Waffle mode mitigation in adaptive optics systems: 
a constrained Receding Horizon Control approach

Mikhail Konnik\(^1\) and José De Doná\(^2\)

Abstract—The wavefront sensors that are used in adaptive optics to sense the atmospheric turbulence have blind modes. One of those unseen modes, a \textit{waffle mode}, occurs when the phase between actuators is sensed to be zero while in reality there is a piston difference between actuators. In this paper we discuss a way of waffle mode mitigation by setting additional constraints in the Receding Horizon Control. The results on numerical simulations are presented. The increase in the computational load with the values of constraints is estimated. Output disturbance rejection performance for the constrained case is evaluated. Our numerical simulations suggest that the constrained receding horizon controller is feasible with present day computational capabilities for small and medium-sized adaptive optics systems.

I. INTRODUCTION

Adaptive optics (AO) attempts to compensate atmospheric turbulence in real-time by measuring a wavefront, reconstructing it, and adjusting the shape of a deformable mirror (DM). A controller tracks a zero reference (that is, a flat wavefront) and generates signals for actuators of a deformable mirror to counteract the atmospheric turbulence. However, the commonly-used methods of wavefront reconstruction (the Fried geometry \cite{1}) are insensitive to a chequerboard-like pattern \cite{2} called \textit{waffle}. When this mode goes unsensed, it can accumulate within the closed-loop operation of the system. The problem may be significant enough that it may be necessary to open the AO feedback loop to clear the DM commands and alleviate the waffle \cite{3}.

Efforts have been made to mitigate waffle and other “blind” reconstructor modes \cite{4}–\cite{6}. Various weighting schemes for least squares wavefront reconstructors were proposed \cite{2} for penalization of the waffle behaviour. Modifications of the Hudgin reconstruction geometry were proposed \cite{7} along with a spatially filtered wave-front sensor \cite{8}. Modal removal in an FFT-based wavefront reconstruction method can be used as well \cite{9}. Filtering out waffle mode from the control signals \cite{3} or by modification of reconstruction matrix \cite{10} can be used for waffle modes attenuation.

It is envisaged that more advanced control techniques such as Receding Horizon Control (RHC) will produce substantial improvements in waffle mitigation. The main feature of RHC is that one can incorporate specific constraints to mitigate waffle mode in addition to the constraints on the amplitude of the control signal. It was previously reported in \cite{11} that the RHC technique is feasible for adaptive optics systems of a moderate size.

The results of numerical simulations of constrained RHC control for waffle mode reduction are presented in the present article. The increase in the computational load with the tightness of the constraints is estimated. Output disturbance rejection performance for the constrained case is also evaluated, and various lengths of the prediction horizons in RHC control are considered.

II. THE PROBLEM: WAFFLE MODE IN ADAPTIVE OPTICS FROM THE CONTROL POINT OF VIEW

We start with the description of the optical problem (waffle mode) and translate it to the control problem as this phenomenon is important for this work.

A. Adaptive optics from the control perspective

The \textit{wavefront} can be thought as a surface of all light rays coming from a distant star. If there is no atmospheric turbulence, the wavefront surface will be flat, which means that all the rays have travelled the same distance. Thus wavefront is non-planar when it reaches the ground due to atmospheric turbulence. The deviation from planar of a wavefront is known as wavefront error, and causes the blurring of astronomical images.

![Fig. 1. Principle of the Shack-Hartmann wavefront sensor: (top) undistorted wavefront, and (bottom) distorted by atmospheric turbulence.](image-url)
the aperture (sub-aperture) and forms an image of the light source. When the wavefront is distorted, the light spots become displaced from their nominal positions. Displacements of image centroids in two orthogonal directions \( x, y \) are proportional to the average wavefront over the sub-apertures.

From the control perspective, the atmospheric turbulence can be represented as an output disturbance and described by a discrete-time state space model driven by a white noise. A controller should therefore track a zero reference (that is, a flat wavefront) and generate signals for actuators of DM from the WFS data in closed-loop.

### B. What is the waffle mode?

A Shack-Hartmann sensor has **blind modes**: wavefront functions that can yield zero or very small response in the Shack-Hartmann WFS output [2]. The commonly-used wavefront reconstruction method called Fried geometry [1] is insensitive to a chequerboard-like pattern of phase errors called **waffle**. The waffle mode resembles a breakfast waffle: equal magnitude alternating variations of the wavefront over the mirror actuator locations [13], as seen in Fig. 2. That is, if one regards the square WFS lenslets as a checkerboard, the mean phase over the black squares will not equal the mean phase over the white squares [6].

