Offset-free Model Predictive Controller of a Heat Pump

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Abstract

This work presents an offset-free model predictive control (OF-MPC) design for energy efficient control of a heat pump. Open-loop system dynamics are examined first to determine the appropriate control structure leading to identification techniques being used to construct linear models of an experimentally-validated heat pump model. Subsequently, a framework consisting of a model predictive controller, cascaded upon a lower level PI controller, is designed. The MPC includes an augmented model (including disturbance states) and an associated Luenberger observer to estimate the prediction disturbance (plant-model mismatch at steady state). Simulation results subject to realistic disturbances and measurement noise demonstrate that energy savings of 2.3% can be achieved with the proposed OF-MPC design compared to a traditional control approach.

1. Introduction

Building comfort control is a complex task, consisting of not only conditioning but ventilation and transportation of a medium to/from regions requiring comfort maintenance. This task is completed by various mechanical and electrical components interacting usually in a hierarchical manner, all in an attempt to simultaneously meet comfort and indoor air quality objectives. The operation of the heating, ventilating and air-conditioning (HVAC) systems is critical to achieving these objectives in an energy-efficient manner.

In the recent past there have been a significant number of simulation and experimental-based studies on the modeling and control of HVAC systems including refrigeration systems for food preservation, roof top cooling units, and chillers (see, e.g., [1, 2] and the references therein). In this work, our focus is on a heat pump, an HVAC unit commonly used to condition (by supplying both heating and cooling) building air. Here, we consider only the cooling mode which uses a vapor compression cycle (VCC) for cooling. Models (validated using experimental data) covering various regimes of operation for both vapor compression cycles and heat pumps have been developed (see [3] and [4] for an excellent review) lending themselves well to providing a reliable environment in which control designs can be both developed and evaluated.

The control objectives which are usually addressed are centered in the fluids exiting the indoor coil of the heat pump. This includes ensuring that the air temperature (the cooling medium) at this outlet (denoted as the supply air temperature) is able to satisfy the building cooling demand while also ensuring the outlet refrigerant is maintained as superheated vapor. This latter objective is of utmost importance in preventing physical damage in downstream components, as liquid refrigerant can damage the mechanical components used in the compressor and result in either a reduction or complete halt in conditioning. At the same time, the two-phase refrigerant region within the indoor coil maximizes the energy efficiency of the cycle due to the superior heat transfer properties of a liquid-vapor refrigerant mixture relative to refrigerant solely in the vapor form, resulting in a tradeoff existing between optimality and system longevity. In addressing such control objectives typical manipulated variables include the compressor speed, the air flow rates across both the indoor and outdoor coils as well as the expansion valve opening.

Current single-input-single-output (SISO) control approaches fail to account for system interactions, such as those present in the interacting loops within a VCC, and also poorly handle competing constraints which can result in control performance deterioration. Energy use is also not usually featured as an objective in the control performance and is handled instead by choosing set-points based on the technician’s or operator’s experience, which usually evolves on a local unit-to-unit basis.

Model predictive control (MPC), a form of advanced control, has achieved effect control performance
in many fields, including, more recently the HVAC and building fields. The key components of a model predictive controller are 1) a model (linear or nonlinear) that allows prediction of the process states for candidate input trajectories and 2) a description of the objective function/constraints that reflect the desired (or as limited by physical constraints) behavior of the process. MPC applications have become increasingly prevalent as a result of their ability to handle the multivariable nature of system dynamics, constraints and optimality all in one integrated fashion, lending themselves well to effective closed-loop performance (see e.g., [5] and references therein for recent results).

Utilizing recent MPC developments, a linear model based (identified from test data) predictive controller is presented in [6] and implemented on a multi-evaporator compression cooling cycle to achieve energy efficient operation in both the presence and absence of common time-varying disturbances. Other HVAC MPC applications have included a nonlinear model predictive controller (using a reduced order, first principles, nonlinear model) designed and implemented for an experimental VCC as in [7] and also the work of [8], where system nonlinearity for an experimental VCC is addressed in a slightly different fashion using instead a gain-scheduling approach.

More recently, MPC applications have started to use mechanisms specific to plant-model mismatch, known as offset-free MPC designs (see [9, 10, 11]). These mechanisms for many applications handle system nonlinearities through a coupled approach which sees a model, linear or nonlinear, empirical or first-principles-based, accompanied with a state estimator (to estimate plant-model mismatch, assumed constant) to achieve offset-free control performance. In the HVAC and building fields, this has included the work of [12, 13] where a reduced order coupled model-estimator structure has shown to effectively capture the nonlinear dynamics and provide both robust and energy efficient control performance for both VCC and building-based MPC approaches. There is however, continued need to demonstrate the ability of advanced control approaches to improved performance in HVAC systems with detailed simulation models being used for the purpose of replicating the complex HVAC dynamics.

