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Anomaly matching for the QCD string

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ABSTRACT

A criterion to be satisfied by a string theory of QCD is formulated in the ultraviolet regime. It arises from the trace anomaly of the QCD stress tensor computed using instantons. It is sensitive to asymptotic freedom. It appears to be related to the trace anomaly of the QCD string. Our current understanding of noncritical strings in physical dimensions is limited, but remarkably, a formal treatment of the bosonic string yields numerical agreement both in magnitude and sign for the gauge group SU(2).

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It is a long held belief that QCD has a formulation based on string theory. There are both experimental and theoretical reasons to expect this. On the experimental side, the success of dual models in describing some of the low energy phenomenon is quite compelling[1]. Support from the theoretical side comes from 't Hooft's $1/N$ analysis of SU(N) QCD that has a topological expansion like in string theory[2]. Wilson's lattice gauge theory in the strong coupling expansion suggests area confinement and leads to a sum of surfaces reminding us of strings[3]. Other reasons include a picture of quark confinement via flux tubes that are expected to behave like strings at least qualitatively.

It is not clear whether QCD has an exact formulation based on strings. No satisfactory string model currently exists that could describe all the phenomenon of strong interactions. The absence of massless particles in the low energy spectrum of QCD, if not tachyons, makes it difficult to relate it to the known string theories. Further, it is not clear how asymptotic freedom that is an integral part of QCD could be incorporated into the string picture. It could be that the simplest possibility, the Nambu-Goto string, is not capable of describing QCD. An analytical continuation of the confining phase to high temperature suggests that this might be the case[4]. A string theory of QCD, if one exists, is likely to be unconventional. It has escaped us perhaps due to our limited understanding of noncritical strings in physical dimensions.

In this letter, we do not address these problems. Rather, we look for any common threads within the two pictures proceeding under the assumption that some kind of a QCD string does exist at all distance scales. Thanks to asymptotic freedom, this is expected to be easier in the ultraviolet regime. Surprisingly, it is found possible to formulate a short distance criterion to be satisfied by a string theory of QCD. This arises from the trace anomaly of the QCD stress tensor computed using instantons. Being proportional to the beta function, it carries one important information we have on QCD, its asymptotic freedom. A string candidate for QCD is expected to exhibit this anomaly. It appears to be calculable in terms of the trace anomaly of the string. Due to our insufficient understanding of noncritical strings in physical dimensions, this is a difficult test to be verified. Remarkably though, we find numerical agreement both in magnitude and sign for pure SU(2) QCD in a string model treated formally. Because this test appears to be sensitive to the trace anomaly of the string, it is expected to severly limit the effective world sheet degrees freedom in the ultraviolet regime.

Let us first concentrate on the trace anomaly of the QCD stress tensor. We will be working in the Euclidean region and mostly concerned with pure QCD. QCD Lagrangian in the absence of quark masses is scale invariant and the only contribution to the trace θ of the stress tensor is from the anomaly. This is given in terms of the beta function as[5]

$$\theta \; = \; \beta(g)\frac{\partial}{\partial g}L, \tag{1}$$

where L is the QCD Lagrangian and g is the strong coupling constant. This follows from the following observation. A scaling of the coordinates in a Green's function can be implemented by a scaling of the cutoff parameter in a theory like QCD that has no dimensionful coupling constants. But, as is well known for renormalizable theories, the latter scaling can be compensated by a change in the coupling constant and this introduces the beta function. The trace of the stress tensor, being the divergence of the generator of scale transformations, thus picks up an anomaly proportional to the beta function. Our interest is in the vacuum expectation value of θ in the asymptotic regime, that is as $g \rightarrow 0$. More specifically, we are interested in taking the $g \rightarrow 0$ limit of $\langle\theta\rangle/F$ where F is the QCD free energy. This limit would have vanished if not for instantons. It is well known that in the dilute gas approximation the instanton contribution in SU(N) QCD is[6]

