

Circumventing the No-Go Theorem in Noncommutative Gauge Field Theory

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Abstract

Stringent restrictions for model building are imposed by a no-go theorem in noncommutative gauge field theory. Circumventing this theorem is crucial for the construction of realistic models of particle interactions. To this end, the noncommutative construction of tensor representations of gauge groups using half-infinite Wilson lines is extended to allow for gauge groups consisting of an arbitrary number of $U_*(N)$ factors. This as well allows representations other than the ones permitted by the no-go theorem.

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1 Introduction

The study of noncommutative (NC) gauge field theories has been initiated from the early stages of the development of NC quantum field theory, in connection with the observation that the noncommutativity of space-time coordinates appears in string theory in the presence of an NS-NS B -field [1]. It was noted from the very beginning that the only allowed noncommutative gauge groups are the unitary groups. This is due to the fact that in NC field theory with Heisenberg-like commutation relation

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \quad (1.1)$$

where \hat{x}^μ are the space-time coordinate operators, and $\theta^{\mu\nu}$ is an antisymmetric constant matrix, the conventional procedure requires to replace the usual product between any fields with the Moyal star-product

$$(fg)(\hat{x}) \longmapsto (f * g)(x) = \exp \left[\frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu} \right] f(x)g(y) \Big|_{x=y}. \quad (1.2)$$

Due to the Moyal star-product by which the gauge transformation are multiplied, a number of constraints arise, the first being that the closure condition is satisfied only by unitary groups $U_*(N)$ (see next section for details on their construction), while any other groups such as special unitary, orthogonal or symplectic gauge groups do not close. Two more consequences of the noncommutativity of the Moyal star-product deserve to be mentioned in this connection: i) unlike the commutative case, the unitary group $U_*(1)$ is non-Abelian; ii) the group $U_*(N)$ is simple and not semi-simple as in the commutative case, however $U_*(1)$ is still a subgroup of $U_*(N)$, although the quotient $U_*(N)/U_*(1)$ does not exist.

For the allowed $U_*(N)$ gauge groups, further restrictions appear [2, 3, 4], gathered into a no-go theorem in Ref. [4]. The theorem states that: 1) the local $u_*(n)$ algebra only admits the irreducible $n \times n$ matrix representation. Hence the gauge fields are in $n \times n$ matrix form, while the matter fields can only be in fundamental, adjoint or singlet states; 2) for any gauge group consisting of several simple-group factors, the matter fields can transform nontrivially under *at most two* NC group factors. In other words, the matter fields cannot carry more than two NC gauge group charges.

Especially, the last restriction for charges is problematic for model building. The construction of an NC version of the standard model (SM) [5] disclosed the above problems. Since only the unitary group is allowed as a gauge group, the natural minimal extension

of the SM gauge group is $U_*(3) \times U_*(2) \times U_*(1)$. With this choice, unlike in the commutative case, the quarks cannot have three gauge charges. Since matter fields can only carry at most two gauge groups, the quarks can not be charged under the $U_*(1)$ group if they are charged under $U_*(3)$ and $U_*(2)$.⁵ If one wants to construct a model, in general, possessing gauge groups $\prod_i U_*(n_i)$ and matter charged under more than two gauge groups, one would similarly encounter the restriction of the no-go theorem. Therefore, it is crucial to circumvent the restrictions of the no-go theorem in a consistent way.

The progress in the formulation of NC gauge field theory was not put off by the no-go theorem. At different stages, steps were taken towards evading the requirements of this no-go theorem in NC gauge theory. A key ingredient of the scheme is to satisfy the closure condition of direct product group by introducing an NC version of a Wilson line. NC Wilson lines were firstly introduced in the construction of NC gauge invariant operators [8]. Then, an important step was to use a half-infinite NC Wilson line in order to construct tensorial representations of any rank for $U_*(N)$ in [9, 10], where an NC extension of the supersymmetric $U_*(5)$ Grand Unified Theory was proposed.

In this paper, we extend the use of the Wilson line in the NC gauge field theory to construct the action integral formed out of fields carrying any number of charges.⁶ We first give a brief review of the no-go theorem [4]. Then we explain how tensorial representation for the $U_*(N)$ gauge group is constructed by using the Wilson line, whose formulation is essential for our construction. We also present symmetric and anti-symmetric representations by employing the formulation. After that, we construct representation for direct product of an arbitrary number of gauge groups. Finally we give a comment on a mechanism to break a trace $U_*(1)$ part of a $U_*(N)$ gauge group proposed in [6, 7].