![Fig. 2. Waffle mode and their propagation on the DM commands](image)

The waffle mode can accumulate within the closed-loop operation of the adaptive optics system [3]. The impact on the actual mirror can lead to damage of the mirror’s surface. Hence, it is very important to eliminate (or at least to mitigate) the waffle mode.

### III. Adaptive optics system from the control point of view: description of dynamic models

In this paper we use the dynamic model of an adaptive optics system as reported in [11]. The system is considered to be LTI MIMO and is subject to an output disturbance. The goal of the controller is to track a zero reference (a flat wavefront). Measurement noise from the photosensor inside the WFS and time delays are not considered in this preliminary study and will be included in subsequent work.

#### A. Plant dynamics

The plant is a deformable mirror (DM) with a square array \( m \times m \) of actuators attached to the faceplate of the DM. The dynamics of the actuators in a DM is considered to be a first-order transfer function (TF) with a fast pole [14], [15]. In order to account for the fact that actuators slightly differ from one to another, the continuous transfer matrix (TM) was generated taking the values of poles from the Normal Distribution \( p_N \sim N(p, \sigma^2_p) \) with mean \( p \) and standard deviation \( \sigma_p = \alpha \cdot p \). The continuous TF for the \((i,j)\)-th actuator is therefore:

\[
G(s)_{i,j}^{\text{plant}} = \frac{1}{s + p_N},
\]

where \( \alpha = 0.01 \) and the poles mean value is \( p = 1500 \) are considered in this work.

#### B. Coupling between actuators in a deformable mirror

Continuous-faceplate deformable mirrors usually have relatively strong coupling only between the nearest neighbour actuators. In this case the continuous transfer matrix contains off-diagonal elements \( G_{i,j}(s) \):

\[
G(s) = \begin{bmatrix}
G_{1,1} & G_{1,2} & 0 & G_{1,4} & 0 & \ldots \\
G_{2,1} & G_{2,2} & G_{2,3} & 0 & G_{2,5} & \ldots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots
\end{bmatrix}
\]

For example, the coupling transfer function \( G_{5,2}(s) \) between the actuator 5 and actuator 2 is obtained as \( G_{5,2}(s) = \gamma \cdot G_{5,5}(s) \). In this work \( \gamma = 0.1 \) is considered as a static coupling degree between the nearest neighbour actuators.

#### C. Output disturbance dynamics

The output disturbance, which is the atmospheric turbulence, is modelled as a continuous transfer function:

\[
G(s)_{i,i}^{\text{atm}} = \frac{1}{s^2 + s \cdot \kappa_1 + \kappa_2},
\]

The poles of the output disturbance model are usually slower than the poles of the DM and in this paper we used \( \kappa_1 = 2 \) and \( \kappa_2 = 30 \) as mean values of the coefficients.

#### D. Overall model of the system dynamics

The continuous transfer matrices of both plant and disturbance dynamics are formed using (1), (2), and (3). These transfer matrices are converted to state space and augmented to account for both plant and disturbance dynamics:

\[
A = \begin{bmatrix} A_{\text{plant}} & 0 \\ 0 & A_{\text{atm}} \end{bmatrix}, \quad B = \begin{bmatrix} B_{\text{plant}} \\ 0 \end{bmatrix}, \quad C = [C_{\text{plant}} \ C_{\text{atm}}].
\]

Then the state space matrices \( A \), \( B \), and \( C \) for the overall model are discretised using zero-order hold and a sample time of \( T_s = 10^{-3} \) seconds.
E. Formulation of Receding Horizon Control

Consider the state space model of a linear system:

\[ x_{k+1} = Ax_k + Bu_k, \]
\[ y_k = Cx_k + d_k, \]

where \( x_k \) is the state, \( u_k \) is the control input, \( y_k \) is the output, and \( d_k \) is a time-varying output disturbance. Using a standard state space Receding Horizon Control formulation [16], the cost function in vector notation for a state prediction horizon \( N_p \) and control prediction horizon \( N_c \) can be expressed as:

\[ V_{N_p,N_c} = \frac{1}{2} x^T C^T QCx + \frac{1}{2} x^T QX + \frac{1}{2} U^T RU, \]

where \( X = [x_1, x_2, \ldots x_{N_p}]^T \), \( x \) denotes the current state (i.e., \( x = x_0 \)), and \( U = [u_0, u_1, \ldots u_{N_c}]^T \). The matrices \( Q \) and \( R \) are denoted as follows:

\[ Q = \text{diag}(C^T QC, \ldots P) \quad \text{and} \quad R = \text{diag}(R, \ldots R). \]

