therefore the residual symmetries of each classical vacua are those associated to the $SU(N)$ generators which commute simultaneously with V_1 and V_2. The number of classical vacua and the pattern of symmetry breaking depend on the values of m and they will be analyzed in the following section.

3 $SU(N)$ Symmetry Breaking: trivial vs non-trivial 't Hooft flux

The main purpose this section is to find and classify all possible vacua and to describe the residual symmetries for an effective four-dimensional theory obtained from a $SU(N)$ gauge theory on a six-dimensional space-time where the two extra dimensions are compactified on a torus, for both the cases of trivial and non-trivial 't Hooft non-abelian flux.

3.1 Trivial 't Hooft flux: $m = 0$

In the $m = 0$ case, Eq. (13) tell us that the two V_a matrices commute and consequently can be parametrized as:

$$V_a = e^{2\pi i(\alpha_a \cdot H)} \quad , \quad \alpha_a \cdot H \equiv \sum_{j=1}^{N-1} \alpha_a^j H_j \tag{14}$$

with H_j the $(N-1)$ generators of the Cartan subalgebra of $SU(N)$. The periodicity conditions, and consequently the classical vacua, are now characterized by $2(N-1)$ real continuous parameters, $0 \leq \alpha_a^j < 1$. As in the five-dimensional case these parameters are non-integrable phases, which arise only in a topologically non-trivial space and cannot be gauged-away. When all the α_a^i are vanishing the initial symmetry is unbroken. At classical level α_a^i are undetermined. Their values must be dynamically determined at the quantum level where the rank-preserving Hosotani symmetry breaking mechanism can occur.

The mass spectrum of the four-dimensional gauge and scalar components of the 6D gauge field follows straightfully the five-dimensional discussion. In the Cartan-Weyl basis Eq. 5, the twists V_a act in a diagonal way, that is

$$V_a H_j V_a^\dagger = H_j \quad , \quad V_a E_r V_a^\dagger = e^{2\pi i\,(\alpha_a \cdot q_r)}\, E_r\,, \tag{15}$$

and the four-dimensional mass spectrum for gauge/scalar fields reads:

$$m_{(k)}^2 = 4\pi^2 \sum_{a=1}^{2} (n_a + \alpha_a \cdot q_k)^2 \frac{1}{l_a^2} \quad , \quad n_a \in \mathbb{Z}\,. \tag{16}$$

This is the same kind of spectrum seen previously in the five-dimensional case. For gauge (scalar) field components associated to a generator belonging to the Cartan subalgebra, H_j, the spectrum reduce to the ordinary Kaluza-Klein (KK) one. For gauge (scalar) field components associated to the non-diagonal generators, E_r, the mass spectrum is consequently shifted by a factor proportional to the non-integrable phases α_a^j. Therefore, the symmetry breaking induced by the commuting twists, V_a, does not lower the rank of $SU(N)$.

One can easily generalize these results to the $U(N)$ case adding an extra diagonal generator, $H_0 = \mathbb{1}_N/\sqrt{2N}$. Obviously H_0 commute with all the twists V_a and consequently A_M^0 always remains unbroken. The maximal symmetry breaking pattern that can be achieved in the $m = 0$ case, for an $U(N)$ gauge theory is given by:

$$U(N) \sim U(1) \times SU(N) \rightarrow U(1) \times U(1)^{N-1} = U(1)^N. \tag{17}$$

This symmetry breaking mechanism is exactly the same Hosotani mechanism one is used to in a five-dimensional framework.

3.2 Non-trivial 't Hooft flux: $m \neq 0$

In the $m \neq 0$ case, the twists V_a don't commute between themselves and so necessarily they induce a rank-reducing symmetry breaking [12]. The most general solution of the consistency relation Eq. (13) can be parametrized as follows:

$$V_1 = \omega_1\, P^{s_1}\, Q^{t_1} \quad , \quad V_2 = \omega_2\, P^{s_2}\, Q^{t_2}\,. \tag{18}$$

s_a, t_a are integers parameters taking values between $0, ..., (N-1)$ (modulo N) and satisfying the following constraint:

$$s_1\, t_2 \;-\; s_2\, t_1 \;=\; m/\mathcal{K} \equiv \tilde{m}\,. \tag{19}$$

P and Q are $SU(N)$ constant matrices given by

$$P \equiv P_{\widetilde{N}} \otimes \mathbb{1}_K \quad , \quad Q \equiv Q_{\widetilde{N}} \otimes \mathbb{1}_K \tag{20}$$

where $K \equiv$ g.c.d.(m, N) and $\widetilde{N} \equiv N/K$. $P_{\widetilde{N}}$ and $Q_{\widetilde{N}}$ are $\widetilde{N} \times \widetilde{N}$ matrices defined as

