A Simple Free-Flow Traffic Model for Vehicular Intermittently Connected Networks

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Abstract—The performance of Vehicular Data Networks (VDNs) is highly dependent on vehicular traffic. Existing studies on VDNs consider custom-developed traffic models that mimic real-life vehicular traffic behaviour and prepare the ground for accurate VDN performance evaluation. Traffic evolution is affected by numerous random events. Some developed models are microscopic. They independently consider some possible factors (e.g. weather, road geometry, drivers’ skills, etc). These microscopic models are complex and their implementations may be costly. Other models are macroscopic. These revolve around only three major traffic parameters, namely: density, flow and speed. The majority of the existing such models are unrealistic as they are based on restrictive assumptions tailored to their enclosing study. Comparing the performance of VDN protocols becomes adequate if and only if these protocols are all developed on top of the same traffic model. Unfortunately, the opposite is true. Hence, the design of a generic traffic model that serves as a basis for future studies on VDNs is equally urgent and important. This manuscript presents a comprehensive and traffic-theory-inspired macroscopic description of vehicular traffic behaviour over roadway facilities operating under Free-flow traffic conditions. Accordingly, a simple and tractable macroscopic traffic model is proposed. Extensive simulations are conducted to verify the validity of the proposed model and its high accuracy.

I. INTRODUCTION

The conception of vehicular data networks consists of transforming vehicles into intelligent mobile entities that are able to wirelessly communicate with each other as well as with stationary roadside units (SRUs). In this way, a highly dynamic self-organized network that supports a large variety of safety1, convenience and leisure2 applications can be formed.

Pragmatically, researchers, network operators and engineers as well as the large vehicular industry and some governmental authorities have shown a recent interest in this emerging networking conception. [1]–[6]. In fact, the majority of the leading vehicle manufacturers are producing communication-enabled vehicles equipped with small yet powerful wireless devices, global positioning system (GPS) units, navigation systems loaded with digital maps and a large number of real-time monitoring sensors. The U.S. Federal Communications Commission (FCC) has dedicated the 5.9 GHz band for short/medium-range communication services supporting Intelligent Transportation Systems in order to expedite inter-vehicle and vehicle-to-roadside communication [7]–[9].

As opposed to traditional wireless ad hoc networks [10], a vehicular network exhibits volatile connectivity and has to handle a variety of network densities. For example, a vehicular network deployed over a rural roadway or within an urban area is likely to experience higher nodal densities. This is especially true during rush hours (e.g. 8:00 A.M. to 10:00 A.M. and 4:00 P.M. to 7:00 P.M.). However, during late night hours and whenever deployed over large highways or within scarcely populated areas, a vehicular network is expected to suffer from frequent network partitioning and repetitive link disruptions. Over the past couple of years, the networking research community has witnessed many publishable studies revolving around the connectivity analysis as well as the proposal of routing and forwarding schemes that handle the broadcast storm (e.g. [11], [12]) and data delivery (e.g. [13]) in the context of a dense vehicular network. These studies were conducted under the simplified assumption that these vehicular networks are naturally well-connected. In contrast, even though the development of reliable, timely and resource efficient forwarding schemes that support the diverse topologies of Vehicular Intermittently Connected Networks (VICNs) is crucially challenging, it is believed that the immature understanding of network disruption causes and resolution procedures is persistently leading to inadequate scheme designs and inaccurate performance analysis and evaluation.

While the universally known Delay-/Disruption-Tolerant Networking’s store-carry-forward mechanism (refer to [14]) has emerged as a highly effective solution that mitigates VICNs’ link disruptions, the published performance evaluations of various VICN forwarding schemes adopting this mechanism have been shown to be inconsistent with real-life experimental observations. Ever since, the networking research community has been expressing a growing interest in uncovering the major cause of this inconsistency. Recently, several researchers have linked and proved that the reason behind this conflict between the real-world experimental observations and the theoretical analysis is the utilization of unrealistic theoretical vehicular traffic models (e.g. [16], [17]). Following this, every published work enclosed a customized model that attempts to emulate the realistic behaviour of vehicular traffic. The vehicular traffic is affected by a large number of random events (e.g. weather, road geometry, drivers’ skills and habits, haphazard catastrophic incidents etc). Thus far,
the open literature lacks any model that accounts for all such events. However, some of the developed models tend to have a microscopic aspect (e.g. [18], [19]) as they independently consider factors such as weather, road geometry, commuter’s skills and habits, and so forth. These microscopic models are complex which renders them highly theoretical with limited implementation feasibility for simulations. Other models take on the macroscopic (e.g. [20], [21]) aspect. Macroscopic models revolve around three major traffic parameters, namely: the vehicular density, the traffic flow and vehicles’ speeds. Most of the existing models deviate from reality since they are based on highly restrictive assumptions (e.g. all vehicles navigate at a single constant speed, vehicles’ speeds are independent from the vehicular density, etc.) tailored to their enclosing study. Ultimately, since the existing VICN forwarding schemes have different underlying traffic models, comparing their performance is not meaningful.

This manuscript aims at achieving the following three objectives:

1) Present a comprehensive and traffic-theory-inspired macroscopic description of Free-flow traffic conditions (i.e. conditions\(^3\) where vehicular traffic is typically characterized by low to medium vehicular density, arbitrarily high mean speeds and stable flow.) over one-dimensional uninterrupted\(^6\) roadway segments. The purpose of this description is to introduce a generic notation for the above-mentioned three macroscopic traffic parameters and highlight the strong correlation between them.

2) Propose a novel and universal Simple Free-flow Traffic Model (SFTM) that is based on the presented Free-flow traffic behaviour description.

3) Conduct a case study with the purpose of giving more insight into the integration of the proposed SFTM traffic model into the design and analysis of VICN forwarding schemes.

The remainder of this manuscript is organized as follows. In Section II, a selection of major related work is discussed along with the novel contributions enclosed in this manuscript. Section III presents a comprehensive description of the Free-flow traffic model based on which the novel SFTM model is proposed. In Section IV, extensive simulations are conducted to verify the validity and accuracy of the proposed SFTM model. In section V, a case study is conducted to give more insight into the integration of SFTM into the development and performance evaluation of VICN forwarding schemes. Finally, the manuscript is concluded in section VI.

II. RELATED WORK

A. Selective Literature Survey:

The networking community has thus far witnessed the publication of various seminal studies incorporating traffic models

Note that, under such conditions, delay tolerance becomes a major requirement for successful data delivery. This is because low to medium vehicular density coupled with high vehicle speeds causes the network to become sparse and subject to frequent link disruptions.

No grade intersections, traffic lights, STOP signs, direct access to adjoint lands, bifurcations, etc.

3) Conduct a case study with the purpose of giving more insight into the integration of the proposed SFTM traffic model into the design and analysis of VICN forwarding schemes.

1) Stochastic Traffic Models

These models are simplistic and do not account for any of the fundamental principles of vehicular traffic theory. They describe the random mobility of vehicles using graphs that represent roadway topologies. The movement of vehicles is random in the sense that either individual or a group of vehicles navigate at random speeds over any arbitrary one of the paths represented by the graph. The interactive behaviour among vehicles as well as the correlation between the vehicular density, vehicles’ speeds and the overall traffic flow rate is often neglected or over-simplified. The performance of these models is traditionally contrasted to fully random mobility models that impose no constraints on the nodes’ mobility (e.g. Random Walk [22], Random Waypoint [23]). Most stochastic models deviate from reality due to their highly restrictive assumptions.

Examples of stochastic traffic models include the City Section Mobility Model (CSMM) introduced in [24]. Under CSMM all edges of the roadway topology graph are considered bi-directional and one-dimensional roads. All the edges intersect and form a grid. Vehicles select at random one of the intersections as their travel destination. They move towards this destination at constant speed. Motions are either vertical or horizontal. In addition, the model differentiates between two speed levels respectively a high and a low speed.

