Analytical Model and Experimental Validation of Cross Polar Ratio in Polarized MIMO Channels

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Abstract—Cross-polarized antenna systems are used to reduce equipment size in MIMO systems while still achieving low inter-antenna correlation. One fundamental parameter of cross-polarized MIMO systems is the cross-polar discrimination (XPD). This paper proposes a model to determine the XPD as a function of channel condition and under different antenna configurations. The environment is supposed to have a truncated Laplacian Power Angular Spectrum (PAS) that is widely used in standardization bodies. Antenna XPD is shown to be sensitive to different channel conditions, as well as to different receiver orientations. Measurements were conducted at 3.5 GHz to validate the theoretical model. Good agreement is obtained between theoretical XPD and experimental results.

Index Terms—MIMO, cross-polarized, XPD, channel modeling.

I. INTRODUCTION

By using multiple antennas at transmitter and receiver side, MIMO systems create diversity that can be exploited to obtain higher data rates and improved quality of wireless link transmission [1]. One major issue of MIMO systems is that reasonable antenna spacing has to be at least half a wavelength at the mobile terminal, and about 10 wavelengths at the base station to achieve reasonably low correlation, increasing overall equipment size. Cross-polarized antennas can resolve this problem by using perpendicular antennas. Indeed, perpendicular antennas can be co-located and still have sufficiently low correlation, therefore increasing the system’s diversity [2], [3], [4].

Cross-polarization discrimination (XPD) is a fundamental parameter for cross-polarized MIMO systems. A lot of XPD measurements have been reported in previous papers [5], [6], but, to the author’s knowledge, no paper addresses the problem of determining a closed-form expression for the XPD as a function of channel conditions and for different receiver orientations. Although lots of papers give measurement results for the XPD for dual-polarized channels when transmitter and receiver are parallel, it is unclear what happens when the receiver antenna is rotated. When considering dual- and tri-polarized antennas in practical applications (cell phones, laptops, etc.), the user can hardly be asked to keep his mobile fixed in a certain orientation. Therefore it is of importance to understand what happens with the channel parameters when rotating the antenna system.

The aim of this paper is to establish a closed-form expression for the XPD, which can include parameters like channel conditions and receiver orientation, and to validate the theoretical expressions with measurement results.

The paper is organized as follows: section II presents the analytical model and the derivation of the closed-form solution for the XPD. Section III presents results for the XPD when using this new closed-form solution. Finally, section IV presents the measurement setup and measurement results that are used to validate our analytical model.

II. ANALYTICAL MODEL

In this section, we will present the channel model we will use to compute the cross-polar ratio of cross-polarized di- or tri-poles. We will use the following simplifying assumption for the channel model:

- The antennas are supposed to be isotropic
- All waves are coming in the azimuthal plane. Elevation is not considered. This is a common assumption in a lot of channel models like the 802.11n standard channel model [7].
- We suppose uncorrelated scattering: if $E_x$ is the $x$-component of the electric field $\vec{E}$ and $\Omega$ is an angle $(\theta, \varphi)$, then $\langle E_x(\Omega_1) E_x(\Omega_2) \rangle \propto \delta(\Omega_1 - \Omega_2)$, where $\langle \rangle$ denotes the expectation operator. In reality, correlation between waves coming from different directions tends to be low. This is often referred to as the homogenous assumption [8].
- The Power Angular Spectrum (PAS) is a truncated Laplacian [9], which is widely used in standardization bodies [7]. In the general case, all three components of $\vec{E}$ are present in each incoming wave, resulting in similar PAS for each polarization (except for a power factor). Measurement results supporting this assumption can be found in [10].
- When a linearly polarized wave is scattered, a component may appear in the orthogonal polarization. We will suppose both orthogonal polarizations of $\vec{E}$ to be uncorrelated.

Let us suppose we have a fixed coordinate system x-y-z with an incident field $\vec{E}(\Omega)$. If the antennas of a tripole receiver have polarizations $\vec{A}_1$, $\vec{A}_2$ and $\vec{A}_3$ in the x-y-z coordinate...
system, the received signal (in Volts) on antenna \( i \) is given by:

\[
V_i = \int_{\Omega} \vec{A}_i(\Omega) \cdot \vec{E}(\Omega) d\Omega
\]  

(1)

where \( \cdot \) denotes the dot product. For simplicity of notation, the term \( \Omega \) will be omitted for the rest of the development. The antenna XPD between two antennas \( i \) and \( j \) (which we will simply refer to as XPD\(_{ij} \), later on) in dB is given by:

\[
\text{XPD}_{ij}(\text{dB}) = 10 \log_{10} \left( \frac{\langle |\vec{A}_i| \cdot \vec{E} \rangle^2}{\langle |\vec{A}_j| \cdot \vec{E} \rangle^2} \right)
\]

(2)

We will use the fact that the antennas are isotropic and that we have uncorrelated scattering to compute \( \langle |\vec{A}_i| \cdot \vec{E} \rangle \). In the x-y-z coordinate system, this term simplifies to the form given in (3) and (4) (given at the bottom of the page).