Motivated by the above considerations, this work considers the problem of offset-free energy efficient model predictive control (MPC) for a heat pump. To this end, an offset-free MPC, comprising of a coupled PI-MPC offset-free control approach is proposed for an experimentally-validated heat pump model. Open-loop system characteristics motivating the use of this coupled control approach are presented before delving into the details of both the simulation environment and the controller specifics. Offset elimination and enhanced energy efficiency relative to a traditional control approach are demonstrated in a simulation study that considers the heat pump in the presence of both realistic disturbances and measurement noise.

2. Preliminaries

In this section, we briefly review the heat pump model, software environment and highlight the control relevant features of the heat pump.

2.1. Heat Pump Model

A heat pump is bi-modal in nature, able to either release/absorb heat depending on the mode of operation, which, within the design limitations, enables heating/cooling demands to be satisfied using a single unit. In this work, focus is placed only on the cooling mode, where identical dynamics and components to those found in a vapor compression cycle (VCC) are used in cooling an air medium. The heat pump model used here was developed in Modelica by the thermodynamic simulation company, TLK-Thermo GmbH. In developing the individual component models and the overall dynamics, experimental steady-state data was used in validating the dynamics of first principle-based heat exchanger, piping and valve models as well as an empirical-based compressor component model. Note that in the steady state data and the transition, the superheat was always controlled to a set point, and the different points corresponded to different steady state values of the rpm and the superheat set point.

2.2. Simulation Environment

In creating a practically-relevant simulation environment three distinct software clients were required. First, the Building Controls Virtual Test Bed (BCVTB, [14]) was used as an interfacing agent providing an exchange medium for data transfer between control design-based Matlab and heat pump-model-based Dymola. In this setup, the actuator signals, the compressor rpm (ωk) and the superheat setpoint (T_{SH,SP}) are defined first in Matlab, then via BCVTB, received in Dymola, while the heat pump, the supply air temperature (T_{SA}) and the refrigerant superheat temperature (T_{SH}) are measured first in Dymola, then received in Matlab, again via BCVTB. This exchange architecture is displayed in Figure 1.
3. Temperature Control

A temperature control structure built around the PI-assisted heat pump model motivated prior is described in this section. The structure comprises an offset-free MPC (OFMPC), that uses a linear discrete-time prediction model (Section 3.1) coupled with a classic observer-based mismatch estimation (Section 3.2) and provides set points to a lower level superheat controller (as well as adjusting the rpm directly).

3.1. I/D/O Discrete Time Model

For the discrete time model, the process outputs at a specific sampling instant are assumed to depend linearly on the previous process conditions (defined by the process outputs and inputs). Mathematically, the model is described as:

\[
T_{SA}(k) = \Delta T_{SA}(k-1) + Bu(k-1)
\]

where \(u(k)\) is the process input vector at sampling instant \(k\) (respectively), and \(A\) and \(B\) are model coefficients. For the Heat Pump the inputs as previously defined are as follows: \(u = [T_{SH,SP} \ \theta_k]^{T}\), respectively. In identifying the model coefficients, pseudo random binary sequences (PRBS) were generated for each input and subsequently implemented on the heat pump model. Using the System Identification Toolbox in Matlab (which essentially solves the linear regression problem to compute the model coefficient matrices), the model coefficient matrices were identified.

3.2. Offset-free MPC design

The proposed MPC design consists of a standard ‘offset-free’ MPC mechanism (see [9, 10, 11]) coupled with the discrete-time model (3.1) for use in the MPC optimization problem. The specific offset-free mechanism as implemented on the heat pump is described by the following set of equations:

\[
\begin{bmatrix}
T_{SA,k+1} \\
\theta_{k+1}
\end{bmatrix} = \begin{bmatrix} A & G_\theta \\ 0 & I \end{bmatrix} \begin{bmatrix} T_{SA,k} \\
\theta_k
\end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k
\]

\[
y_k = \begin{bmatrix} I \\ 0 \end{bmatrix} \begin{bmatrix} T_{SA,k} \\
\theta_k
\end{bmatrix}
\]