$$\begin{cases} \left(P_{\widetilde{N}}\right)_{kj} &= e^{i\pi \frac{\widetilde{N}-1}{\widetilde{N}}} \delta_{k,j-1} \\ \left(Q_{\widetilde{N}}\right)_{kj} &= e^{-2\pi i \frac{(k-1)}{\widetilde{N}}} e^{i\pi \frac{\widetilde{N}-1}{\widetilde{N}}} \delta_{kj} \end{cases} \qquad k,j = 1, 2, ..., \widetilde{N}, , \tag{21}$$

and satisfying the conditions

$$P_{\widetilde{N}} Q_{\widetilde{N}} = e^{-2\pi i \frac{1}{\widetilde{N}}} Q_{\widetilde{N}} P_{\widetilde{N}} \quad , \quad \left(P_{\widetilde{N}}\right)^{\widetilde{N}} = \left(Q_{\widetilde{N}}\right)^{\widetilde{N}} = e^{\pi i (\widetilde{N}-1)} . \tag{22}$$

When $K = 1$, then $\widetilde{N} = N$ and P, Q reduce to the usual elementary twist matrices defined by 't Hooft in [6].

The matrices ω_a are constant elements of $SU(K) \subset SU(N)$. They commute between themselves and with P and Q. Therefore ω_a can be parametrized in terms of generators H_j belonging to the Cartan subalgebra of $SU(K)$:

$$\omega_a = e^{2\pi i (\alpha_a \cdot H)} \quad , \quad \alpha_a \cdot H \equiv \sum_{\rho=1}^{K-1} \alpha_a^\rho H_\rho \tag{23}$$

Here α_a^ρ are $2(K-1)$ real continuous parameters, $0 \le \alpha_a^\rho < 1$. As in the $m = 0$ case, they are non-integrable phases and their values must be dynamically determined at the quantum level producing a dynamical and spontaneous symmetry breaking.

The $m \ne 0$ four-dimensional mass spectrum is easily obtained using the following basis [12] for the $SU(N)$ generators

$$\tau_{(\rho,\sigma)}(\Delta, k_\Delta) = \begin{cases} \text{if } \begin{cases} \rho = \sigma \\ \Delta = k_\Delta = 0 \end{cases} \Rightarrow \left(\sum_{i=1}^{\rho} \lambda_{(i,i)}^K - \rho \lambda_{(\rho+1,\rho+1)}^K\right) \otimes \mathbb{1}_{\widetilde{N}} \\ \\ \text{else} \qquad\qquad\qquad \Rightarrow \lambda_{(\rho,\sigma)}^K \otimes \tau^{\widetilde{N}}(\Delta, k_\Delta) \end{cases} \tag{24}$$

where Δ, k_Δ are integers assuming values between $0, \ldots, (\widetilde{N} - 1)$ while the indices ρ, σ take values between $1, \ldots, K$, excluding the case $\Delta = k_\Delta = 0, \rho = \sigma$ in which ρ takes values between $1, \ldots, (K-1)$. The matrices $\lambda_{(\rho,\sigma)}^K$ and $\tau^{\widetilde{N}}$ are $K \times K$ and $\widetilde{N} \times \widetilde{N}$ matrices, respectively, defined as:

$$\left(\lambda_{(\rho,\sigma)}^K\right)_{ij} = \delta_{\rho i} \delta_{\sigma j}$$

$$\tau^{\widetilde{N}}(\Delta, k_\Delta) = \sum_{n=1}^{\widetilde{N}} e^{2\pi i \frac{n}{\widetilde{N}} k_\Delta} \lambda_{(n,n+\Delta)}^{\widetilde{N}} . \tag{25}$$

The definition of $\lambda_{(n,n')}^{\widetilde{N}}$ comes straightforwardly.

In this basis, the $SU(K)$ generators that commute with P and Q are simply given by $\tau_{\rho,\sigma}(0,0)$. In particular, the generators belonging to the Cartan subalgebra of $SU(K)$ are given by $H^\rho = \tau_{\rho,\rho}(0,0)$. The following commutation relations are satisfied:

$$\left[\tau_{(\rho,\rho)}(0,0), \tau_{(\rho,\rho)}(0,0)\right] = 0 \quad , \quad \left[\tau_{(\rho,\rho)}(0,0), \tau_{(\sigma,\tau)}(\Delta, k_\Delta)\right] = q_\rho^{(\sigma,\tau)} \tau_{(\sigma,\tau)}(\Delta, k_\Delta) .$$