In [25], the authors investigate the effect of different mobility models on a selection of vehicular networking performance metrics. For this purpose they adopt a Freeway Mobility Model (FMM) and a Manhattan Mobility Model (MMM). Under FMM, freeways are considered to be multi-lane and bi-directional. Furthermore, the vehicular mobility is subject to a set of contraints, namely: a) a vehicle is not allowed to switch lanes, b) the speeds of vehicles are assumed to be uniformly distributed over a specific range, and c) vehicles must be spaced out by a minimum safety distance. Finally, the authors conduct their study under the assumption that no more than one vehicle exists on the considered roadway segment.

2) Traffic Stream Models

Such models interpret vehicular mobility as a hydrodynamic spatiotemporal phenomenon. They fall under the category of macroscopic models. This is especially true since they regard vehicular traffic as a flow and relate the three fundamental macroscopic parameters, namely: i) the vehicular density, ii) the vehicles’ speed and iii) the traffic flow rate. Traffic stream models do not independently consider the per vehicle behaviour. Instead, they describe the collective behaviour of large vehicles streams. This renders them of particular utility for high-level analytical studies of traffic behaviour as part of the design of data delivery schemes for vehicular networks. Nevertheless, the existing macroscopic models in the open literature are based on different restrictive and case specific assumptions. Hence, comparing the performance of designed data delivery strategies built on top of these models becomes not meaningful. The networking research community lacks

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TABLE I
ADVANTAGES AND DISADVANTAGES OF EXISTING TRAFFIC MODELS

<table>
<thead>
<tr>
<th>Traffic Model Category</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic Traffic Models (e.g. [24], [25])</td>
<td>- Very simplistic and easily tractable.</td>
<td>- Overlook the fundamental principles of vehicular traffic theory.</td>
</tr>
<tr>
<td></td>
<td>- Describe the collective behaviour of large vehicle streams.</td>
<td>- Neglect the correlation between speed, flow and density.</td>
</tr>
<tr>
<td></td>
<td>- Relate the three macroscopic traffic parameters: speed, flow and density.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Useful for high-level analytical studies of traffic behaviour.</td>
<td>- Some of these models (e.g. [26], [28]) are based on unrealistic and case specific assumptions.</td>
</tr>
<tr>
<td>Traffic Stream Models (e.g. [26]–[28])</td>
<td></td>
<td>- Other models (e.g. [27]) are highly complex.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car Following Models (e.g. [29], [30])</td>
<td>- Account for individual vehicle behaviour relatively to a vehicle ahead.</td>
<td>- Remarkably complex.</td>
</tr>
<tr>
<td></td>
<td>- Analytically delineate vehicular traffic dynamics.</td>
<td>- Computational power and resource exhaustive.</td>
</tr>
<tr>
<td></td>
<td>- Flexible and account for a large number of parameters.</td>
<td>- May easily become analytically intractable (e.g. [30]).</td>
</tr>
</tbody>
</table>

A universal macroscopic model that is simple, realistically accounts for the fundamental principles of vehicular traffic theory and hence constitute the primary building block in the design of vehicular networking data delivery schemes.

The simplest model of this kind was proposed in [20] where the authors assume that the velocity is a function of the density. This model is particularly capable of modelling kinematic waves and has been used over the past couple of years by researchers in the field of vehicular networking.

The work of [26] addresses the joint connectivity and delay-control problem in the context of a highly restrictive macroscopic vehicular mobility model where vehicles navigate at only two speed levels respectively high speed $V_H$ and low speed $V_L$. Precisely, the authors assume that a vehicle may assume a speed level $V_H$ ($V_L$) for an exponentially distributed amount of time before switching to $V_L$ ($V_H$) independently of the traffic flow and density the values of which seemed to be chosen arbitrarily.

In [27], the authors exploit inter-vehicular communication to establish continuous end-to-end connectivity. However, throughout their study, the authors propose to approximate the macroscopic vehicular traffic dynamics using the combination of: a) a fluid model, b) a stochastic model and c) a density-dependent velocity profile. Even though their proposed approach is remarkably accurate, it is however highly complex.

The authors of [28] adopt the Markov Decision Process (MDP) approach in their design of a data delivery scheme that has the objective of minimizing the transit delay. In addition to the remarkable complexity of their MDP framework, the authors neglect the correlation between the vehicular flow and speed. Moreover, they assume that vehicle speeds and inter-arrival times are drawn from known but unspecified probability distributions. These assumptions render their work highly theoretical with limited practicality.

3) Car Following Models

Such models describe the individual behaviour of each vehicle relatively to a vehicle ahead. Car following models (e.g. [29]) fall under the category of microscopic models which are the most commonly employed to analytically delineate vehicular traffic dynamics. In the majority of car following models, a vehicle’s speed and/or acceleration is expressed as a function of factors such as the distance to a front vehicle and the actual speeds of both vehicles. As such, these models implicitly account for the finite driver’s reaction time.

Car following models are very flexible. They may account for a large number of parameters that pertain, for example to vehicle technicalities, commuters’ skills and habits and weather constraints resulting in a remarkable increase of their degree of accuracy as well as their level of realism. Furthermore, car following models incorporate lane changing routines that allow for the regulation of vehicles’ mobility in between lanes. Consequently, these models can easily describe the vehicular traffic behaviour over individual multi-lane roadways. Car following models may be also used to simulate traffic dynamics on independent roadways of an urban scenario. However, in simulations, the interactions between traffic flows at road junctions must be handled with care. In other words, intersections crossing rules in the presence of stop/priority signs and traffic lights have to be defined within the simulation framework. Defining such rules within analytical frameworks is highly complex and often infeasible. This is especially true since the joint complex description of the acceleration of different vehicles, lane changing and intersection management result in mathematically intractable problems [30]. Compared to macroscopic models, microscopic ones in general and car following models in particular are characterized by a high level of precision. However, they are highly computationally expensive especially whenever the number of simulated vehicles becomes large. It is observed that, in practice, car following models are avoided when large scale simulations are conducted. Instead, discrete time models similar to the one adopted in this manuscript are employed. Detailed discussions and comparisons on the implementation of different car following models may be found in [31]–[33].

A concise summary of the above described traffic model categories together with their advantages and disadvantages are laid out in Table I.

B. Novel Contributions:

Enlightened by rudimentary principles borrowed from vehicular traffic theory [36], the first contribution of this
manuscript appears in the layout of a concise yet comprehensive study of the Free-flow traffic behaviour. Precisely, this study captures the macroscopic vehicular traffic features as described by traffic theorists and characterizes the random, density dependent behaviour of traffic flow, vehicle speeds and travel times using appropriate and highly accurate probability distributions.

Following the macroscopic vehicular traffic study, the second contribution of this manuscript manifests itself in the foundation of a highly accurate, queueing-theory-inspired and simple free-flow traffic model (SFTM). Particularly, it is observed that, under Free-flow traffic conditions, the probability that a given road segment attains full capacity\(^5\) is zero. Hence, such a road segment may be modelled as an infinite-server queueing system and each vehicle navigating over that segment as a job occupying one of the available servers for a finite amount of time. This amount of time is equivalent to the vehicle’s residence time (i.e. the amount of time this vehicle will take to travel the entire segment’s length) and depends on the vehicle’s speed and the length of the segment. Notice that the computation of the mean vehicle’s residence time using the proposed vehicle speed distribution inspired by vehicular traffic theory is a complex task. This is especially true since these distributions lead to integral expressions that have no closed-form solutions. To work around this problem, we propose to approximate this distribution using a two-phase Coxian distribution and we show that the proposed approximation leads to highly accurate results. In addition, we characterize one of the major performance measures of this model which is the instantaneous number of vehicles residing within the considered road segment. This is equivalent to the instantaneous number of busy servers. Also, the steady-state distribution of the number of busy servers is determined.