In (3), \( E_{x/y/z} \) is the x/y/z-component of \( \vec{E} \). Equation (4) is composed of three terms:

- vector \( \vec{A}_{ij} \) that contains all information on antenna \( i/j \) (orientation, cross-polar isolation)
- central matrix \( \mathbf{P}_H \) which depends only on the channel (not the antenna configuration). Diagonal elements represent channel powers and non-diagonal elements represent channel correlations

By using this notation, channel and antenna properties are split. When rotating the antenna system, it is unnecessary to re-compute matrix \( \mathbf{P}_H \), but only to modify \( \vec{A}_{ij} \). In order to resolve the integrals in (3), we will use the spherical components of \( \vec{E} \). A plane wave coming from a direction with co-elevation \( \theta \) and azimuth \( \varphi \) has the following x-y-z components:

\[
\begin{align*}
E_x &= \cos \theta \cos \varphi E_0 - \sin \varphi E_\varphi \\
E_y &= \cos \theta \sin \varphi E_0 + \cos \varphi E_\varphi \\
E_z &= -\sin \theta E_0
\end{align*}
\]

(5)

where \( E_0 \) and \( E_\varphi \) are the \( \vec{1}_0 \) and \( \vec{1}_\varphi \) components of \( \vec{E} \). Since all waves are supposed to arrive in the azimuthal plane, we can simplify (5) to

\[
\begin{align*}
E_x &= -\sin \varphi E_\varphi \\
E_y &= \cos \varphi E_\varphi \\
E_z &= -E_0
\end{align*}
\]

(6)

When we use (6) to determine the elements of matrix \( \mathbf{P}_H \) in (3), we will have to compute the terms \( \langle E_0 E_0 \rangle \), \( \langle E_\varphi E_\varphi \rangle \), \( \langle E_x E_x \rangle \) and \( \langle E_x E_\varphi \rangle \). Because of the Laplacian PAS, we can write:

\[
\begin{align*}
\langle E_0 E_0 \rangle &= P_0 p(\varphi) \\
\langle E_\varphi E_\varphi \rangle &= P_\varphi p(\varphi)
\end{align*}
\]

(7)

(8)

where \( P_0 \) and \( P_\varphi \) are the powers of the \( \vec{1}_0 \) and \( \vec{1}_\varphi \) components of \( \vec{E} \) (in the case without considering elevation, \( P_0 \) is the power of the vertical field component, and \( P_\varphi \) is the power of the horizontal field component). The term \( p(\varphi) \) is given by:

\[
p(\varphi) = \frac{Q}{\sigma \sqrt{2}} \exp \left[ -\frac{\sqrt{2}|\varphi - \varphi_{\text{mean}}|^2}{\sigma^2} \right]
\]

(9)

with \( \varphi \in [\varphi_{\text{mean}} - \Delta \varphi, \varphi_{\text{mean}} + \Delta \varphi] \). In this equation \( Q \) is a normalization factor, \( \varphi \) is the azimuth angle, \( \varphi_{\text{mean}} \) is the mean arrival angle, \( \sigma \) is the azimuthal spread and \( \Delta \varphi \) is the truncation spread. We have also supposed orthogonal components of \( \vec{E} \) to be uncorrelated, so

\[
\langle E_\varphi E_\varphi \rangle = \langle E_\varphi E_\varphi \rangle = 0
\]

(10)

When using the results (7) - (10), the integrals of \( \mathbf{P}_H \) in (3) can be resolved. \( \mathbf{P}_H \) then has the following form:

\[
\mathbf{P}_H = \begin{pmatrix}
P_{H,11} & P_{H,12} & 0 \\
P_{H,21} & P_{H,22} & 0 \\
0 & 0 & P_{H,33}
\end{pmatrix}
\]

(11)