The state penalty matrix for (8) is \( \Gamma = V \) and control penalty matrix \( \Omega = I \). Then we define matrices \( \Gamma \) and \( \Omega \) as:

\[ \Gamma = \begin{bmatrix} B & \ldots & 0 \\ AB & \ldots & 0 \\ \vdots & \ddots & \vdots \\ A^{N_p-1}B & \ldots & A^{N_p-N_c}B \end{bmatrix}, \quad \Omega = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^{N_p} \end{bmatrix} \]

The dynamics in (5) can be expressed over the prediction horizon in vector form as:

\[ X = \Gamma U + \Omega x. \]

Substituting (9) into the cost function (7) we obtain:

\[ V_{N_p,N_c} = \tilde{V} + \frac{1}{2} x^T \Xi U + U^T \Xi x, \]

where the term \( \tilde{V} \) is independent of \( U \), and the Hessian matrix \( \Xi \) and matrix \( F \) are defined as follows:

\[ \Xi = \Gamma^T Q \Gamma + R, \quad F = \Gamma^T Q \Omega. \]

We form a Quadratic Programming (QP) problem to minimise the cost function (10) and therefore evaluate a constrained control output:

\[ \min U \quad \frac{1}{2} U^T \Xi U + U^T \Xi x, \quad \text{subject to} : M \cdot U \leq \Lambda \]

where \( U \) is a vector of current and future inputs. The Hessian matrix \( \Xi \) is positive definite, hence the quadratic optimisation problem is convex. In turn, this means that the constrained solution exists and is unique.

IV. NUMERICAL SIMULATION GOALS AND INITIAL CONDITIONS

The numerical simulations were run on MATLAB 2007b\(^1\). The Dantzig-Wolfe [17] QP solver was used from the MATLAB Model Predictive Control Toolbox. We consider two cases of waffle mode: spatial and temporal. By spatial waffle we mean the movements of nearest neighbour actuators in opposite directions within the same sampling time instant. On the other hand, the control commands that force the same actuator to move in opposite directions from one time instance to the next, are called temporal waffle.

We introduce waffle mode as an exogenous “input disturbance” signal\(^2\). That is, we add to the current control signal \( u_0 \) a portion of the current control with inverted signs:

\[ u_0 = u_0 + u_0 \cdot W \]

where the current control signal \( u_0 \) is multiplied element-wise by the waffle matrix \( W \), which is:

\[ W = \begin{bmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & W_0 \end{bmatrix}, \quad W_0 = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} \]

![Fig. 3. Waffle mode injection into control signals at the sampling time 450. Note that the waffle pattern in the lower right part of the control signals for this case (5 x 5 actuators DM) corresponds to the W matrix from (14).](image)

![Fig. 4. Waffle mode added to control signals (bottom) and appeared as uncompensated output disturbance (top) at sampling instances 400 and 450.](image)

The vector of current control signal \( u_0 \) is reshaped to a matrix of appropriate dimensions during the waffle mode injection. That is, for the model of a deformable mirror with 5 x 5 actuators we form a 5 x 5 matrix \( W \) from (14) and put the waffle \( W_0 \) into specified part (bottom right part in this case, as seen in Fig. 3). Such a method of waffle injection

\(^1\)Simulations were run on the Lenovo T420 notebook: Intel Core i7-2640M Processor 2.80GHz, 8 GB DDR3 SDRAM, 500GB 7200rpm HDD, Debian GNU/Linux i386 v 6.0 and MATLAB 2007b for UNIX.