⁵It was shown that, provided that trace- $U_*(1)$ subgroups of $U_*(3) \times U_*(2) \times U_*(1)$ are properly broken, the gauge boson of the residual $U_*(1)$ symmetry (corresponding to hypercharge $U(1)$ group in the commutative limit) couples to all the matter fields (placed into representations of $U_*(3) \times U_*(2) \times U_*(1)$ strictly according to the no-go theorem) through the proper hypercharges (for further discussion, see [6, 7]).

⁶This construction was hinted at in the formulation of the NC minimal supersymmetric SM [7].

2 Group representations with half-infinite Wilson lines

2.1 The no-go theorem in noncommutative gauge theory

For the self-consistency, we give a brief account of the no-go theorem [4] in NC gauge field theory. Let us consider the NC version of gauge transformation for a gauge field:

$$A_\mu \rightarrow U * (A_\mu - i\partial_\mu) * U^{-1}. \quad (2.1)$$

Here $U = e_*^{-i\lambda}$ is a gauge group element with insertion of the Moyal star-products between the matrix valued functions, and $\lambda = \lambda^a T^a$, where T^a is the matrix for a representation of the gauge group. The infinitesimal gauge transformation is

$$\delta A_\mu = \partial_\mu \lambda - \frac{i}{2}[T^a, T^b](\lambda^a * A_\mu^b + A_\mu^b * \lambda^a) - \frac{i}{2}\{T^a, T^b\}(\lambda^a * A_\mu^b - A_\mu^b * \lambda^a). \quad (2.2)$$

As one can see, this gauge transformation is not closed, i.e., δA_μ is not Lie algebra-valued, unless $\{T^a, T^b\}$ is a linear combination of T^c . For example, it is obvious that special unitary group does not satisfy the condition. The only allowed gauge group is an NC version of unitary group, $U_*(N)$, for which the above requirement is automatically true.

In addition to these restrictions, the representations of the $u_*(n)$ Lie algebra are restricted to $n \times n$ hermitian matrices. Hence the gauge fields are in $n \times n$ matrix form, while the matter fields can only be in fundamental (F), anti-fundamental (\bar{F}), adjoint ($F \times \bar{F}$) and bi-fundamental ($F \times \bar{F}'$). Furthermore, matter fields can only transform non-trivially under at most two simple subgroups of any gauge group consisting of a product of simple groups. In other words, the matter fields cannot carry more than two NC gauge group charges. For $U_*(1)$ this restriction means that the charges of the matter fields are quantized to just 0, +1 or -1 [11].

2.2 Tensor representations of $U_*(N)$

An obvious requirement for the NC gauge group representations is to satisfy the closure property of the gauge group. For the fundamental representation of the scalar matter field denoted as the column vector ϕ^i , the gauge transformation is defined by

$$\phi^i \rightarrow (\phi^U)^i = U^i_j * \phi^j. \quad (2.3)$$

This satisfies gauge group multiplication law

$$(\phi^U)^V = \phi^{V*U} , \quad (2.4)$$

where V is another gauge group element. One can also check that this property is satisfied for the anti-fundamental, adjoint and bi-fundamental representations of matter fields. However, representations other than them such as higher rank tensorial representations are not allowed. For instance, let us consider a rank-2 representation of the single gauge group $U_*(n)$, $\phi^{ij}(x)$. In this case, one can readily see that the NC gauge transformation for this field

$$\phi^{ij} \rightarrow U_{i'}^i * U_{j'}^j * \phi^{i'j'} , \quad (2.5)$$

does not satisfy the group multiplication law (2.4).

The construction of the tensorial representation was proposed in Ref. [10]. Since the basic ingredients of this construction are at the core of the extension to a direct product of groups, we shall briefly review them here.

The idea is to modify the gauge transformation (2.5) in a non-trivial gauge-field-dependent way so that the group multiplication law holds (2.4). Here we introduce the NC version of a half-infinite Wilson line,

$$W_C(x) = P_* \exp \left(ig \int_0^1 d\sigma \frac{d\zeta^\mu(\sigma)}{d\sigma} A_\mu(x + \zeta(\sigma)) \right) , \quad (2.6)$$

where the integration is along the contour C from ∞ to x ,

$$C = \{ \zeta(\sigma) , 0 \leq \sigma \leq 1 \mid \zeta(0) = \infty , \zeta(1) = 0 \} , \quad (2.7)$$