The elements of \( \mathbf{P}_H \) are given in Table I. \( T_x \) and \( T_y \) in Table I

\[
\begin{align*}
P_{H,11} &= \frac{P_{0e} - \frac{\Delta \varphi}{4(1+2\sigma^2)}}{4(1+2\sigma^2)} T_x \\
P_{H,22} &= \frac{P_{0e} - \frac{\Delta \varphi}{4(1+2\sigma^2)}}{4(1+2\sigma^2)} T_y \\
P_{H,33} &= P_0 \left( 1 - e^{-\frac{\Delta \varphi}{4(1+2\sigma^2)}} \right) \\
P_{H,12} &= P_{H,21} = -P_0 e^{-\frac{\Delta \varphi}{4(1+2\sigma^2)}} \left( \cos(\varphi_{\text{mean}}) \sin(\varphi_{\text{mean}}) \right) \times \left[ e^{\frac{\Delta \varphi}{4(1+2\sigma^2)}} - \cos(\Delta \varphi) + \sqrt{2} \cos(\Delta \varphi) \sin(\varphi_{\text{mean}}) \right]
\end{align*}
\]

**TABLE I**

**ELEMENTS OF MATRIX \( \mathbf{P}_H \)**
I are given by

\[
T_x = 2e^{\Delta \varphi \sqrt{2} \sigma} (1 + 2\sigma^2 - \cos(2\varphi_{\text{mean}})) - 2(1 + 2\sigma^2) + 2\cos(\Delta \varphi) \cos(2\varphi_{\text{mean}})
\]

\[
-2\sqrt{2}\sigma \sin(\Delta \varphi) \cos(2\varphi_{\text{mean}})
\]

\[
T_y = 2e^{\Delta \varphi \sqrt{2} \sigma} (1 + 2\sigma^2 + \cos(2\varphi_{\text{mean}})) - 2(1 + 2\sigma^2) - 2\cos(\Delta \varphi) \cos(2\varphi_{\text{mean}})
\]

\[
+2\sqrt{2}\sigma \sin(\Delta \varphi) \cos(2\varphi_{\text{mean}})
\]

Now the elements of \( \mathbf{P}_H \) are known, \( \mathbf{P}_H \) can be substituted in (3), that can then be used to compute the XPD in (2). A closed-form solution is obtained to determine the antenna XPD as a function of channel conditions and receiver orientation. Note that this analytical model can also be used to compute the inter-antenna correlation, which will be subject to another publication.

III. SIMULATION RESULTS

The model that was presented in section II explicitly split the antenna effects and the channel effects in (3). The channel effects included azimuthal spread \( \sigma \), truncation spread \( \Delta \varphi \), mean arrival angle \( \varphi_{\text{mean}} \) and both orthogonal components powers \( P_\theta \) and \( P_\varphi \). We will introduce the relative power between orthogonal polarizations of the electrical field vector \( \mathbf{E} \) (which we will call the cross-polar ratio (XPR)):

\[
\text{XPR (dB)} = 10 \log_{10} \left( \frac{P_\theta}{P_\varphi} \right)
\]  (12)

In the case where elevation is not considered, a positive XPR corresponds to a mainly vertically polarized wave, while a negative XPR corresponds to a mainly horizontally polarized wave. We will first study the effect of antenna rotation on the different XPDs of the system. A tripole antenna made of \( \lambda/2 \) antennas is given in Figure 1. The antenna is rotated around the antenna 1 axis. For each configuration of antenna/channel parameters there are three XPD values: \( \text{XPD}_{21} \), \( \text{XPD}_{31} \) and \( \text{XPD}_{32} \). Depending on the studied effect, one of these XPDs is only a 90°-shifted version of another XPD, as we will explain later on.

![Rotation of antenna tripole around axis A1](image)

Fig. 1. Rotation of antenna tripole around axis A1.

The effect of antenna rotation on \( \text{XPD}_{32} \) and \( \text{XPD}_{31} \) is given for different values of XPR in Figure 2 and 3. When we rotate the antenna system around the axis of antenna 1, antenna 2 has a 90° delay over antenna 3. For that reason, the \( \text{XPD}_{21} \) is just a 90°-shifted version of \( \text{XPD}_{31} \). For a horizontal/vertical antenna configuration (0°), we can see that when a mainly vertically polarized wave (positive XPR) arrives at the antenna system, it will mainly be detected by the vertical antenna, resulting in high XPD. For a +45°/−45° antenna configuration (45°), a mainly vertically polarized wave will have identical contribution to antennas 2 and 3, resulting in 0 dB XPD. For such a wave to have identical contribution to antennas 1 and 3, the antenna has to be rotated 90°. The reasoning is similar for mainly horizontally polarized waves, corresponding to negative values of XPR.