Finally, a case study is presented with the objective to demonstrate how the proposed SFTM model prepares the ground for an adequate design of vehicular networking data delivery schemes. Under Free-flow traffic conditions, the vehicular network becomes highly prone to link disruptions. This type of vehicular networks is referred to as a Vehicular Intermittently Connected Network (VICN). In the presented case study, we borrow a two-hop VICN scenario from [28] and [41] where connectivity is to be established between two isolated stationary roadside units, a source \(S\) and a destination \(D\). In the absence of any kind of networking infrastructure, vehicles passing by \(S\) will transport its data bundles to \(D\). For this purpose, a Bulk Bundle Release Scheme (BBRS) is built on top of the proposed SFTM. Rigorous mathematical analysis and extensive simulations are conducted in order to highlight the impact of the underlying traffic model on the performance analysis of data delivery schemes such as BBRS.

III. Vehicular Traffic Analysis

A. Free-Flow Traffic Characteristics:

Consider a roadway segment \([AB]\) such as the one depicted in Figure 1. \([AB]\) has a length \(L_{AB}\) (meters). Let \(v_i\) be the mean vehicle length. The capacity of \([AB]\) is defined as \(C_{AB} = \frac{L_{AB}}{v_i}\) (vehicles), [36]. The mean vehicular density, \(\rho_v\) (vehicles/ meter), is defined as the mean number of vehicles per unit length. Thus, the maximum vehicular density is \(\rho_{\text{max}} = \frac{C_{AB}}{L_{AB}} = \frac{1}{v_i}\). The vehicular flow rate, \(\mu_v\) (vehicles/second), is defined as the mean number of vehicles passing a fixed point on \([AB]\) per unit time\(^6\). Without loss of generality, this fixed point is assumed to be the entry point to the segment (i.e point \(A\)). In the sequel, the event of a vehicle entering \([AB]\) at point \(A\) is referred to as a vehicle arrival. Therefore, \(\mu_v\) is interpreted as the vehicle arrival rate whose maximum is denoted by \(\mu_{\text{max}}\). Let \(S_{\text{max}}\) denote the speed limit over the segment \([AB]\).

The observation of \([AB]\) begins at a certain point in time \(t_0\) (e.g. very early morning) set as the origin of the time axis (i.e. \(t_0 = 0\)) and \([AB]\) is empty (i.e. no vehicles are navigating over \([AB]\), \(\rho_v = 0\) and \(v_i = 0\)). After some time, vehicles start arriving to \([AB]\) causing \(\rho_v\) to gradually increase with time. \(\mu_v\) also exhibits a gradual stable\(^7\) increase as a function of \(\rho_v\). However, there exists a critical density value \(\rho_c\) that, once reached, vehicle platoons start forming all over the road segment \([AB]\). This indicates that: a) \([AB]\) has become considerably congested and b) the vehicular flow has attained its maximum \(\mu_{\text{max}}\). At this point, \([AB]\) becomes highly unstable (see [36]) since the slightest traffic perturbation may either re-stabilize the traffic flow or cause a transition into a state of over-forced flow where \(\mu_v\) starts decreasing while \(\rho_v\) increases further. Eventually, at \(\rho_{\text{max}}\), \(\rho_v = 0\) indicating that \([AB]\) is experiencing a traffic jam.

From the point of view of vehicular ad-hoc networks (VANETs), the formation of an end-to-end path between an arbitrary pair of nodes becomes highly probable whenever the vehicular density is high (i.e. \(\rho_c < \rho_v < \rho_{\text{max}}\)) regardless if those nodes are fixed (e.g. stationary roadside units) or moving along the road segment (i.e. vehicles equipped with wireless devices). In this situation, delay tolerance is no longer a requirement and typical wireless protocols can be used over inter-vehicular-enabled VANETs to establish a multi-hop connectivity between a particular data source and destination. Obviously, this is not the case whenever the road segment is operating under Free-flow traffic conditions (i.e. \(0 < \rho_v < \rho_c\)) where the network becomes sparse and prone to link disruptions. Therefore, cases of over-forced vehicular traffic are ignored in this present study.

As shown in Figure 1, an arbitrary vehicle \(i\) enters \([AB]\) at time \(t_i\), resides within \([AB]\) for a period \(R_i = \frac{L_{AB}}{v_i}\) and exits at time \(e_i = t_i + R_i\). Subsequently, vehicle \(i+1\) with speed \(s_{i+1}\) arrives at time \(t_{i+1}\), resides within \([AB]\) for a period \(R_{i+1}\) and departs at time \(e_{i+1}\). In traffic theory, the time headway is defined as the time interval between successive vehicles crossing the same reference point on a road segment, [36]. In the present study, it is assumed that the reference point is the entry point to \([AB]\) (i.e. point \(A\)). Thus, the time headway becomes equivalent to the vehicle inter-arrival time that is denoted by \(I = t_{i+1} - t_i\). Selecting

\(^5\)A segment of a road has a well determined length. Consequently, only a finite number of vehicles may simultaneously navigate within that segment. This number is referred to as the capacity of the road segment.

\(^6\)In this manuscript time is measured in units of seconds

\(^7\)The flow of vehicles into and out of \([AB]\) are equal.
a distribution for $I$ is a delicate task that has to be handled carefully.

In [15], the authors have conducted thorough experiments over highways surrounding the city of Madrid in Spain. They have collected large sets of realistic traces during two separate time intervals, namely: a) Rush hours from 8:30 A.M until 9:00 A.M and b) Non-rush hours from 11:30 A.M until 12:00 P.M. After thorough analysis of their collected data sets, the authors found that $I$ is best modelled by a weighted Exponential-Gaussian distribution mixture. Indeed, this finding is of notable importance. In fact, this model particularly characterizes inter-vehicular behavioral dependencies under strict free-flow conditions. For this purpose, the primary objective of this manuscript is the development of a distribution for $I$. Over highways surrounding the city of Madrid in Spain, they have collected large sets of realistic traces during two separate parts. It is, indeed, a tangible proof that $I$ is exponentially distributed.

Note that the mean vehicle inter-arrival time, $\bar{T} = E[I]$, is inversely proportional to the vehicle arrival rate $\mu_v$. It follows that the probability density function of $I$ can be expressed as:

$$f_I(t) = \frac{1}{\mu_v} e^{-\frac{t}{\mu_v}}, \text{ for } t \geq 0$$

(1)

Denote by $\bar{S}$ the mean of vehicle speeds observed over $[AB]$. It is established in [36] that:

$$\bar{S} = S_{\text{max}} \left(1 - \frac{\rho_v}{\rho_{\text{max}}}\right)$$

(2)

Define $\bar{R} = \frac{L_{AB}}{N}$ as the mean vehicle residence time within $[AB]$ and $N$ as the mean number of vehicles in $[AB]$. Hence, the following relationship is established using Little’s Law:

$$\mu_v = \frac{N}{\bar{R}} = \frac{N \cdot \bar{S}}{L_{AB}} = \rho_v \cdot \bar{S} = -\frac{S_{\text{max}}}{\rho_{\text{max}}} \rho_v + S_{\text{max}} \rho_v$$

(3)

From (3) it is clear that $\mu_v = 0$ at both $\rho_v = 0$ and $\rho_v = \rho_{\text{max}}$. Also, the maximum flow rate $\mu_{\text{max}} = \frac{S_{\text{max}} \rho_{\text{max}}}{4}$ occurs at the critical density value $\rho_v = \frac{S_{\text{max}}}{\rho_{\text{max}}} = \rho_c$. The critical speed is defined as $S_c = S_{\text{max}} \rho_c = S_{\text{max}}$. Recall that this study considers only Free-flow traffic conditions (i.e. $\rho_v \in [0; \frac{S_{\text{max}}}{\rho_{\text{max}}}]$). According to [36], under Free-flow traffic conditions, the speed $s_i (i > 0)$ of an arbitrary arriving vehicle $i$ is a Normally distributed random variable with a probability
density function given by:

\[ f_S(s_i) = \frac{1}{\sigma_S \sqrt{2\pi}} e^{-\left(\frac{s_i - \bar{S}}{\sigma_S \sqrt{2}}\right)^2} \tag{4} \]