$$F \; \propto \; g^{-4N}\exp(-8\pi^2/g^2), \tag{2}$$

The desired limit is now easily taken,

$$\frac{\langle\theta\rangle}{F} \; = \; \beta(g)\frac{\partial}{\partial g}\ln F \; \rightarrow \; -\frac{11}{3}N, \tag{3}$$

making use of the one loop beta function of pure QCD,

$$\beta(g) \; = \; -\frac{11N}{48\pi^2}g^3. \tag{4}$$

Thus $\langle\theta\rangle/F$ has a finite limit as $g \rightarrow 0$. It is enough to know the one loop beta function to reach this limit. Dilute gas approximation for instantons is likely to be sufficient to compute this limit exactly.

Instanton contributions are negligibly small in the limit $g \rightarrow 0$ compared to any perturbative ones. But we retained only those in the process of taking the limit. This is because our interest is in the part of QCD that is of relevance to the string picture. Note that the QCD string is a nonperturbative phenomenon expected to describe the confining phase of

QCD. Stringy features are thus going to be singular at $g = 0$. The leading contribution to this singular part is likely to be from instantons. In the rest of this letter, this singular part is referred to simply as QCD free energy.

We looked at $\langle\theta\rangle$ divided by the free energy because that is more interesting than $\langle\theta\rangle$ itself. It has a natural meaning in the string picture. Note the partition function of the string is expected to give us not the QCD partition function but its free energy. Hence, it is $\langle\theta\rangle/F$ that has an interpretation as a world sheet expectation value. It should agree with the expectation value of an analogous object evaluated on the world sheet. Because this quantity has a finite ultraviolet limit in QCD and is sensitive to asymptotic freedom, it should be regarded as a strong test for the QCD string candidates.

It is thus important to check whether there exist any string theories exhibiting the same trace anomaly. Unfortunately, there are no known consistent string candidates for QCD. For the purpose of illustrating how this anomaly could arise in the string picture, we here address a bosonic string governed by a two dimensional field theory of the space time coordinates X and the ghosts b and c defined on the world sheet, namely a two dimensional sphere[7]. For the usual action, this is not a well defined theory since its BRST operator is not nilpotent outside 26 dimensions[8]. However, we leave open the possibility that there could be a suitable theory that cures this problem. This is more so in the regime we are looking at since one expects important corrections to the usual action that become relevant at short distances[9]. A metric h in the background governs the geometry of the world sheet with an overall scale factor determining the size. There is another metric on the world sheet induced by an embedding into space time. Metric h is expected to be correlated to an effective value of this induced metric. It is hence a convenient measure of the distance scale being probed. Note also that the integration measure for the X fields, a measure of the quantum fluctuations of the world sheet, depends on h through the norm

$$||\delta X||^2 \;=\; \int \sqrt{\det\, h}\, |\delta X|^2\,, \tag{5}$$

where the integral is over the sphere. Thus, a scaling of the coordinates could be implemented by a scaling of the metric $h \to e^\sigma h$.

Important contributions to this scaling arise from the integration measures $D_h(X)$, $D_h(b)$ and $D_h(c)$ of the X, b and c fields respectively. As is well known, these measures scale as

$$D_h(X)D_h(b)D_h(c) \;\to\; D_h(X)D_h(b)D_h(c)\, \exp(-S_\sigma), \tag{6}$$

where

$$S_\sigma \;=\; \frac{26-D}{48\pi}\int\left(\frac{1}{2}\partial\sigma\partial\sigma + R\sigma + m^2 e^\sigma\right). \tag{7}$$