and the path ordering involves the Moyal star-product between any functions. Under the NC gauge transformation (2.1), the Wilson line transforms as

$$W_C(x) \rightarrow U(x_1) * W_C(x) * U^{-1}(x_2) , \quad (2.8)$$

where x_1 and x_2 are endpoints of the contour. Without loss of generality, we can restrict spatial components of x_1 to be at infinity, which we simply denote as $x_1 \rightarrow \infty$. Furthermore we restrict the allowed gauge transformation $U(x)$ to those which approach a constant U_∞ as $x_1 \rightarrow \infty$:

$$W_C(x) \rightarrow U_\infty W_C(x) * U^{-1}(x) . \quad (2.9)$$

Note that the gauge transformation (2.1) with boundary condition $A_\mu(x) \rightarrow 0$ as $x \rightarrow \infty$ means $U_\infty = \text{constant}$. We choose this constant as $U_\infty = 1$

$$W_C(x) \rightarrow W_C(x) * U^{-1}(x) \quad (2.10)$$

by ignoring the global transformation at infinity which can be attributed to a normalization of the fields.

By using the NC Wilson line, let us find the modified gauge transformation law. To do this, it is convenient to define the quantity

$$\Phi^{ij} = W_{C_1}{}^i{}_{i'} * W_{C_2}{}^j{}_{j'} * \phi^{i'j'}, \quad (2.11)$$

where the subscripts C_1 and C_2 denote two contours which have the same endpoints (2.7). By requiring this quantity to be gauge invariant, one obtains the gauge transformation of ϕ^{ij} as

$$\phi^{ij} \rightarrow (\phi^U)^{ij} = (U * W_{C_2}^{-1})^j{}_k * U^i{}_l * W_{C_2}{}^k{}_m * \phi^{lm}. \quad (2.12)$$

This gauge transformation satisfies the closure condition (2.4) [10], so that it is a suitable gauge transformation. In the $\theta^{\mu\nu} \rightarrow 0$ limit, the Wilson lines in (2.12) cancel each other and the gauge transformation reduces to (2.5).

A few comments are in order. For a single index representation, the gauge transformation law reduces to the normal NC gauge transformation

$$\phi^i \rightarrow (U * W_C^{-1})^j{}_l * W_C{}^l{}_k * \phi^k = U^i{}_j * \phi^j. \quad (2.13)$$

since the Wilson lines cancel.

The gauge transformation for the rank-2 tensor ϕ^{ij} (2.12) cannot be decomposed into symmetric and antisymmetric representations like as commutative case since the gauge transformation does not commute with the interchange of the indices

$$(\phi^U)^{ij} \rightarrow (\phi^U)^{ji} = (U * W_{C_2}^{-1})^i{}_k * U^j{}_l * W_{C_2}{}^k{}_m * \phi^{lm} \neq (U * W_{C_2}^{-1})^j{}_k * U^i{}_l * W_{C_2}{}^k{}_m * \phi^{ml}. \quad (2.14)$$

In other words, in the NC case rank-2 tensor is not reducible, and we cannot treat $\phi^{(ij)} = \frac{1}{2}(\phi^{ij} + \phi^{ji})$ as symmetric representation (similarly to antisymmetric case). Instead of it, one can construct the following symmetric gauge invariant tensor

$$\Phi^{(ij)} = W_{C_1}{}^i{}_a * W_{C_2}{}^j{}_b * \phi^{ab} = \frac{1}{2}(W_{C_1}{}^i{}_a * W_{C_2}{}^j{}_b + W_{C_1}{}^j{}_a * W_{C_2}{}^i{}_b) * \phi^{ab}, \quad (2.15)$$

where ϕ^{ab} follows the gauge transformation law (2.12) while $\Phi^{(ij)}$ is a symmetric tensor. The antisymmetric tensor $\Phi^{[ij]}$ is given by

$$\Phi^{[ij]} = W_{C_1}^{[i} * W_{C_2}^{j]} * \phi^{ab} = \frac{1}{2}(W_{C_1}^{i_a} * W_{C_2}^{j_b} - W_{C_1}^{j_a} * W_{C_2}^{i_b}) * \phi^{ab}. \quad (2.16)$$

Similarly, one can define the modified gauge transformation for fermions. For example, the gauge transformation for the fermionic 2-tensor ψ^{ij} is given by

$$\psi^{ij} \rightarrow (\psi^U)^{ij} = (U * W_{C_2}^{-1})^j_k * U^i_l * W_{C_2}^k_m * \psi^{lm}, \quad (2.17)$$

corresponding to the gauge invariant quantity $\Psi^{ij} \equiv W_{C_1}^{i_{i'}} * W_{C_2}^{j_{j'}} * \psi^{i'j'}$. Hereafter we will restrict our attention to scalar fields as it is straightforward to apply for fermions.