The authors of [35] assume justifiably that \( \sigma_S = k\bar{S} \) and that \( s_i \in [S_{\text{min}}, S_{\text{max}}] \), where \( S_{\text{min}} = \bar{S} - m_\sigma \bar{S} \) and the two-tuple \((k, m)\) depend on the ongoing traffic activity over the observed roadway segment and are determined based on experimental data. Accordingly, in the rest of this manuscript a truncated version of \( f_S(s_i) \) in (4) shall be adopted. It is defined as:

\[ \tilde{f}_S(s_i) = \frac{f_S(s_i)}{\int_{S_{\text{min}}}^{S_{\text{max}}} f_S(s_i) \, ds_i} = \frac{2f_S(s_i)}{\text{erf} \left( \frac{S_{\text{max}} - \bar{S}}{\sigma_S \sqrt{2}} \right) - \text{erf} \left( \frac{S_{\text{min}} - \bar{S}}{\sigma_S \sqrt{2}} \right)} \tag{5} \]

for \( S_{\text{min}} \leq s_i \leq S_{\text{max}} \). Furthermore, a seminal study conducted in [37] together with extensive real-life experiments and data acquisition over numerous roadways show that, \( s_i \) is constantly maintained during the vehicle’s entire navigation period on the road. Let \( \tilde{F}_S(v) \) and \( F_R(\tau) \) denote the respective cumulative distribution functions of the vehicle’s speed and residence time. It can be easily shown that:

\[ F_R(\tau) = 1 - \tilde{F}_S \left( \frac{L_{AB}}{\tau} \right) = 1 - \frac{K}{2} \left[ 1 + \text{erf} \left( \frac{L_{AB}}{\tau \cdot \bar{S}} - \frac{\bar{S}}{\sigma_S \sqrt{2}} \right) \right] \quad \tag{6} \]

where \( K = 2 \left[ \text{erf} \left( \frac{S_{\text{max}} - \bar{S}}{\sigma_S \sqrt{2}} \right) - \text{erf} \left( \frac{S_{\text{min}} - \bar{S}}{\sigma_S \sqrt{2}} \right) \right]^{-1} \).

Hence the vehicle’s residence time has a probability density function that is expressed as:

\[ f_R(\tau) = \frac{K \cdot L_{AB}}{r^2 \sigma_S \sqrt{2\pi}} e^{-\left(\frac{L_{AB}}{\tau \cdot \bar{S}} - \frac{\bar{S}}{\sigma_S \sqrt{2}}\right)^2}, \quad \tau \in \left[ \frac{L_{AB}}{S_{\text{max}}}, \frac{L_{AB}}{S_{\text{min}}} \right] \tag{7} \]

B. A Simple Free-flow Traffic Model (SFTM):

Under Free-flow traffic conditions, the road segment \([AB]\) experiences low to medium vehicle arrival rates (from (3), \( 0 \leq \mu_v \leq \mu_{\text{max}} \)) while the observed vehicle speeds are high (from (2), \( S_c \leq \bar{S} \leq S_{\text{max}} \)). [35]-[37]. Hence, the probability that \([AB]\) attains full capacity under such conditions is zero. In light of the above, \([AB]\) can be modelled as an \( M/G/\infty \) queueing system where: \( i \) vehicle arrivals follow a Poisson process with parameter \( \mu_v \), \( ii \) the number of busy servers at time \( t \) is equivalent to the number of vehicles within \([AB]\) at time \( t \) which is denoted by \( N(t) \) and \( iii \) the busy period of an arbitrary server \( i \) is equivalent to the residence time of vehicle \( i \) within \([AB]\) whose p.d.f. is given in (7), \( N(t) \) is one of the major characteristic measures of this system.

**Theorem 3.1:** The number of vehicles within \([AB]\) is Poisson distributed with a parameter \( \mu_v \bar{R} \).

**Proof:** Define the following:

- \( P_n(t) = Pr[N(t) = n] \).
- \( A_j(t) = Pr[j \text{ vehicles arrived in } (0, t)] = \frac{(\mu_v t)^{j} e^{-\mu_v t}}{j!} \).
- \( P_{n|j}(t) = Pr[N(t) = n| j \text{ arrivals in } (0, t)] \).

Therefore:

\[ P_n(t) = \sum_{j=0}^{\infty} P_{n|j}(t) \cdot A_j(t) \tag{8} \]

The probability that an arbitrary vehicle \( i \) that arrived at time \( t_i \) is found within \([AB]\) at time \( t \) is \( 1 - F_R(t - t_i) \). Recall that vehicle arrivals follow a Poisson process. Hence, the distribution of the vehicle arrival times conditioned by \( j \) arrivals during time interval \((0, t)\) is identical to the uniform distribution of \( j \) points over \((0, t)\). Accordingly, the probability that any of the \( j \) vehicles that arrived in \((0, t)\) is found within \([AB]\) at time \( t \) is given by:

\[ q(t) = \int_{0}^{t} [1 - F_R(t - t_i)] \frac{dt_i}{t} = \frac{1}{t} \int_{0}^{t} [1 - F_R(t_i)] \, dt_i \tag{9} \]

Consequently, the probability that a vehicle that arrived to \([AB]\) during the time interval \((0, t)\) would have departed from \([AB]\) at time \( t \) is:

\[ 1 - q(t) = \frac{1}{t} \int_{0}^{t} F_R(t_i) \, dt_i \tag{10} \]

Knowing \( q(t) \), it is easy to show that:

\[ P_{n|j}(t) = \begin{cases} \left(\frac{j}{n}\right) (q(t))^n [1 - q(t)]^{j-n} & , n \leq j \\ 0 & , n > j \end{cases} \tag{11} \]

Using (11), equation (8) can be re-written as:

\[ P_n(t) = \sum_{j=n}^{\infty} \left(\frac{j}{n}\right) (q(t))^n [1 - q(t)]^{j-n} \cdot \frac{(\mu_v t)^{j} e^{-\mu_v t}}{j!} = \frac{[\mu_v t \cdot q(t)]^n e^{-\mu_v t}}{n!} \tag{12} \]

Notice that \( \lim_{t \to \infty} [t \cdot q(t)] = \bar{R} \). Let \( N = \lim_{t \to \infty} N(t) \). Thus, the limiting probability of having \( N = n \) vehicles within \([AB]\) is:

\[ P_n = \lim_{t \to \infty} [P_n(t)] = \left(\mu_v \bar{R}\right)^n e^{-\mu_v \bar{R}} \frac{1}{n!} \tag{13} \]

**Remark:** \( P_n \) is independent of \( f_R(\tau) \).

At this stage, recall that the p.d.f. of \( R \) is given in (7). Thus:

\[ \bar{R} = \int_{0}^{\infty} r \cdot f_R(r) \, dr = \int_{0}^{\infty} \frac{K \cdot L_{AB}}{r^2 \sigma_S \sqrt{2\pi}} e^{-\left(\frac{L_{AB}}{r \cdot \bar{S}} - \frac{\bar{S}}{\sigma_S \sqrt{2}}\right)^2} \, dr \tag{14} \]

The complex integral in (14) has no closed-form solution. The squared coefficient of variation \( c^2_v = \frac{\sigma^2_v}{\mu_v^2} \) captures the degree of variability of \( R \) where \( \sigma^2_v \) is the variance of \( R \) and \( \mu_v \) is the square of its mean. Simple numerical analysis show that \( c^2_v > 1 \). Hence, following the recommendation of [40], \( f_R(r) \) may be approximated by a two-phase Coxian density function \( f_R^{Cox}(r) \) that is given by:

\[ f_R^{Cox}(r) = m_1 \cdot \mu_1 e^{-\mu_1 r} + (1 - m_1) \cdot \mu_2 e^{-\mu_2 r} \tag{15} \]

where \( \mu_1 = 2\mu_R \) and \( \mu_2 = \frac{\mu_v}{\bar{R}} \) and \( m_1 = 1 + \frac{\mu_v}{\sigma^2_v \cdot (\mu_2 - \mu_1)} \). Let \( \bar{R} \) denote an approximated version of \( \bar{R} \) computed as:

\[ \bar{R} = \int_{0}^{\infty} r \cdot f_R^{Cox}(r) \, dr = \frac{m_1}{\mu_1} + \frac{1 - m_1}{\mu_2} \tag{16} \]
where \( \tilde{P} \) is substituted by \( \tilde{R} \). Also, let \( \tilde{N} \) represent the approximated version of \( N \). Hence:

\[
\tilde{N} = \sum_{n=0}^{\infty} n \cdot \tilde{P}_n = \mu_v \tilde{R}
\]

where \( \tilde{R} \) is substituted by \( \tilde{R} \). Also, let \( \tilde{N} \) represent the approximated version of \( N \). Hence:

\[
\tilde{N} = \sum_{n=0}^{\infty} n \cdot \tilde{P}_n = \mu_v \tilde{R}
\]