![Effect of antenna rotation and XPR on XPD between antennas 2 and 3](image)

Fig. 2. Effect of antenna rotation and XPR on XPD between antennas 2 and 3 (\( \sigma = 35°, \Delta \varphi = 180° \) and \( \varphi_{\text{mean}} = 45° \))

![Effect of antenna rotation and XPR on XPD between antennas 1 and 3](image)

Fig. 3. Effect of antenna rotation and XPR on XPD between antennas 1 and 3 (\( \sigma = 35°, \Delta \varphi = 180° \) and \( \varphi_{\text{mean}} = 45° \))

The effect of mean azimuthal arrival angle is given on Figure 4. In this case, \( \sigma \) is set to 35°, \( \Delta \varphi \) is set to 180°, and we consider a horizontal/vertical antenna configuration.
Mean arrival angle $\varphi_{\text{mean}}$ and XPR are varied. For broadside arrival angle (0°), both orthogonal components are projected onto both antennas, resulting in lower XPD. For endfire mean arrival angle (90°), only the vertical component contributes to the signal received by the antennas, resulting in higher XPD. The results for XPD$_{31}$ are a 90°-shifted version of XPD$_{32}$. XPD$_{21}$ has a similar variation but do not depend on XPR since only the horizontally polarized wave contributes to antennas 1 and 2.

**Fig. 4.** Effect of mean arrival angle and XPR on XPD between antennas 2 and 3 ($\sigma = 35^\circ$, $\Delta \varphi = 180^\circ$ and H/V antenna configuration)

### IV. EXPERIMENTAL VALIDATION

#### A. Experimental Setup

A measurement campaign has been conducted to validate the model and results presented in II and III. The measurement setup is given in Figure 5. We used a Rhode&Schwarz ZVA-24 vector network analyzer (VNA), with a distance of approximately 10 m between transmitter and receiver, with the door closed in order to avoid line of sight (LOS) conditions. The transmitter used a vertically polarized antenna, and the receiver used a tripole, composed of three monopole antennas. Each receiving antenna has a dipole-like radiation pattern, with good cross-polar isolation on about 320° of the H-plane. The working frequency was 3.5 GHz, with a bandwidth of 100 MHz. The dynamic range of the VNA was about 30-50 dB for our experiment. A grid of 100 measurement points was measured at the receiver in order to determine the XPD between all antenna pairs of the receiving tripole and the power angular spectrum for each polarization. Because of the radiation pattern of the antennas of the tripole and the imperfect cross-polar isolation of each antenna, some difference is expected between theoretical curves and experimental results. The effect of cross-polar isolation of each antenna can be accounted for in the analytical model by including cross-polar isolation effects in $\vec{A}_i$ and $\vec{A}_j$ in (4).

**Fig. 5.** Floor plan and measurement setup

#### B. Measurement Results

The azimuthal power profile is estimated with a Bartlett beamforming algorithm, following the procedure given in [10], and is given in Figure 6 (for one polarization). Although it is possible to work with several clusters (by computing $P_\text{H}$ for each cluster and then superimposing them), we will only consider the main cluster. We can see that the Laplacian PAS assumption is verified, with a mean arrival angle $\varphi_{\text{mean}}$ of 52.6° (which corresponds to the direction of the door of the lab) and a azimuthal spread $\sigma$ of 20°. Measurement results are given in Figure 7, 8 and 9 for XPD$_{32}$, XPD$_{31}$ and XPD$_{21}$. The tripole antenna is rotated around the axis of antenna 1 (see Figure 1). The parameters that are used for the model are the ones determined through the Laplacian fitting ($\varphi_{\text{mean}} = 52.6^\circ$, $\sigma = 20^\circ$). Experimental results have been fitted with theoretical curves for different values of XPR. The value of

**Fig. 6.** Power angular spectrum and Laplacian fitting.

...
XPR that best fit the experimental results is 3.5 dB. The differences between the theoretical model and the experimental values are due to imperfect antenna cross-polar isolation, non-isotropic antenna radiation pattern and the fact that only one cluster is considered in the model.

V. CONCLUSION

A closed-form solution for determining the XPD of a cross-polarized antenna systems has been proposed, based on an analytical model that supposes truncated Laplacian azimuthal cluster spread and uncorrelated orthogonal polarization components. XPD showed to be sensitive to receiver orientation, azimuthal spread and environment depolarization behavior (XPR). A measurement campaign has been conducted at 3.5 GHz to validate the analytical model. Measurements have shown that XPD is sensitive to receiver orientation, and that the variation of this coefficient can be predicted using our closed-form solution.

ACKNOWLEDGMENT

The authors would like to thank the Vanburen Foundation.

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