2.3 Fields charged under an arbitrary number of $U_*(N)$ groups

Now we extend the above discussion into construction of representations for a direct product of any number of $U_*(N)$ with different N . We start by considering a direct product of two groups $U_*(M) \times U_*(N)$ and a field charged under these two factors, ϕ^{mn} where m and n denote gauge indices for fundamental representations of $U_*(M)$ and $U_*(N)$, respectively. Performing the simple NC version of gauge transformation for fundamental representation for the gauge group $U_*(M) \times U_*(N)$, we have

$$\phi^{mn} \rightarrow (\phi^{mn})^U = (U_N)^n_{n'} * (U_M)^m_{m'} * \phi^{m'n'}. \quad (2.18)$$

This does not satisfy the closure condition (2.4), i.e., $((\phi^{mn})^U)^V \neq (\phi^{mn})^{V*U}$.

Thus, we would like to modify the gauge transformation law so that it satisfies (2.4) similarly to the case of the tensorial representation for a single $U_*(N)$ gauge group. As in (2.11) of the previous subsection, we require gauge invariance of a quantity,

$$\Phi^{mn} = (W_M)^m_{m'} * (W_N)^n_{n'} * \phi^{m'n'}, \quad (2.19)$$

where W_M and W_N are the Wilson lines for the gauge group $U(M)$ and $U(N)$, respectively.

Each Wilson line has a contour with end points as in Eq. (2.7) and follows the gauge transformation law (2.10). The exact shape of the contour may be different for M and N . We then define the gauge transformation law so that (2.19) is gauge-invariant:

$$\phi^{mn} \rightarrow (\phi^U)^{mn} = (U_N * W_N^{-1})^n_k * (U_M)^m_l * (W_N)^k_p * \phi^{lp}. \quad (2.20)$$

For notational convenience, we write this in the tensor notation:

$$\phi^U = (1 \otimes U_N * W_N^{-1}) * (U_M \otimes W_N) * \phi \quad (2.21)$$

This gauge transformation law satisfies the group multiplication law (2.4), and it is therefore a suitable NC gauge transformation. Note that this transformation law includes only the Wilson line W_N not but W_M .

There is another possible form of the gauge invariant object

$$\Phi^{mn} = (W_N)_{n'}^n * (W_M)_{m'}^m * \phi^{m'n'} . \quad (2.22)$$

Gauge transformation associated with (2.22) is given by

$$\phi^{mn} \rightarrow (\phi^{mn})^U = (U_M * W_M^{-1})_k^m * (U_N)_{n'}^n * (W_M)_p^k * \phi^{p n'} . \quad (2.23)$$

This gauge transformation also satisfies the closure condition (2.4) and in this case includes the Wilson line of the $U_*(M)$ group not $U_*(N)$. Both of the gauge transformation (2.20) and (2.23) fall into the ordinary gauge transformation in the commutative limit. Now the noncommutativity seems to split the ordinary-space representation into two distinct NC representations. However, as we will see in the next section, they would lead to the same physical result. Therefore, in the following, we will adopt (2.20) .

We generalize the representation (2.20) into one for a direct product of n unitary gauge groups. In tensor notation, we obtain the gauge transformation as

$$\begin{aligned} \phi_{[n]}^U &= (U_{M_n} * W_{M_n}^{-1} \otimes 1 \otimes \cdots \otimes 1) * (1 \otimes U_{M_{n-1}} * W_{M_{n-1}}^{-1} \otimes 1 \otimes \cdots \otimes 1) \\ &\quad * \cdots * (1 \otimes \cdots \otimes U_{M_1} * W_{M_1}^{-1}) * (W_{M_1} \otimes \cdots \otimes W_{M_n}) * \phi_{[n]} , \end{aligned} \quad (2.24)$$

where W_{M_i} is the Wilson line for $U(M_i)$ gauge group. If ϕ has any anti-fundamental indices, they can be taken to transform from the right. Thus we have obtained the field $\phi_{[n]}$ carrying n charges. Here the corresponding gauge invariant object is

$$\Phi_{[n]} = W_{M_1} \otimes W_{M_2} \otimes \cdots \otimes W_{M_n} * \phi_{[n]} . \quad (2.25)$$

For anti-fundamental indices one finds a similar gauge invariant quantity by multiplying with the corresponding Wilson lines from the right instead of the left. In this case, it is also easy to see that taking the commutative limit, the gauge transformation (2.24) reduces to the commutative one since the Wilson lines cancel.