IV. Numerical Analysis and Simulations

A Java-based discrete event simulator was developed to examine the validity and accuracy of the proposed SFTM model. The model’s characterizing metrics were evaluated for a total of \( 10^7 \) vehicles and averaged out over multiple simulator runs to ensure the realization of a 95% confidence interval. The following input parameter values were assumed: i) \( \rho_v \in [0.005; 0.1] \), ii) \( L_{AB} = 200 \) and iii) \( (k, m) = (0.3, 3) \).

Figures 3(a) and 3(c) plot \( f_R(r) \) together with \( f_R^{Cos}(r) \) as given respectively in (7) and (15). Similarly, Figures 4(a) and 4(c) plot \( P_n \) as given in (13) concurrently with its approximated counterpart \( \tilde{P}_n \). The accuracy of \( f_R^{Cos}(r) \) and that of \( \tilde{P}_n \) were respectively tested for all values of the vehicular density in the range \([0.005; 0.1]\). The results corresponding to \( \rho_v = 0.01, \rho_v = 0.07 \) and \( \rho_v = 0.1 \) are shown. These results constitute tangible proofs of the validity and high accuracy of the established approximations. This is especially true since Figures 5(a) and show that the highest mean squared error (MSE) resulting from the approximation of \( f_R(r) \) by \( f_R^{Cos}(r) \) is 1.67% and Figure 5(b) shows that the largest (MSE) resulting from the approximation of \( P_n \) by \( \tilde{P}_n \) is 0.6%. Finally, extensive simulations were conducted to evaluate SFTM’s characteristics in terms of the mean vehicle residence time, and the mean number of vehicles within the road segment. Figures 6(a) and 6(b) show an increase of the mean vehicle’s residence time and the mean number of vehicles within \([AB]\) as a function of \( \rho_v \). This is explained as follows. As \( \rho_v \) increases, the mean vehicle speed decreases. Concurrently, the flow of vehicles increases. As a result, \([AB]\) will experience faster vehicle arrivals and the arriving vehicles will be spending more time within \([AB]\).

V. Case Study

This section presents a practical example where the SFTM model may be applied.

A. Networking Scenario:

Consider the scenario illustrated in Figure 7 which depicts an uninterrupted highway along which two isolated stationary roadside units (SRUs), a source \( S \) and destination \( D \), are deployed. Both \( S \) and \( D \) have a communication range that covers a segment of the road of length \( L_{AB} \). Moreover, these two SRUs are separated by a distance \( L_{SD} >> L_{AB} \). Connectivity is to be established between \( S \) and \( D \). In the absence of all sorts of networking infrastructure, wireless nodes mounted over mobile vehicles serve as opportunistic store-carry-forward devices that transport bundles from \( S \) to \( D \). Vehicles have random speeds and enter the coverage range of \( S \) at random time instants. No inter-vehicle communications may occur. Under such conditions, an intermittence-free end-to-end \( S-D \) path does not exist. A network of this type belongs to a subclass of vehicular networks that is conveniently referred to as Two-Hop Vehicular Intermittently Connected Networks (TH-VICNs).

B. Motivation:

Major wireless operators in the U.S. (e.g. AT&T and Verizon) have recently reported substantial data traffic growth in their networks which is only partly driven by the utilization of smart-phones (e.g. iPhone, BlackBerry, etc). According to Cisco, wireless networks in North America carried approximately 17 Petabytes per month in 2009. It is projected that, in 2014, these networks will carry around 740 Petabytes. That is a 40-fold increase. This traffic growth is due to the increased adoption of Internet-connected mobile computing devices and increased data consumption per device. The aggregate impact of these devices on demand for wireless broadband access as well as the load they will incur on the service providers’ infrastructure}[41].
### Table II

**MAIN SFTM PARAMETERS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{AB}$</td>
<td>Length of segment $[AB]$</td>
</tr>
<tr>
<td>$l_v$</td>
<td>Mean vehicle length</td>
</tr>
<tr>
<td>$C_{AB}$</td>
<td>Capacity of segment $[AB]$</td>
</tr>
<tr>
<td>$\rho_v, \rho_c, P_{max}$</td>
<td>Mean, critical and maximum vehicular densities over $[AB]$</td>
</tr>
<tr>
<td>$\mu_v, \mu_c, \mu_{max}$</td>
<td>Mean, critical and maximum vehicular flow rates over $[AB]$</td>
</tr>
<tr>
<td>$s_i, f_S(s_i), F_S(t)$</td>
<td>Minimum, maximum and critical speeds over $[AB]$</td>
</tr>
<tr>
<td>$\xi_i, e_i$</td>
<td>Arrival and departure times of vehicle $i$</td>
</tr>
<tr>
<td>$t_i, f_R(t), F_R(\tau)$</td>
<td>Vehicle residence time, its density and cumulative distribution functions</td>
</tr>
<tr>
<td>$I, f_I(t)$</td>
<td>Inter-arrival time to $[AB]$, its mean and density function</td>
</tr>
<tr>
<td>$S, R, N$</td>
<td>Mean speed, residence time and number of vehicles within $[AB]$</td>
</tr>
<tr>
<td>$N(t), P_n(t), P_{n</td>
<td>j}(t)$</td>
</tr>
<tr>
<td>$N, P_n, N, P_n$</td>
<td>Limiting values of $N(t)$ and $P_n(t)$ as $t \to \infty$ and their approximated versions</td>
</tr>
<tr>
<td>$A_j(t)$</td>
<td>Probability of $j$ vehicle arrivals within the interval $(0, t)$</td>
</tr>
<tr>
<td>$q(t)$</td>
<td>Probability that any of the $j$ vehicles that arrived in $(0, t)$ is found within $[AB]$ at time $t$</td>
</tr>
<tr>
<td>$\sigma^2_R, \mu^2_R, \mu_{max}^2$</td>
<td>Variance, squared mean and squared coefficient of variation of the vehicle residence time</td>
</tr>
<tr>
<td>$f_R^{Cox}(r)$</td>
<td>Coxian approximation of $f_R(r)$</td>
</tr>
</tbody>
</table>

**End of Table II**

---

(a) $\rho_v = 0.01$

(b) $\rho_v = 0.07$ (vehicle/meter)

(c) $\rho_v = 0.1$

---

(a) $\rho_v = 0.01$

(b) $\rho_v = 0.07$ (vehicle/meter)

(c) $\rho_v = 0.1$

---

Fig. 3. $f_R(r)$ V.S. $f_R^{Cox}(r)$ for different values of $\rho_v$.

Fig. 4. $P_n$ V.S. $\overline{P}_n$ for different values of $\rho_v$.
networks (SPNs) are expected to be enormous. Despite the recent advancements in wireless communication technologies, the improvement of both the capacity and coverage of wireless networks has been the limiting factor for unleashing the wireless broadband capabilities. Motivated by the work in [40], we target the exploitation of mobile vehicles as a means of boosting the capacity of legacy wireless networks and extending their coverage ranges. Given their intrinsic tendency to grow to irregular large scales, vehicular networks present unparalleled opportunistic connectivity solutions that contribute in satisfying the exponentially growing user demands for all-time-anywhere connectivity irrespective of the spatiotemporal limitations as well as offloading data traffic and relieving SPNs from congestions.

