Target Mass Corrections to the Spin-dependent Structure Functions[1]

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At the energies at which most data in polarized deep-inelastic scattering have been taken the ratio M^2/Q^2, where M is the mass of the nucleon, is not negligible. The target mass corrections should be taken into account to analyze the Q^2 dependence of structure functions at low Q^2 and to extract information of dynamical higher twist effects at moderate energies.

We calculated the target mass corrections for spin dependent structure functions (SF) both for neutral and charged current interactions using the operator product expansion in lowest order in QCD. In this paper we present only the twist-2 contribution to the structure functions[‡]. We considered the light-cone expansion of the forward Compton amplitude in momentum space. Using the corresponding dispersion relations [2] the moments of structure functions can be expressed by the xmatrix elements of twist-2 operators. The target mass effects are calculated using the method proposed by Georgi and Politzer [3]. Following this way we expressed the moments of the SF as series of $(M^2/Q^2)^j$ and functions of reduced operator matrix elements. After performing the inverse Mellin transform we get the following expressions for the SF:

$$g_1^{\pm}(x) = x\frac{d}{dx}x\frac{d}{dx}\left[\frac{x}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_1^{\pm}(\xi)}{\xi}\right], \qquad (1)$$

$$g_2^{\pm}(x) = -x\frac{d^2}{dx^2}x\left[\frac{x}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_1^{\pm}(\xi)}{\xi}\right], \qquad (2)$$

$$g_3^{\pm}(x) = 2x^2\frac{d^2}{dx^2}\left[\frac{x^2}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_2^{\pm}(\xi)}{\xi^2}\right], \qquad (3)$$

$$g_4^{\pm}(x) = -x^2\frac{d}{dx}x\frac{d^2}{dx^2}\left[\frac{x^2}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_2^{\pm}(\xi)}{\xi^2}\right], \qquad (4)$$

$$g_5^{\pm}(x) = -x\frac{d}{dx}\left[\frac{x}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_3^{\pm}(\xi)}{\xi}\right] + \frac{M^2}{Q^2}x^2\frac{d^2}{dx^2}\left[\frac{x^2}{(1+4M^2x^2/Q^2)^{1/2}}\frac{G_2^{\pm}(\xi)}{\xi^2}\right]. \qquad (5)$$

The functions $G_1^{\pm}(y)$, $G_2^{\pm}(y)$ and $G_3^{\pm}(y)$ are defined via the matrix elements of the twist-2 operators appearing in the light-cone expansion of the forward scattering amplitude.

The expressions for the SF $g_1(x,Q^2)$ and $g_2(x,Q^2)$, Eqs. (1) and (2), can be rewritten as

$$g_1(x) = x\frac{d}{dx}\mathcal{F}(x) + x^2\frac{d^2}{dx^2}\mathcal{F}(x), \qquad (6)$$

$$g_2(x) = -2x\frac{d}{dx}\mathcal{F}(x) - x^2\frac{d^2}{dx^2}\mathcal{F}(x). \qquad (7)$$

Here $\mathcal{F}(x)$ denotes the function in the brackets of Eqs. (1) and (2). ¿From (6) and (7) one obtains

$$g_2(x) = -g_1(x) - x\frac{d}{dx}\mathcal{F}(x). \qquad (8)$$

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[‡]The details, also for the operators of twist-3, are given in Ref. [1].

Integrating the Eq. (6) we get

$$x\frac{d}{dx}\mathcal{F}(x) \quad = \quad -\int_x^1 \frac{dy}{y} g_1(y). \tag{9}$$

Eqs. (8) and (9) lead to the Wandzura-Wilczek relation [4]

$$g_2(x) \quad = \quad -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y). \tag{10}$$

Therefore, the Wandzura-Wilczek relation is not affected by target mass corrections. This result was obtained in Ref. [5] before. Moreover, it was shown that the Wandzura-Wilczek relation is valid for the quarkonic operators even in the massive quark case [1]. Although the expression for $g_2(x, Q^2)$, Eq. (7), is formally consistent with the Burkhardt-Cottingham sum rule [6]

$$\int_0^1 dx g_2^i(x, Q^2) = 0, \tag{11}$$

the $0th$ moment of the structure functions $g_2^i(x, Q^2)$ is not described by the local operator product expansion.

The relation between the structure functions $g_3(x)$ and $g_4(x)$ obtained by Blümlein and Kochelev [2] is also valid at any order in M^2/Q^2. Integrating Eq. (4) one gets

$${g_3}^i(x, Q^2) = 2x \int_x^1 \frac{dy}{y^2} {g_4}^i(y, Q^2). \tag{12}$$

Unlike the above relations, the Dicus relation between $g_4(x)$ and $g_5(x)$, cf. Ref. [2], is violated by the target mass corrections. Note that the Dicus relation corresponds to the Callan-Gross [7] relation for unpolarized structure functions. The spin dependence enters in front of the corresponding structure functions as an overall factor $S \cdot q$. It is worth mentioning that the Callan-Gross relation is also violated by target mass corrections [3].

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