Dependence on the metric h is implicit in this expression. $D=4$ is the space time dimension. R is the scalar curvature of the sphere. The number 26 comes from the ghosts b and c, and D is from the coordinates X. In our case σ is a constant. The m^2 term could be dropped because it is proportional to the world sheet area expected to be negligible in the ultraviolet limit. Now, using the following well known theorem on the sphere,

$$\int R \;=\; 8\pi, \tag{8}$$

we find that the measure contributes a factor

$$\exp\left[-(26-D)\sigma/6\right]. \tag{9}$$

The exponent gives the trace anomaly of the world sheet stress tensor. Because we have treated the background metric as a measure of the distance scale being probed, it gets tied to the trace anomaly of the space time stress tensor. It is to be compared with our result for QCD. First, note that the scaling $h \to e^\sigma h$ corresponds to a scaling of the coordinates by $e^{\sigma/2}$. Hence the exponent of interest is $(26-D)/3$. Remarkably, in four dimensions, this exponent coincides with our result $11N/3$ for pure SU(2) QCD.

It turns out that the above agreement for SU(2) is both in magnitude and sign. To see this, let us relate the two anomalies given f (the world sheet integral of) the free energy of the string and F that of QCD. Recall that the string partition function $\exp(-f)$ is expected to agree with the QCD free energy. In the presence of some kind of a source, the relation is upto a constant

$$f[x,h] \;=\; -\ln F[x,\mu,g(\mu)]. \tag{10}$$

We have specified the dependence on the source locations collectively as x. Also, we have included in f a dependence on the background metric h, and in F a dependence on the renormalization point μ and the running coupling constant $g(\mu)$. Now, a scaling of x can be passed on to that of h and μ by dimensional analysis, and this leads to

$$f\left[x, e^\sigma h\right] \;=\; -\ln F\left[x, e^{-\sigma/2}\mu, g(\mu)\right]. \tag{11}$$

Renormalizability of QCD is a statement that the QCD free energy is μ independent. Hence, a scaling of μ can be compensated by a change in the coupling constant g, that is

$$F\left[x, e^{-\sigma/2}\mu, g(\mu)\right] \;=\; F\left[x,\mu, g\left(e^{\sigma/2}\mu\right)\right]. \tag{12}$$

Using this in (11) and going to infinitesimal σ, we get the result

$$h\frac{\partial}{\partial h}f \;=\; -\frac{1}{2}\beta(g)\frac{\partial}{\partial g}\ln F, \tag{13}$$

where the beta function is introduced through $\beta(g) = \mu dg(\mu)/d\mu$. In deriving this, we have ignored any other dimensionful parameters besides h that could be present in f. But it is reasonable to assume that those parameters become insignificant at short distances. Dimensionless parameters if present do not interfere in our discussion. Note that we do not have to change them as we did for $g(\mu)$.

As noted earlier, the right side of the above equation comes from the trace anomaly of the QCD stress tensor. On the left side, we have (an integral of) the trace of the world sheet stress tensor giving its trace anomaly. Hence, this is an anomaly matching consistency check for a string picture of QCD. It should be valid in the absence of sources. We have already computed the two sides of this equation in the ultraviolet limit. The right side is $11N/6$ in pure QCD. The left side is $(26-D)/6$. In four dimensions, the two agree both in magnitude and sign for the gauge group SU(2). This is remarkable given that the two anomalies were computed from completely different approaches. The agreement in sign is a pleasant surprise since the sign is crucial for asymptotic freedom in QCD. The restriction to SU(2) is understandable given that introducing an N dependence on the world sheet must involve some yet unknown physics. Fortunately for SU(2) all the numerical factors combine to give agreement.

There are reasons to suspect that the numerical agreement could be just a coincidence. We have ignored the contributions coming from the action since we do not know what that constitutes at short distances. Independence over the background metric has not been ensured. If it were to become a dynamical variable, with its conformal factor becoming a field variable on the world sheet, we would get a Liouville field theory. The highly nontrival integration measure of this field could modify our conclusions. The numerical agreement is only for SU(2) and not for SU(N) of higher N. But, whatever the reasons, one can not escape the fact that an ultraviolet test of this kind is a useful tool to uncover string physics from QCD.