The TH-VICN of Figure 7 becomes of particular utility in rural or other sparsely populated areas where the setup of a wired networking infrastructure may be highly expensive, [7]. In these scenarios, SRUs (also known as information relay stations or data posts) are deployed near disconnected sites and low cost opportunistic end-to-end connectivity is established through vehicles plying between these SRUs. Note that, very few of these SRUs called gateways may be connected to the Internet through minimal infrastructure. All others are completely isolated (even with no direct connectivity) and powered by batteries or small solar cells\(^\text{10}\). Data traffic is then aggregated at source SRUs and routed appropriately via vehicles to destination SRUs. Hence, here the SRUs can act as both routers or wireless access points in hot spots.

\(^\text{10}\)Energy consumption is important in this case but is outside the scope of our research.

In other scenarios, two sites may be connected through microwave links that may suffer from data traffic overload as well as from loss of connectivity due to humidity, rain, storms, clouds, mist, fogs and so forth. Hence, deploying SRUs and exploiting the vehicular infrastructure to forward traffic from one site to the other will not only significantly contribute in reducing the load on the microwaves but also provide a protection channel upon their failure under bad atmospheric conditions.

C. Primary Objective:

The open literature encloses several proposals of bundle release schemes that aim at achieving delay-minimal bundle delivery in the context of the above described TH-VICN scenario, [28], [41]. While these schemes are particularly appealing, their corresponding analytical performance evaluations are of reduced accuracy since they are based on restrictive traffic models. In fact, the authors of [28] assume a general distribution of vehicle speeds and do not account for the correlation between the earlier described macroscopic traffic parameters. In [41] the authors assume uniformly distributed vehicle speeds. This assumption only holds in particular cases of very light traffic. Under such conditions, vehicles may navigate independently at arbitrarily high speeds.

The primary objective of this present case study is to give more insight into how the simple Free-flow traffic model (SFTM) may be used to evaluate the performance of release schemes such as those proposed in [28], [41]. Particularly, it is of interest to determine the mean bundle end-to-end delay denoted by \(E_d\) and defined as the mean time it takes for an arriving bundle at the source SRU \(S\) to be delivered to
the destination SRU D. Observe that \( F_d \) is composed of two factors, namely: i) the mean bundle queueing delay \( \overline{Q}_d \) defined as the mean time period a bundle spends in S's buffer and ii) the mean bundle transit delay \( \overline{T}_d \) defined as the mean time a bundle spends in the buffer of its carrying vehicle until it is delivered to D.

D. Basic Assumptions:

For the purpose of this case study, it is assumed that:

- A1: The bundle arrivals at the source SRU follow a Poisson process with parameter \( \lambda \) bundles per second.
- A2: All bundles have fixed size of \( s_b \) bytes.
- A3: The source SRU's transmission rate is \( T_R \) bps.
- A4: The source SRU has an infinite queue size.

Note that assumptions (A1) through (A4) are extracted from [28] and [41] where they have been rigorously justified.

E. Adopted Bundle Release Scheme:

The advancements in wireless technology have allowed for data transmission rates in the order of tens of Mbps resulting in a negligible bundle transmission time when compared to the vehicle residence time\(^{11}\). Therefore, it becomes more efficient to release as many bundles as possible during the entire vehicle residence time instead of releasing only a single bundle per vehicle. Therefore, for the sake of this study, a Bulk Bundle Release Scheme (BBRS) is adopted. Under BBRS, the source SRU S uniformly selects one of the vehicles present within its communication range and continuously releases bundles to that vehicle until it goes out of range. Consequently, every vehicle leaving the coverage range of S will be carrying a bulk of bundles to be delivered to D. In the sequel, a bulk of bundles will be simply referred to as a bulk. The size of a bulk is a random variable that highly depends on the number of buffered bundles at the source and the bundle admission capabilities of the selected vehicle.

F. Modelling and Analysis of BBRS:

1) Vehicle Residence Time

In this present case study, the essence of Delay-Tolerant Networking is preserved. This is especially true since it is established that the source SRU S has no a priori knowledge of vehicle arrival times and speeds. However, similar to [28] and [41], it is assumed that S is equipped with sensors that enable the determination of the speeds of arriving vehicles only at the time of their arrival. Hence, upon the arrival of a vehicle i at time \( t_i \), S determines its speed \( s_i \) and residence time \( R_i = \frac{\overline{t}_s}{s_i} \). Recall from equation (5) that \( S_{\text{min}} \leq s_i \leq S_{\text{max}} \). Hence, the maximum and minimum residence times are respectively \( R_{\text{max}} = \frac{\overline{t}_s}{S_{\text{min}}} \) and \( R_{\text{min}} = \frac{\overline{t}_s}{S_{\text{max}}} \). \( R_i \) has an approximated probability density function \( f_R \) as expressed in equation (15).

2) Bundle Admission Capability of a Selected Vehicle

The bundle admission capability of a vehicle i is defined as the maximum number of bundles \( K_i \) that vehicle may receive during its entire residence time \( R_i \). Based on assumptions (A2) and (A3), the bundle transmission time is \( T_b = \frac{s_b}{T_R} \). Therefore, knowing \( R_i \) and \( T_b \), the source S computes \( K_i = \lceil \frac{R_i}{T_b} \rceil \). Notice that \( K_i \) depends on \( R_i \) and takes on positive integer values \( k (k \in \mathbb{Z}^+) \). Also, it has the respective upper and lower bounds \( K_{\text{max}} = \lceil \frac{R_{\text{max}}}{T_b} \rceil \) and \( K_{\text{min}} = \lceil \frac{R_{\text{min}}}{T_b} \rceil \). Hence, the probability mass function of \( K_i \) is:

\[
\widetilde{f}_{K_i}(k) = \int_{\frac{kT_b}{R_i}}^{\frac{(k+1)T_b}{R_i}} f^{\text{Cox}}(r)dr
\]

\[
= m_1e^{-\mu_1kT_b} \left( 1 - e^{-\mu_1T_b} \right) + (1 - m_1)e^{-\mu_2kT_b} \left( 1 - e^{-\mu_2T_b} \right) \quad (19)
\]

It follows that the mean of \( K_i \) denoted by \( \widetilde{K}_i \) is computed as:

\[
\widetilde{K}_i = m_1 \left( 1 - e^{-\mu_1T_b} \right) \sum_{k=K_{\text{min}}}^{K_{\text{max}}} k e^{-\mu_1kT_b}
\]

\[
+ (1 - m_1) \left( 1 - e^{-\mu_2T_b} \right) \sum_{k=K_{\text{min}}}^{K_{\text{max}}} k e^{-\mu_2kT_b} \quad (20)
\]

3) Bulk Size

The bulk size is a random variable denoted by \( B_i \) and depends on both, X representing the number of bundles buffered at S, and \( K_i \). \( B_i \) may take on values that depend on the following three identified cases:

- Case 1: If \( X = 0 \), then \( B_i = 0 \).
- Case 2: If \( 0 < X \leq K_i \), then \( B_i = X \).
- Case 3: If \( X > K_i \), then \( B_i = K_i \).

The above three cases imply the following. For a known value of \( K_i \), bundles are buffered at S and up to \( K_i \) of them, if they exist, might be released. Consequently, if \( X < K_i \), then all of the X bundles are going to be released leaving behind an empty queue. Otherwise, if \( X \geq K_i \), only \( K_i \) of them are released. Once S completes the transmission of these \( K_i \) bundles to vehicle i (which has now departed), it will select a new vehicle, if available, and start handling the remaining bundles in its queue. If no vehicles are readily available then all remaining bundles will be held in S's buffer until a vehicle arrives and so forth. Ultimately, S cannot release more than \( K_{\text{max}} \) bundles. This only occurs whenever \( X \geq K_{\text{max}} \) but the arriving vehicle's speed \( s_i = S_{\text{min}} \). To this end, a source operating under BBRS can be represented by an M/M/1 queuing system with bulk bundle release. It is therefore of interest to resolve this system in light of the new traffic model of section III and derive closed-form expressions for X that represents the mean number of bundles in the queue. Then, the mean bundle queueing delay is computed using Little’s Law.