\(^2\)The waffle modes can be introduced in the end-to-end simulations with phase screens, but for the controller there should be no difference on how those modes appeared. The point of this contribution is to mitigate the waffle mode in the control signal no matter its origin. Therefore, it is a reasonable approximation to inject the waffle mode directly in the control inputs.
using element-wise multiplication allows introducing the waffle mode in the specific part of the current control signal \( u_0 \). The waffle modes were injected at sampling time instants 400 and 450, as seen in Fig. 4. The lines in Fig. 4 correspond to the control channels shown in matrix form in Fig. 3. The injected waffle mode was attenuated by including constraints in the Receding Horizon Control (matrices \( M \) and \( \Lambda \) in (12)).

V. SIMULATION RESULTS: TEMPORAL WAFFLE MODE REDUCTION

Unlike spatial waffle that is a movement of nearest neighbour actuators in opposite directions, we consider a temporal waffle to be a movement that forces the same actuator to go in the opposite directions from one time instance to the next. This can damage the deformable mirror and therefore such control commands should be penalised. With Receding Horizon Control one can put constraints \( \delta U_{max} \geq |u_{t,k} - u_{t-1,k}| \) between a current control signal \( u_{t,k} \) in \( k \)-th channel and the previous signal \( u_{t-1,k} \) to reduce the magnitude of the temporal waffle mode in control signals for a DM. We provide the results of simulations to show the effect of the constraints and computation burden of the corresponding QP in the further subsections.

A. Constraints for temporal waffle mode reduction

The constraints on temporal control signal increment can be formulated as follows:

\[
\delta u = \begin{bmatrix} u_{-1} + \delta U_{max} \\ u_{-1} + \delta U_{max} \\ \vdots \\ u_{-1} + \delta U_{max} \end{bmatrix}, \quad \delta u_{min} = \begin{bmatrix} u_{-1} - \delta U_{min} \\ u_{-1} - \delta U_{min} \\ \vdots \\ u_{-1} - \delta U_{min} \end{bmatrix}
\]

(15)

where the \( \delta U_{max} \) and \( \delta U_{min} \) are maximal and minimal increment in a control signal, and \( u_{-1} \) is the previous value of the control signal. Minimal and maximal constraints from (15) can be merged into a single matrix:

\[
L = \begin{bmatrix} M_0 \\ -M_0 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \delta u_{max} \\ -\delta u_{min} \end{bmatrix}
\]

(16)

where \( M_0 = I_{N_c \cdot m} \) is the \( N_c \cdot m \times N_c \cdot m \) identity matrix, with \( N_c \) as the control prediction horizon and \( m \) the number of inputs. In this work, the constraints are assumed to be symmetrical, that is \( \delta u_{max} = -\delta u_{min} \). This is justified since the operational point of the plant (namely, deformable mirror) is usually adjusted to be between the maximum and the minimum stroke during the calibration procedure.

The simulations were performed for different values of incremental constraints \( \delta u \) from \( \delta u = 0 \) (tight constraints) to \( \delta u = 1 \) (loose constraints). Various prediction horizons \( N_p \) and \( N_c \) were considered for each of the constraint sets.

B. Results for the temporal waffle reduction: disturbance rejection performance

The results in Fig. 5 indicate that taking longer prediction horizons beyond moderate values actually gives similar results of the output disturbance rejection.

The data in Fig. 5 have been taken from the channels that were affected by the waffle mode injection, and the results represent the average tendency of the residual disturbance magnitude. One can see, however, that the residual magnitude of the disturbance is slightly lower for the case of \( N_p = 2, N_c = 2 \). Taking the prediction horizons \( N_p > 2 \) and \( N_c > 2 \) does not provide noticeable improvement in the waffle mode reduction but requires considerably more computational time (see Fig. 7). The case of \( N_p = 2, N_c = 2 \), however, provides slightly improved performance in waffle mode reduction, but requires at least 5x times more computation time for the RHC to compute the control signal.

An additional fact to consider is that the stricter incremental constraints can lead to inferior disturbance rejection performance. That is, if the waffle mode will not occur, we still pose the additional constraints on the control signal, which 1) slows down the QP solution, and 2) may excessively restrict the control signal and therefore reduce the disturbance rejection performance. One can see in Fig. 6 that the stricter constraints on \( \delta u \) reduce the amplitude of waffle...
mode, but the question is: how to estimate a reasonable constraint on $\delta u$? The answer will depend on the parameters of the disturbance (in the case of adaptive optics systems this is an atmospheric turbulence), which can be time-variant. This can be a topic of further research and is beyond the scope of the current contribution.