where \(T_{SA} \in R^1, u \in R^2\) are the model output and inputs which correspond to the variable definitions given for Equation 1. In this work, only one dynamic ‘disturbance’ state, \(\theta \in R^1\) is needed to ensure that the \(T_{SA}\) prediction error converges to zero. A reformulation of the matrices and vectors used in (2) results in:

\[
\dot{x} := [T_{SA} \ \theta], \quad A_\Sigma := \begin{bmatrix} A & G_\theta \\ 0 & I \end{bmatrix},
\]

\[
B_\Sigma := \begin{bmatrix} B \\ 0 \end{bmatrix}
\]

where the discrete-time system in (2) equipped with a Luenberger observer provides augmented state estimates as defined by:

\[
\dot{\hat{x}}(k+1) = A_\Sigma \hat{x}(k) + L(x_m(k) - \hat{x}(k))
+ B_\Sigma u(k)
\]

The augmented disturbance model provides the ability to initially separate the tuning of the plant-model mismatch correction mechanism and the control design. In particular, open loop tests can be first carried out to determine appropriate values of the offset-free mechanism tuning parameters \((G_\theta, L_\theta, L_{TSA})\). This led to open-loop simulations being performed where each system input, \(\theta_k\) and \(T_{SH,SP}\), were varied independently while comparing the \(T_{SA}\) predictions from the augmented model with their measured counterpart. This open-loop test environment was used in an attempt to diagnose the individual effects of each input on both the speed and oscillatory nature exhibited in the \(T_{SA}\) predictions, while also providing a means that could be used to understand the individual effects of each tuning parameter. This included learning that as a result of \(G_\theta\) being a scalar, its specific value is not particularly important and can be thus, chosen as 1 (in the case where \(G_\theta\) is a matrix, the relative values of the entries of the matrix determine the multivariable interactions in the observer system). Also of value, is the known fundamental tradeoff which must be considered when tuning the poles. Too aggressive poles, in the presence of noise or once the system is closed are likely to result in high frequency oscillatory behavior, while if 'sluggish' poles (high values) are used, the offset elimination will be slow. Using this tuning framework with the effect of pole magnitude in hand, a pole combination which provided reasonably fast elimination of the offset in a minimally oscillating manner were the final pole values chosen before closing the loop.

3.3. Heat Pump MPC design

Next, the state estimator coupled with the discrete time model were integrated into a traditional linear
MPC framework cascaded upon the lower level superheat PI controller as displayed in Figure 2.

Figure 2: Model Predictive Control structure equipped with offset-free mechanism

Subsequently, at a sampling instance k, when a new measurement of $T_{SA}(k)$ is received, the optimization problem below is solved to compute the current (and for the next $P$ time steps) superheat setpoint deviation ($T_{SH,SP}(k+l)$, $l=0, \ldots, P-1$), and compressor frequency ($\omega_k(k+l)$, $l=0, \ldots, P-1$) through the control structure displayed above.

$$\min_{u(l)} \sum_{l=0}^{P-1} u_2(l) + ||\hat{T}_{SA}(l) - T_{SA,SP}||_q + ||\Delta u(l)||_R$$

subject to: $\Delta u_{\text{min}} \leq \Delta u(l) \leq \Delta u_{\text{max}}$

$$u_{\text{min}} \leq u(l) \leq u_{\text{max}}$$

$T_{SA,SP} - \sigma_{SA} \leq \hat{T}_{SA}(l) \leq T_{SA,SP} + \sigma_{SA}$

where:

$$\hat{T}_{SA}(l+1) = A\hat{T}_{SA}(l) + Bu(l) + G_{\theta}\dot{\theta}(l)$$

\[ \dot{\theta}(l+1) = \dot{\theta}(l), \]

with \[ \hat{T}_{SA}(0) = \hat{T}_{SA}(k), \]

\[ \dot{\theta}(0) = \dot{\theta}(k) \]

where \[ \hat{T}_{SA}(k) = A\hat{T}_{SA}(k-1) + Bu(k-1) \]

\[ + G_{\theta}\dot{\theta}(k-1) + L_{\dot{T}_{SA}}[T_{SA}(k-1) - \bar{T}_{SA}(k-1)] \]

\[ \dot{\theta}(k) = L_{\theta}[T_{SA}(k-1) - \bar{T}_{SA}(k-1)] \]