4) Mean Number of Buffered Bundles

Taking X as a state variable, the state-transition diagram in Figure 8 represents the behaviour of the queueing system under study. Let \( S_x (x = 0, 1, 2,...) \) denote the \( x^{th} \) state indicating that \( X = x \). Observe that all states except \( S_0 \) are entered both from their left-hand neighbour upon the occurrence of a bundle arrival with a mean rate \( \lambda \) and their \( K_{i}^{th} \) neighbour to the right upon the occurrence of a bulk departure with a mean rate \( \mu \). These states are exited upon the occurrence of either an arrival or a departure. However, state
$S_0$ can only be entered from any one of its immediate right $K_i$ neighbours upon a departure and exited upon an arrival. At this point, it is important to note that, $\mu$ is a function of the vehicle flow rate $\mu_v$, the probability that there are no vehicles within the coverage range of the source $P_0$ and $K_i$. In fact, after completing the transmission of the most recently released bulk and with a probability $P_0$, $S$ will find no available vehicles within its coverage range. Therefore, it will have to wait for the occurrence of the next vehicle arrival in order to start releasing the next bulk. In this case, the bulk departure rate is $\tilde{P}_0 \mu_v$. In contrast, with a probability $1 - \tilde{P}_0$, after completing the transmission of the most recent bulk, $S$ will readily find other vehicles within its coverage range. Hence, it will immediately select one of them, compute its bundle admission capability and start the releasing the corresponding bulk. Under such conditions, the bulk departure rate becomes $\frac{1 - \tilde{P}_0}{K_i T_b}$. It follows that the overall bulk departure rate can be expressed as:

$$
\mu = \tilde{P}_0 \mu_v + \left(1 - \tilde{P}_0\right) \frac{1}{K_i T_b} \tag{21}
$$

Without loss of generality, assume that the choice of $S$ falls on a vehicle $i$. The bundle admission capability corresponding to this vehicle is $K_i$. Knowing $K_i$, denote by $P_x | K_i$, the long-term probability of finding $x$ bundles in the system. Therefore, the diagram shown in Figure 8 leads to the following set of balance equations:

$$
\lambda P_{0|K_i} = \mu \sum_{i=1}^{K_i} P_{i|K_i}, \quad \text{for } x = 0 \tag{22}
$$

$$
(\lambda + \mu) P_{x|K_i} = \lambda P_{(x-1)|K_i} + \mu P_{(x+K_i)|K_i}, \quad \text{for } x \geq 1 \tag{23}
$$

Next, the conditional probability mass function of the number of bundles in the queue\(^1\) is derived.

**Theorem 5.1:** For a known value of $K_i = k$, the conditional probability mass function of the number of bundles in the queue is given by:

$$f_{X|K_i}(x) = \left(1 - \frac{1}{z^*(k)}\right) \left(\frac{1}{z^*(k)}\right)^n, \quad x \geq 0 \tag{24}
$$

**Proof:** Let $\tilde{X}(z|K_i) = \sum_{x=0}^{\infty} z^x P_{x|K_i}$ denote the probability generating function of $X$ given $K_i$ and $\rho = \frac{\lambda}{\mu}$. Using [39] and proper manipulation of (20) and (21), it is shown that:

$$
\tilde{X}(z|K_i) = \frac{\sum_{x=0}^{K_i-1} (z^x - z^{K_i}) P_x | K_i}{\rho z^{K_i+1} - (1 + \rho) z^{K_i} + 1} \tag{25}
$$

It can be easily shown using *Rouche’s Theorem* that the denominator of equation (25) has $K_i + 1$ zeros of which exactly one occurs at $z = 1$, exactly $K_i - 1$ are such that $|z| < 1$ and only one that we denote by $z^*(K_i)$ will be such that $|z^*(K_i)| > 1$. In addition, observe that the numerator of (25) is a polynomial in $z$ of degree $K_i$. One of the roots of this numerator is $z = 1$. Recall one of the fundamental properties of probability generating functions stating that $\tilde{X}(z|K_i)$ is bounded by the region $|z| < 1$. As a result, the remaining $K_i - 1$ zeros of the numerator in (25) must exactly match the $K_i - 1$ zeros of the denominator for which $|z| < 1$. Consequently, the respective polynomials of degree $K_i - 1$ of the numerator and denominator must be proportional. That is:

$$
\alpha \sum_{x=0}^{K_i-1} (z^x - z^{K_i}) P_x | K_i = \frac{\rho z^{K_i+1} - (1 + \rho) z^{K_i} + 1}{(1 - z) (1 - z^{*(K_i)})} \tag{26}
$$

where $\alpha$ is a proportionality constant. Cancelling common factors in the numerator and denominator of (26) leads to:

$$
\tilde{X}(z|K_i) = \frac{1}{\alpha (1 - z^{*(K_i)})} \tag{27}
$$

At this point, the constant $\alpha$ may be found by setting $\tilde{X}(1|K_i) = 1$. This results in having:

$$
\tilde{X}(z|K_i) = \frac{1 - \frac{1}{z^{*(K_i)}}}{1 - \frac{1}{z^{*(K_i)}}} \tag{28}
$$

Inverting (28) leads to the probability mass function of $X$ conditioned by $K_i = k$:

$$f_{X|K_i}(x) = \left(1 - \frac{1}{z^*(k)}\right) \left(\frac{1}{z^*(k)}\right)^n, \quad x \geq 0 \tag{29}$$

\(^1\)That is, the probability that $X = x$ given that $K_i = k$. We denote this probability mass function as $f_{X|K_i}(x)$. 

---

\[\begin{array}{c}
\text{Fig. 8. State-transition-rate diagram representing the behavior of } S \text{ under BBRS.}
\end{array}\]
Recall that \( K_j \in [K_{\min}; K_{\max}] \). Hence, the unconditional probability mass function of \( X \) is expressed as:

\[
f_X(x) = \sum_{k=K_{\min}}^{K_{\max}} \left[ m_1 e^{-\mu_1 k T_b} \left( 1 - e^{-\mu_1 T_b} \right) + (1 - m_1) e^{-\mu_2 k T_b} \left( 1 - e^{-\mu_2 T_b} \right) \right] \times \left( 1 - \frac{1}{z^*(k)} \right) \left( \frac{1}{z^*(k)} \right)^x, x \geq 0
\]

(30)

Accordingly, the mean number of bundles in \( S \)'s queue is:

\[
\tilde{X} = E[X] = \sum_{x=0}^{\infty} x \cdot f_X(x)
\]

(31)

5) Mean Bundle Queueing Delay

Using Little’s Law, the mean bundle queueing delay is:

\[
\tilde{Q}_d = \frac{\tilde{X}}{\lambda}
\]

(32)

6) Mean Bundle Transit Delay

The transit delay experienced by a bulk of bundles carried by a vehicle \( i \) with speed \( s_i \) is \( T_i = \frac{L_{SD}}{s_i} \). \( T_i \) has a probability density function that is given by:

\[
f_{T_i}(t) = \frac{K \cdot L_{SD}}{t^2 \sigma_s \sqrt{2\pi}} e^{-\frac{(\frac{t}{\sigma_s} - \frac{L_{SD}}{\sigma_s})^2}{2}}, t \in \left[ \frac{L_{SD}}{\sigma_s}; \frac{L_{SD}}{\sigma_s} \right]
\]

(33)

Notice that \( f_{T_i}(t) \) has exactly the same structure as \( f_B(r) \) given in equation (7) with the only difference being that \( L_{AD} \) is substituted by \( L_{SD} \). Hence, the approximated density function of \( T_i \) is given by:

\[
f_{T_i}^{\text{Cox}}(t) = h_1 \cdot \beta_1 e^{-\beta_1 t} + (1 - h_1) \cdot \beta_2 e^{-\beta_2 t}
\]

(34)

where \( \beta_1 = 2 \mu_{T_i} \) and \( \beta_2 = \frac{\beta_1}{2} \) and \( h_1 = 1 + \frac{\beta_1}{2\sigma_s^2 (\beta_1 - \beta_2)} \). Note that \( \mu_{T_i} = \frac{L_{SD}}{\sigma_s^2} \) where \( \sigma_s \) is the mean vehicle speed experienced under a given vehicular density \( \rho \). Also, \( c_v^2 = \frac{\sigma_v^2}{\mu_{T_i}} \) is the squared coefficient of variations where \( \sigma_v^2 \) is the variance of \( T_i \).