The action of the twists V_a on this basis is given by

$$V_a \, \tau_{(\rho,\sigma)}(\Delta, k_\Delta) \, V_a^\dagger \;\; = \;\; e^{\frac{2\pi i}{N}(s_a \Delta + t_a k_\Delta) + 2\pi i \, (\alpha_a \cdot q^{(\rho,\sigma)})} \; \tau_{(\rho,\sigma)}(\Delta, k_\Delta) \,, \tag{26}$$

and the four-dimensional mass spectrum takes the following form:

$$m^2_{(\rho,\sigma)}(\Delta, k_\Delta) = 4\pi^2 \sum_{a=1}^{2} \left(n_a + \frac{1}{\widetilde{N}} \left(s_a \, \Delta \; + \; t_a \, k_\Delta \right) \; + \; \alpha_a \cdot q_{(\rho,\sigma)} \right)^2 \frac{1}{l_a^2} \quad , \tag{27}$$

with $n_a \in \mathbf{Z}$. Therefore, beside the usual KK mass term, there are other two additional contributions. The first one, quantized in terms of $1/\widetilde{N}$, is a consequence of the non-trivial commutation rule of Eq. (22) between P and Q that induces the $SU(N) \to SU(\mathcal{K})$ symmetry breaking. Since s_a, t_a cannot be simultaneously zero, the spectrum described by Eq. (27) always exhibits some (tree-level) degree of symmetry breaking. Given a set of s_a, t_a and for all the $\alpha_a^\rho = 0$ (that is $\omega_a = 1$), only the gauge bosons components associated to $\tau_{(\rho,\sigma)}(0,0)$, the generators of $SU(\mathcal{K})$, admit zero modes. This is an explicit breaking. The second contribution to the gauge mass comes from the ω_a and it depends on the continuous parameters α_a^ρ. For $\mathcal{K} > 1$ and all the non-integrable phases $\alpha_a^\rho \neq 0$, the only massless modes correspond to the gauge bosons associated to the Cartan subalgebra of $SU(\mathcal{K})$, i.e. $\tau_{(\rho,\rho)}(0,0)$. The symmetry breaking pattern induced by the ω_a produce a Hosotani symmetry breaking [9] that does not lower the rank of $SU(\mathcal{K})$.

The maximal symmetry breaking pattern that can be achieved for an $U(N)$ gauge theory with matter fields in the fundamental is, in the $m \neq 0$ case, given by:

$$U(N) \sim U(1) \times SU(N) \to U(1) \times U(1)^{\mathcal{K}-1} = U(1)^{\mathcal{K}}. \tag{28}$$

Obviously, when $\mathcal{K} = 1$ the $SU(N)$ subgroup is completely broken, the only unbroken symmetry being the $U(1) \in U(N)$. This symmetry breaking pattern has no analogous in 5-dimensional frameworks and it's peculiar of higher dimensional models where (topological) fluxes can appear.

As a final comment on the spectrum, notice that in both the cases of trivial and non-trivial 't Hooft flux, the classical effective four-dimensional spectrum depends on the gauge indices but it does not depend on the Lorentz ones. This implies that at the classical level the 4D scalar fields A_a, arising from the extra-components of a six-dimensional gauge fields, are expected to be degenerate with the 4D gauge fields A_μ with the same gauge quantum numbers. This degeneracy is always removed at the quantum level [9].

4   Conclusions

In this paper we have analyzed possible symmetry breaking mechanism in the context of Gauge-Higgs unification scenario. The introduction of general five-dimensional SS boundary conditions can drive a 4D gauge symmetry breaking through the dynamical mechanism, conventionally known as Hosotani mechanism. One-loop contributions to the scalar sector can shift the minimum of the effective potential and generate a non-vanishing vev for the Higgs field. This symmetry breaking is spontaneous and rank preserving.

In six dimensions, SS boundary conditions have to satisfy a consistency condition. We discussed in details the $U(N)$ case where a novel ingredient appears: the non-abelian 't Hooft flux. This flux is a topological quantity intimately connected with the U(1) (quantized) magnetic flux. In the case of trivial ($m = 0$) 't Hooft flux the gauge symmetry breaking obtained thought SS boundary condition is the usual rank preserving Hosotani mechanism. In the case of non-trivial ($m \neq 0$) 't Hooft flux one can have, instead, two simultaneous symmetry breaking mechanism. A explicit, rank reducing, symmetry breaking associated to the non-commutativity of the twists

leading to the $SU(N) \to SU(\mathcal{K})$ breaking. On top of that, for $\mathcal{K} > 1$, the residual symmetry group can be further reduced through a spontaneous, rank preserving, Hosotani mechanism.

The simultaneous presence of rank preserving and rank reducing symmetry breaking mechanism makes the non-trivial 't Hooft flux case particularly interesting from a model building point of view.

Acknowledgments

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