3 Gauge Invariant Action Integral

Now that we have obtained fields carrying any number of charges, let us construct a gauge invariant action integral. In what follows we focus on the rank-2 representation (2.20) for a direct product of two gauge groups. We introduce the gauge invariant vector field,

$$\mathcal{A}_\mu^L \equiv A_\mu^{W_L} = W_L * (A_\mu^L - i\partial_\mu) * W_L^{-1}, \quad (3.1)$$

where $L = (M, N)$. With this vector field, we define the covariant derivative:

$$\mathcal{D}_\mu^{MN} = (1 \otimes 1)\partial_\mu + i(\mathcal{A}_\mu^M \otimes 1) + i(1 \otimes \mathcal{A}_\mu^N). \quad (3.2)$$

By using them, we can write down the gauge invariant kinetic term:

$$S = \int d^4x \operatorname{tr} |\mathcal{D}_\mu^{MN} * \Phi|^2 = \int d^4x \operatorname{tr} |D_\mu^{MN} * \phi|^2, \quad (3.3)$$

where

$$D_\mu^{MN} = (W_M \otimes W_N)^{-1} * ((1 \otimes 1)\partial_\mu + (\mathcal{A}_\mu^M \otimes 1) + (1 \otimes \mathcal{A}_\mu^N)) * (W_M \otimes W_N). \quad (3.4)$$

This gauge invariant kinetic term is an NC extension of the commutative gauge invariant kinetic term for the fundamental representation for direct product of the two gauge groups. Taking the $\theta^{\mu\nu} \rightarrow 0$ limit in (3.3), the Wilson lines cancel and it reduces to the ordinary gauge invariant kinetic term

$$S = \int d^4x \operatorname{tr} |\{(1 \otimes 1)\partial_\mu + i(A_\mu^M \otimes 1) + i(1 \otimes A_\mu^N)\}\phi|^2. \quad (3.5)$$

Recall that the Wilson line is a gauge group element. From the gauge transformation (2.20), one sees that the gauge invariant quantity (2.19) can be expressed in terms of a gauge transformation with $U_M = W_M$ and $U_N = W_N$:

$$\Phi = \phi^{U_M=W_M, U_N=W_N}. \quad (3.6)$$

The gauge invariant field is obtained through a gauge transformation with $U(x) = W(x)$. This means that Φ and $\mathcal{A}_\mu^{M(N)}$ lie on the same gauge orbit with ϕ^{mn} and $A_\mu^{M(N)}$, respectively. Therefore the gauge invariant kinetic term is of the same form as the ordinary-space one but with the Moyal star-product between any field. This is actually a gauge fixing procedure as indicated for the tensorial representation for a single simple gauge group

in Ref. [10]. Thus as we mentioned, (2.22) is physically equivalent to (2.19). It is also straightforward to obtain the action integral of gauge theory coupled to the rank- n field $\phi_{[n]}$ with the gauge transformation (2.24) and to fermionic rank- n fields.

Considering the fact that Φ^{mn} itself is gauge invariant, one can easily construct other gauge invariant candidates for the kinetic term, for example:

$$S = \int d^4x \partial_\mu \Phi^{mn} * \partial^\mu \Phi_{mn}^\dagger. \quad (3.7)$$

After the gauge fixing procedure described above, this would give the usual kinetic term of a gauge invariant NC scalar field. On the other hand there is no obvious theoretical reason to prefer the (invariant) covariant derivatives (3.2) in construction of the kinetic term as the composite field Φ is actually invariant and one could do simply with ordinary derivatives. We simply adopt the covariant derivatives (3.2) for phenomenological applications.