C. Results for the temporal waffle reduction: computational time growth

All the measurements were performed for the model of deformable mirror with $5 \times 5$ actuators. The computational time was measured for each sampling time instant between 400 and 450, i.e., where the waffle injection takes place and constraints are violated the most.

It can be seen from Fig. 7 that a control prediction horizon longer than $N_c = 2$ takes unacceptably long time to compute. Moreover, the computation time for long control prediction horizons grows quickly up to 10-20 ms, which is unacceptably long.

![Fig. 7. Temporal waffle mode reduction: dependence of mean computation time versus constraints values for various prediction horizons. The case of $5 \times 5$ actuators deformable mirror is considered.](image)

The computational time of the QP solution for temporal waffle mode reduction depends also on the percentage of constraints being active. One can see from Fig. 8 that the time to solve QP grows slowly with the number of active constraints. The size of the dots in Fig. 8 corresponds to the probability of the computational time, i.e., the bigger the dot, the more likely that the QP solution will take the specified time. The variance of computational time becomes unacceptably long.

![Fig. 8. Computational time for temporal waffle mode reduction versus percentage of activated constraints for different sizes of the DM. The size of the dot on the plot corresponds to the likelihood of the computation time (constraints are set $\delta U_{sw,max} = 2$, which are considered as tight).](image)

A. Constraints for spatial waffle mode reduction

The spatial waffle pattern occurs between the nearest actuators that move in opposite directions, as seen from Fig. 2. Hence we can formulate the constraints to penalise the opposite control signals using the waffle matrix $W_{0,sw} = [1, -1; -1, 1]$ (see also Eq. (14)). For example, the constraints for the actuator 1 in a $3 \times 3$ DM are:

$$|u_1 - u_2| < U_{sw,max}$$  (17)

$$|u_1 - u_4| < U_{sw,max}$$  (18)

$$|u_1 + u_5| < U_{sw,max}$$  (19)

where $U_{sw,max}$ is the maximal allowable difference between nearest control signals for spatial waffle pattern. For a waffle matrix $W_{0,sw}$ of size $a \times b$, there will be $(a+b-1)$ rules like in (20). Converting those rules for spatial waffle pattern to the matrix $M_0$ for arbitrary square DM with $k \times k$ actuators, we will get a matrix of size $N_c \times (a + b - 1) \times N_c \times k$ and

$$L = \begin{bmatrix} M_0 \\ -M_0 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} U_{sw,max} \\ -U_{sw,max} \end{bmatrix}.$$  (20)

In this work, the constraints for the spatial waffle are assumed to be symmetrical and only for closest neighbours (i.e., the matrix $W_{0,sw} = [1, -1; -1, 1]$). The main purpose of the spatial constraints is to prevent the waffle modes from uncontrollable grow and slow down their spread across the mirror’s commands.

One should notice that since the matrices $L$ and $\Lambda$ do not depend on the control values; it is not necessary to recalculate them every time, unlike the matrices for the temporal waffle. Therefore, spatial waffle constraints can significantly increase the computation speed of the RHC solution.

B. Results for the spatial waffle reduction: disturbance rejection performance

Numerical simulations include different level of constraints and prediction horizons to test both disturbance
rejection performance and the computational load. The constraints on the control signal amplitude are varied from loose constraints $U_{sw,max} = 1500$ to tight constraints $U_{sw,max} = 150$. Prediction horizons are varied from the shortest $N_p = 2, N_c = 1$ possible to relatively long horizons $N_p = 4, N_c = 4$. Such a choice of parameters is motivated by practical considerations: tighter constraints will lead to inferior waffle rejection, and longer prediction horizons are infeasible from the computational point of view.

We have to find a trade-off between the disturbance rejection performance and computational speed. From Fig. 11 it can be seen that the control prediction horizon $N_c = 2$ can be considered as a practical computational limit.

![Fig. 9. Spatial waffle reduction: dependence of the output disturbance rejection performance versus the strictness of the incremental constraints.](image)

More important is that taking longer prediction horizons is not actually beneficial for the waffle mode reduction, as seen in Fig. 9. Actually, in the case of spatial waffle modes reduction, the shortest horizon ($N_p = 2, N_c = 1$) gives slightly better results in disturbance rejection.

One can see from Fig. 10 the effect of the constraints for the spatial waffle pattern. Similar to the temporal waffle case, excessive constraints for spatial waffle can slow down the QP solution and restrict the control signal. The latter can lead to inferior disturbance rejection performance.