where the manipulated inputs, $u(l) = [T_{SH,SP}(l) \quad \omega_k(l)]$, are the superheat setpoint (in deviation form) and compressor frequency (in deviation form), respectively, while state predictions (within the MPC) for both the supply air ($T_{SA}$) and fictitious states ($\theta$) are denoted by $\hat{X}$. $\hat{T}_{SA}(k)$ and $\dot{\theta}(k)$ are the estimates of $T_{SA}$ and the fictitious state $\theta$ provided by the Luenberger design at the time instant $k$. Note that all the variables in the above MPC formulation are deviation variables. The control action to be implemented on the process is set as $u(k) = u(0)$ and the process repeated at the next sampling instance. This implies that the following values are implemented on the process: $T_{SH,SP}(k) = u_1(0) + T_{SH,SP,\text{nom}}$ and $\omega_k(k) = u_2(0) + \omega_{\text{nom}}$. $\sigma_{SA}$ is the allowable $T_{SA}$ setpoint (SP) drift and $G_{\theta}$ and the Luenberger gain elements, $L_{\theta}$ and $L_{\dot{T}_{SA}}$, are tuning parameters. $P$ is the prediction horizon (equal in length to the control horizon in this case) and $q$, $r$ and $R$ are the ratios used to trade off the importance of supply air tracking, the compressor frequency and the rate of change of both inputs, respectively.

The objective function includes a linear term that penalizes the compressor RPM. The use of this term is motivated by the characteristic that the compressor power is a linear function of the RPM, causing any decrease in the RPM to result in a decrease in the energy consumption of the heat pump. Another key feature is to incorporate a constraint on the supply air temperature deviation from the setpoint, allowing it to float in an acceptable band rather than restricting it to tracking a fixed value. The width of the band ($2 \times \sigma_{SA}$), is a user-defined parameter that can be set to satisfy the supply air drift requirements which can fluctuate depending on the heat pump application. In our case, a band of 0.1°C ($\sigma_{SA} = 0.05 \, ^{\circ}\!C$) was chosen to assimilate well to the test conditions of our baseline control strategy. The MPC framework also includes both realistic rate of change and operating region constraints for both the RPM and superheat setpoint so as to ensure the simulated heat pump is operated in close proximity to that of an experimental unit.

The direct implication of such a formulation would be that the optimization will move the RPM to as low a value as possible, while allowing the supply air temperature to hit the allowable upper bound. The formulation, however, represents a very ‘aggressive’ form of the MPC design, and results in oscillations and noisy behavior due to the presence of plant model mismatch, and when tested against time-varying disturbances and supply air set point profiles. To mitigate these effects, ‘damping terms’ were added to the objective function which include a squared $T_{SA}$ setpoint (SP) deviation term and a penalty on the rate of change of the inputs. A significantly higher weight was assigned to the change in the RPM relative to that of the weight for the $T_{SH,SP}$ rate of change to achieve a smoother control action. Through attempting a variety of weighting combinations for these final two penalty terms, a combination was obtained which was able to provide aggressive, yet minimal oscillatory control action.

Finally, the PI tuning parameters for the $T_{SH}$-valve controller were adjusted (from the initial tuning put in place for the model identification purposes). After introducing disturbances to the system (detailed in depth in the following section), it was observed that the PI controller lagged in it’s response to rejecting these disturbances. At this point, tuning became interconnected where PI and MPC tuning parameters would be tuned jointly due to their interacting nature. A decision was made to lessen the superheat loop aggressiveness to ensure the supply air tracking requirements held precedence over strict superheat regulation.
3.4. Testing against realistic disturbances and measurement noise

In this section, the OF MPC strategy is implemented on the standalone heat pump model in the presence of realistic variations in both the return (ret) and ambient (amb) conditions in order to satisfy the previously outlined control objectives. This included temperature and relative humidity (φ) profiles for both the return air and ambient air based on profiles developed within the building simulation software, EnergyPlus and then imported into the Modelica environment. The ambient air profiles correspond to weather variations common to a summer day in Chicago, while the return profiles incorporated a scaled version of variations common to one specific building zone from a simulated five zone single-storey office building. These profiles spanned the office peak hours, 8 am to 5 pm, while the supply air set-point profiles were a scaled basis of the supply air profiles of a built-in cooling unit within the EnergyPlus building model and corresponding to regulating the zone temperature at 22 degrees centigrade. Both the return temperature and supply air set-point profiles were scaled to account for the size mismatch which existed amongst the cooling capacities of the built-in EnergyPlus cooling unit and the Dymola-based heat pump model. These measured disturbance profiles are presented in Figure 3.