Finally, we would like to make a comment concerning the so-called Higgsac mechanism [6, 7]. As we explained in the introduction, the minimal NC extension of the SM gauge group is $U_*(3) \times U_*(2) \times U_*(1)$. In order to realize the commutative SM at low energies, one has to break the trace $U_*(1)$ parts of these groups. The Higgsac mechanism was proposed to realize such a breaking in a manner that respects unitarity and the requirements of the no-go theorem. The basic ingredient of this mechanism is a scalar field $\phi_{[n]} = \phi^{i_1 i_2 \dots i_n}$ that is a rank- n tensor under the $U_*(N)$ gauge group (whose extension into the case of direct product of any number of gauge group is straightforward, following the construction explained in section 2.3). Similar to the case of rank-2 tensor in section 2.2, the gauge transformation for the rank- n tensor $\phi_{[n]}$ and the associated gauge invariant object are given as

$$\begin{aligned} (\phi_{[n]})^U &= (U * W^{-1} \otimes \dots \otimes 1) * (1 \otimes U * W^{-1} \otimes \dots \otimes 1) \\ &* \dots * (1 \otimes \dots \otimes U * W^{-1}) * \phi_{[n]}, \end{aligned} \quad (3.8)$$

and

$$\Phi = \frac{1}{n!} \epsilon_{i_1 i_2 \dots i_n} W_{j_1}^{i_1} * W_{j_2}^{i_2} * \dots * W_{j_n}^{i_n} * \phi^{j_1 j_2 \dots j_n}. \quad (3.9)$$

The latter is called the Higgsac field. Here we have used the same Wilson line for simplicity differently from the discussion in section 2.2. With the use of the field (3.9) it was suggested that the following Lagrangian caused a spontaneous breaking of the trace $U_*(1)$ part of the $U_*(N)$

$$\mathcal{L} = \partial_\mu \Phi^\dagger * \partial^\mu \Phi + m^2 |\Phi|^2 - \frac{\lambda}{2} |\Phi|^4, \quad (3.10)$$

In this Lagrangian, the scalar field Φ has a non-zero vacuum expectation value $\langle \Phi \rangle = \langle \phi \rangle = \sqrt{m^2/\lambda}$ where $\phi \equiv \frac{1}{n!} \epsilon_{i_1 i_2 \dots i_n} \phi^{[i_1 i_2 \dots i_n]}(x)$ and $A_\mu = 0$. Expanding Φ with respect to θ and the gauge coupling constant, one finds

$$\partial_\mu \Phi = (\partial_\mu + ing A_\mu^0) \phi + ig \partial_\mu \phi \int_0^1 d\sigma \frac{d\zeta^\mu}{d\sigma} A_\mu^0(x + \zeta(\sigma)) + \mathcal{O}(\theta) + \mathcal{O}(g), \quad (3.11)$$

where A_μ^0 is the trace part of A_μ . From this expression, it appears that the gauge field A_μ^0 has a mass in the presence of the non-zero vacuum expectation value of Φ .

However, according to our discussion above, all the gauge fields included in the scalar field Φ are gauged away by fixing the gauge. Therefore, no coupling between the scalar field and the gauge field occurs and there cannot exist any mass term for symmetry breaking. Thus the symmetry breaking proposed in [6, 7] would be an artifact of using a truncated expansion.

4 Conclusion

We have proposed a possible way out of the restrictions in the no-go theorem of NC gauge theories. We have constructed fields carrying charges of any number of $U_*(N)$ factors. A key ingredient for achieving such a representation is to satisfy the closure condition by modifying the gauge transformation using the half-infinite NC Wilson line [10]. We have constructed the action integral formed out of fields carrying any number of charges. The resultant action is of the same form as the ordinary-space one but with the Moyal star-product between any field, taking into account that the Wilson lines in the gauge invariant quantity are gauged away. This fact leads to a result that within this construction the Higgs mechanism discussed in [6, 7] would not work. One of key issues in the Higgs mechanism is that there are interactions between gauge fields in the Wilson line and scalar fields and trace $U_*(1)$ part of the gauge field acquire a mass upon the condensation of the scalar fields. However, the Wilson lines are completely gauged away. The gauge fields no longer couple to the scalar giving a mass and no gauge symmetry breaking occurs.

It is interesting that, although the no-go theorem for noncommutative gauge fields can be circumvented to a great extent, there is one aspect - the construction of representations of the $U_*(1)$ subgroup of $U_*(N)$ gauge group - which cannot be solved. We believe that this is connected to the fact that the quotient $U_*(N)/U_*(1)$ does not exist. If one could construct a representation of the subgroup $U_*(1)$, then by a correspondingly charged

field one could break spontaneously the $U_*(1)$ subgroup. However, after such a breaking, there is no noncommutative gauge symmetry left, since $SU_*(N)$ does not exist. These aspects strongly remind the situation encountered in quantum groups, when upon the deformation, a subalgebra of an algebra is no more subalgebra of the deformed algebra.

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