C. Results for the spatial waffle reduction: computation time growth

Estimations of computational speed for various prediction horizons and constraints values are presented in Fig. 11. Expectedly, the computational time for the minimal control prediction horizon $N_c = 1$ is the fastest. More important is that the growth rate in case of spatial waffle is considerably faster than for temporal waffle, as clearly seen from a comparison of Fig. 11 and Fig. 7.

It is also interesting to see how the computational demand grows with the number of actuators in a deformable mirror.

The control signal $U$ can be of large amplitude, but the $\delta U$ is relatively small for the relatively slow disturbance and fast sample time. That is, the incremental change of a control signal is small; however the amplitude of the control signal itself can be large.

![Fig. 10. Spatial waffle mode attenuation for different constraints and $N_p = 2, N_c = 1$: tight constraints $U_{sw,max} = 300$ (top), less tight $U_{sw,max} = 500$ (middle), loose constraints $U_{sw,max} = 1500$ (bottom).](image)

One can see from Fig. 12 how the solution of QP is quite demanding and the required computation time grows quickly with the size of the DM. Our simulations show that a spatial waffle mode reduction with RHC control can be performed only for relatively small mirrors like $7 \times 7$ actuators DM. However, the QP algorithm used is not the fastest and further acceleration is possible.

![Fig. 11. Spatial waffle mode reduction: dependence of mean computation time versus constraints value for the case of $5 \times 5$ actuators in a DM.](image)

Also one can see from Fig. 12 how the time for solving QP problem depends on the percentage of the active constraints (spatial waffle modes in this case). We should note that in real-world adaptive optics systems the control signal rarely reaches the constraints. Typically only a few constraints may be active (e.g., 3-5% of constraints), and even this is a rare event (like 1-2% of the operational time). Nonetheless, it is of interest to know the growth rate of computational time even for unlikely cases of very strong turbulence (when many constraints are active).

Note that the computation time for the QP problem solution (at least for the Dantzig-Wolfe algorithm used in
this work\textsuperscript{4}) has some probability distribution. The size of the dots in Fig. 12 corresponds to the probability of the computational time, i.e., the bigger the dot, the more likely that the QP solution will take the specified time. For the case of relatively big problems like 7 × 7 DM and many active constraints (like 20\%), one can see that the spread of computational time can be as big as ±1 msec.

VII. RESULTS DISCUSSION AND CONCLUSION

This paper discusses feasibility of mitigation of waffle mode in adaptive optics using constrained Receding Horizon Control. Putting specific constraints on actuators movements, one can efficiently reduce waffle-like behaviour of actuators in a deformable mirror. The simulations show that the computational burden is lower than one might expect.

\textit{a) Temporal waffle mode attenuation:} The best performance in the reduction of the waffle modes can be achieved with short prediction horizon of \( N_p = 2, N_r = 1 \). Longer prediction horizons take unacceptably long time for the QP to compute the optimal control output delivering similar results of output disturbance rejection. The temporal waffle mode reduction is computationally inexpensive and can be done in reasonable time for relatively large DM like 9 × 9 actuators.

\textit{b) Spatial waffle mode attenuation:} The spatial waffle reduction is more computationally demanding that the temporal waffle. The control prediction horizon \( N_r = 2 \) can be considered as a practical limit from the computational point of view. The required computational time grows quickly with the size of the DM, so the waffle mode reduction can be done for relatively small mirrors like 7 × 7 actuators DM.

Although in real systems the event of reaching constraints is rare, proper (and optimal) handling of saturated operation can be thought as a useful safeguard. Our numerical simulations suggest that constrained RHC is feasible for small and medium-sized adaptive optics systems. We also note that the results reported in this paper are not the fastest possible: using more efficient QP solvers, at least 4x acceleration of the constrained control calculation is attainable.

\textbf{REFERENCES}


\[16\] Graham Clifford Goodwin, Maria Marta Seron, and José De Doná, Controlled Control and Estimation: An Optimisation Approach. Springer, Berlin, 2005.


\textsuperscript{4}The spread of computational time heavily depends on the particular implementation of the QP algorithm. For example, one can use Dual Active Set algorithms that seems to have less variance in computation time even for many active constraints. The reason we use Dantzig-Wolfe is because it is readily available in MATLAB.