Some of the problems may have natural solutions at short distances. The action could be scale invariant or its variation negligible. The background metric could be simply a classical solution for the induced metric. Note that a rigid string[9] with a scale invariant action involving the extrinsic curvature yields as a classical solution a round metric with an overall scale undetermined. The overall scale may play the role of a renormalization point so that independence over it could be ensured in the presence of a running coupling constant. If we were to have a Liouville field theory, it is possible that the nonzero modes are suppressed at short distances. Extension to $N > 2$ may require an N dependent theory, or a theory in a suitable phase may introduce a factor N into the scaling arguments.

There are also reasons to suspect that the numerical agreement could be more than a coincidence. The existence of a finite ultraviolet limit for the trace anomaly in the presence of instantons requires a matching one in the string picture. The number 11 that appears in the one loop beta function for QCD and also as a factor in the trace anomaly of the string in four dimensions does not arise often in computations. It is nontrivial that the rest of the factors combine neatly to give agreement for SU(2). Topological results are needed in both the cases, instantons in QCD and Gauss-Bonnet theorem on the sphere. There are ghosts contributing to the anomaly in both the pictures. It is thus reasonable to expect a relation of some kind for a QCD string, if one exists.

Note that the trace anomaly $\propto 26 - D$ strongly influences our conclusions. Because this anomaly depends on the number of effective degrees of freedom living on the world sheet, it is perhaps unlikely that more of them are excited at short distances as argued in ref. [4]. Asymptotic freedom is crucial as it is responsible for the agreement in the signs. Our analysis is for pure QCD and having quark degrees of freedom in the theory makes asymptotic freedom less severe. It is interesting to note that the same behavior is observed in string theory where the number $26 - D$ decreases for more degrees of freedom. For every quark, we expect a decrease by two units for the anomalies to match.

The number $11N/3$ we have tried to match coming from the trace anomaly is proportional to N. It is not clear how such an N dependence could arise in the string picture based on 't Hooft's $1/N$ analysis. Perhaps this is because the latter is implicitly based on a perturbative argument in the QCD coupling constant and one needs to incorporate instanton effects into it before addressing this issue. It is not surprising that instantons play a major role in our discussion. The string picture is expected to hold in the confining phase of QCD with the instantons possibly contributing some of the nonperturbative effects. That instantons could have implications for the QCD string is also apparent from the results of ref. [10] which identifies the Yang-Mills instantons with the instantons of a two dimensional theory taking values in the loop group. Clearly, there is a lot to be learnt.

To summarize, it is shown in this letter that there exists a short distance criterion to be satisfied by a string theory of QCD. This arises from the trace anomaly of the QCD stress tensor in the presence of instantons. Being proportional to the beta function, it is sensitive to asymptotic freedom. It appears to be related to the well known trace anomaly of string theory. Because of our limited understanding of noncritical strings in physical dimensions, this is a difficult test to be verified. However, a formal treatment of a string model leads to remarkable numerical agreement between the two pictures for the gauge group SU(2) both in magnitude and sign.

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References

[1] See for instance P. D. Collins, "Regge Theory and High Energy Physics", Cambridge University Press, Cambridge 1977.

[2] G. 't Hooft, Nucl. Phys. B 72, 461 (1974).

[3] K. G. Wilson, Phys. Rev. D 10, 2445 (1974).

[4] J. Polchinski, Phys. Rev. Lett. 68, 1267 (1992).

[5] J. C. Collins, A. Duncan and S. D. Joglekar, Phys. Rev. D 16, 438 (1977) and references therein.

[6] See for instance S. Coleman, "Aspects of Symmetry", Cambridge University Press, Cambridge 1985.

[7] See for instance M. B. Green, J. H. Schwartz and E. Witten, "Superstring Theory" Vol. I, Cambridge University Press, Cambridge 1987.

[8] M. Kato and K. Ogawa, Nucl. Phys. B 212, 443 (1983).

[9] A. M. Polyakov, Nucl. Phys. B 268, 406 (1986), and "Gauge Fields and Strings", Harwood Academic Publishers, 1987.

[10] M. F. Atiyah, Comm. Math. Phys. 93, 437 (1984).