A multi-loop PI control strategy was used as a baseline to compare the closed-loop performance of the MPC with. In this baseline the superheat-valve PI controller as present in the MPC, is coupled with a second PI loop which regulates the supply air temperature with that of the compressor RPM. This latter loop was deviated into two separate tuning configurations where both a high and low gain supply air-RPM PI controller were considered for supply air tracking purposes. For each baseline configuration, the set-point used in the superheat-valve PI controller was chosen so as to correspond to the most energy efficient superheat setting.

In quantifying the closed-loop performance, three measures were used. An integral of squared error measure. $\text{ISE}_{\text{SA}}$, between the supply air and it’s band limits (for both the MPC and baseline PI control approaches) was used to quantify tracking, while a second, cumulative power measure, captured the total energy consumed for each while in the closed-loop. This latter measure was estimated directly through the built-in power function within the heat pump model. Finally, a cumulative timer was introduced to sum the length of time the superheat temperature fell below zero degrees centigrade for each control strategy.

To test against measurement noise, in both the supply air and Dymola-based superheat temperatures, a random number generated from a normal distribution and drifted relevant to the uncertainty present in common temperature sensors was added to the respective values. Prior to the controller receiving the output measurements, each “noisy” temperature measurement is filtered separately using a first-order (FO) filter where each filter gain and time constant pairing were tuned so as to both avoid significant actuator ‘chattering’ and significant tracking degradation. With the filters active the closed-loop profiles and measures are presented in Figure 4 and Table 1, respectively.

![Figure 3: Measured disturbance profiles for the heat pump system](image)

![Figure 4: Closed-loop input/output profiles in the presence of measurement noise](image)

| Table 1: Closed-loop Performance Measures subject to Measurement Noise |
|---------------------------|-----------------|----------------|----------------|
| Control Structure         | $\text{PL}_{\text{Hi}}$ | $\text{PL}_{\text{Lo}}$ | OF MPC         |
| $\text{T}_{\text{SA}}$ ISE ($\text{s}^2 \cdot \text{C}^2$) | 2.33            | 13.91          | 2.83           |
| Cumulative Energy (kJ)    | $4.79 \times 10^6$ | $4.80 \times 10^6$ | $4.68 \times 10^6$ |
| $\text{T}_{\text{SA}}$ Time Below Zero (s) | $1.33 \times 10^5$ | $7.40 \times 10^4$ | $6.74 \times 10^4$ |

In the presence of measurement noise the proposed MPC approach was able to achieve similar tracking performance and improved energy efficient performance relative to the baseline PI control strategies. Specifically, this corresponded to the supply air being maintained in closer proximity to the supply air drift band during periods of abrupt demand changes while also
doing so in a more energy efficient manner (2.3% reduction relative to High gain PI and 2.5% reduction relative to Low gain PI). As a result of the MPC being more adept at handling higher frequency disturbances not only is effective tracking and improved energy conservation realized, but also the superheat is maintained above the safety-straining 0°C level for less time than both the high and low gain PI control approaches.

One instance of superior control ability is apparent in Figure 5, when an abrupt change in the prescribed supply air set-point takes place (resulting primarily from the fact the occupants vacate the office at that time, thereby requiring lesser cooling) just after the 8th hour. The MPC framework is better equipped to handle such variations as a result of not only, the flexibility in which it maintains the superheat with, but also in the manner which it uses to incorporate future set-point variations a priori in computing the control action. Note that the use of MPC at the outer level (utilizing weather forecast and occupancy patterns) would enable the outer level to send a set-point trajectory to the heat pump controller. The MPC controlling the heat-pump can then use this information in computing the control action. As a result, the MPC begins to account for this sudden set-point variation prior to its arrival (seen at the time instant marked with a grey, dashed vertical line on subplot (c)) and as a result, the supply air is maintained in a closer proximity to the ramping set-point change resulting in more effective tracking. This is in contrast to maintaining the set-point over the entire prediction horizon at a value which aligns with the current time. Note that this is achieved not only by the predictive nature in which the MPC computes the control action in, but also the additional degree of freedom present through the floating superheat set-point.

**Figure 5:** Magnified closed-loop input/output profiles near abrupt supply set-point variation

In summary, the proposed ‘offset-free’ mechanism was able to achieve effective tracking performance and superior energy efficiency relative to both high and low gain classical PI control approaches.

**Acknowledgment**

Financial support from Ontario Graduate Scholarship Award program for Matt Wallace and the Natural Sciences and Engineering Research Council of Canada through the Collaborative Research and Development Program (in collaboration with Johnson Controls Inc.) is gratefully acknowledged.

**References**