Let \( \tilde{T}_d \) denote an the approximated mean bundle transit delay which is computed as:

\[
\tilde{T}_d = \int_0^\infty t \cdot f_{T_i}^{\text{Cox}}(t) dr = \frac{h_1}{\beta_1} + \frac{1 - h_1}{\beta_2}
\]

(35)

7) Mean Bundle End-To-End Delay

After computing \( \tilde{Q}_d \) and \( \tilde{T}_d \), the final step is to compute the mean bundle end-to-end delay \( \tilde{E}_d \). The mean bundle end-to-end delay is equal to the sum of the mean bundle transit delay and the mean bundle queueing delay. Hence:

\[
\tilde{E}_d = \tilde{Q}_d + \tilde{T}_d
\]

(36)

G. Benchmark Traffic Model:

The first objective of this case study is to show how bundle release schemes for two-hop vehicular intermittently connected networks can be designed in light of the proposed traffic model in section III. In fact, the above mathematical modelling of the Bulk Bundle Release Scheme (BBRS) constitutes a sample of a larger theoretical modelling and analysis framework pertaining to more sophisticated bundle release schemes. In addition, the second objective of this cases study is to highlight the impact of the underlying traffic model on the performance of such bundle release schemes. For this purpose, in this subsection a Benchmark Traffic Model (BTM) is borrowed from [41]. Under BTM vehicular speeds are assumed to be uniformly distributed in the range \([S_{\min}; S_{\max}]\). In addition, the correlation between the macroscopic vehicular traffic parameters (i.e. speed, density and flow) is neglected. The reader is referred to [41] for more details about BTM. Also, it is important to mention that the above conducted analysis of BBRS can be easily refined in order to fit with BTM. However, in order to focus on the main objective of this case study, these refinements are omitted.

H. Simulations and Performance Evaluation:

In order to highlight the impact of traffic models on the performance of bundle release schemes, the BBRS scheme adopted in this case study will be tested using the two traffic models SFTM and BTM. Particularly, a discrete event simulation framework is developed for the purpose of examining the performance of BBRS-SFTM and BBRS-BTM in the context of the sample TH-VICN shown in Figure 7. The adopted performance metrics are: i) the mean queueing delay, ii) the mean transit delay and iii) the mean end-to-end delay.

1) Simulator’s Input Parameters Values

BBRS-SFTM is tested under Free-flow traffic conditions corresponding to vehicular density values \( \rho \), in the range of 0.01 to 0.07 \( (\text{vehicles per meter}) \) and flow rate values \( \mu \) in the range of 0.5 to 2.5 \( (\text{vehicles per second}) \) (or equivalently an mean vehicle inter-arrival time \( T \in [4; 20] \text{ seconds} \)). The typical IEEE 802.11 protocol is used for SRU-to-vehicle communication and vice versa with a data rate of 1 \( (\text{Mbps}) \). The source is assumed to have a coverage range \( L_{AB} = 200 \text{ (meters)} \) and the source-destination distance \( L_{SD} = 20000 \text{ (meters)} \). The bundle arrival rate was taken to be \( \lambda = 1 \text{ (bundle per second)} \). This ensures a fairly heavy offered data load to the source. The bundle size is assumed to be fixed and equal to the maximum transmission unit (MTU) \( (i.e. 1500 \text{ bytes}) \). Following the guidelines of [35], \( k = 0.3 \) and \( m = 3 \). The same settings apply for BBRS-BTM except that for the BTM traffic model, vehicle speeds are uniformly distributed in the range \([10; 50] \text{ (meters per second)} \).

2) Discussion of Results

The above-listed delay metrics were evaluated for a total of \( 10^7 \) bundles and averaged out over multiple simulator runs to ensure the realization of a 95% confidence interval. Figures 9(a)-(c) plot the theoretical mean bundle queueing, transit and end-to-end delays achieved by BBRS-SFTM concurrently with their corresponding simulated counterparts.
Moreover, those figures contrast the different delay performances achieved by BBRS-SFTM to their corresponding one achieved by BBRS-BTM. These figures constitute tangible proofs of the validity of the proposed mathematical analysis of BBRS based on the SFTM traffic model and the high accuracy of the established simulation framework.

Figure 9(a) plots the mean queueing delay achieved by BBRS-SFTM and BBRS-BTM. Both curves are decreasing functions of the vehicular density. In fact, a low vehicular density implies that the vehicular traffic is very light, or alternatively, the vehicle inter-arrival time is large. Consequently, after completing the transmission of an arbitrary bulk of bundles, the source SRU is less likely to readily find another vehicle within its range. Hence, it will have to wait for the arrival of the next vehicle in order to proceed to the release of the next bulk. This additional waiting time contributes to the increase of the mean bundle queueing delay. In contrast, as the vehicular density increases, the vehicular traffic flow increases. Thus, the source SRU’s busy period tends towards continuity as it becomes more likely to readily find vehicles in range and hence continuously release one bulk after the other. Under such conditions, the mean queueing delay decreases.

Now, notice the impact of the traffic model on the queueing delay performance of BBRS. While the mean queueing delay experienced by bundles under BBRS-SFTM is of the order of a couple of seconds, the mean queueing delay under BBRS-BTM is of the order of tens of seconds. In fact, under BBRS-BTM, vehicle speeds are uniformly distributed within a specific range for all values of the vehicular density. Therefore a source SRU is equally likely to select a fast or a slow vehicle. Fast vehicles will reside less in the range of the source and have reduced bundle admission capabilities. Consequently, the source SRU will release to those vehicles an mean number of bundles that is small than that released to slow vehicles. Hence, the mean number of accumulating bundles in the queue will increase and so will the mean queueing delay. This explains the large difference between the queueing delays experienced under BBRS-BTM and BBRS-SFTM.

Figure 9(b) contrasts the performance of BBRS-SFTM to that of BBRS-BTM in terms of the mean transit delay. Since, under BBRS-BTM, vehicle speeds are uniformly distributed over a fixed interval for all vehicular densities, it follows that, irrespective of the vehicular density, the mean speed of bulk transporters is constant and equal to the mean of the chosen interval of speeds. This causes the mean transit delay to become constant for all vehicular densities. However, under BBRS-SFTM, at a low vehicular density, the mean speed of bulk carriers is high. This explains the low mean transit delay. However, the more the vehicular density will increase, the more the speed of transporting vehicles will decrease. Hence, the mean transit delay will increase.

Finally, Figure 9(c) plots the mean end-to-end delay achieved by BBRS-SFTM together with that achieved by BBRS-BTM. This goes without saying that the mean end-to-end delay’s behaviour is clear due to the fact that the it is the sum of the mean queueing delay and the mean transit delay.

VI. Conclusion

This manuscript considered a roadway segment [AB] experiencing Free-flow vehicular traffic. A comprehensive overview of the macroscopic vehicular traffic dynamics constituted the core of a novel and realistic mathematical framework where an observed roadway segment is modelled using an $M/G/\infty$ queuing system. Closed form expressions for this model’s characteristic parameters were developed. Extensive simulations were conducted to examine the validity and accuracy of the presented model. Finally, a simple case study was presented with the purpose of providing more insight into the practical application of the proposed model in a real-life two-hop vehicular intermittently connected network (TH-VICN). It is important to mention that the model proposed in this manuscript has a generic fundamental significance, beyond the specific context of TH-VICNs. Indeed, it can be applied to general systems. Due to this generality, any further results that can be derived have a potential significance for other fields.
REFERENCES


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