

ON THE VACUUM OSCILLATION SOLUTIONS OF THE SOLAR NEUTRINO PROBLEM

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Abstract

We study the stability of the two-neutrino vacuum oscillation solution of the solar neutrino problem with respect to changes of the total fluxes of ^8B and ^7Be neutrinos, Φ_{B} and Φ_{Be} . For any value of Φ_{Be} from the interval $0.7\Phi_{\text{Be}}^{\text{BP}} \leq \Phi_{\text{Be}} \leq 1.3\Phi_{\text{Be}}^{\text{BP}}$ the solar ν_e oscillations into an active neutrino, $\nu_e \leftrightarrow \nu_{\mu(\tau)}$, provide at 95% C.L. a description of the existing solar neutrino data for $\Phi_{\text{B}} \cong (0.35 - 3.4)\Phi_{\text{B}}^{\text{BP}}$, $\Phi_{\text{B}}^{\text{BP}}$ and $\Phi_{\text{Be}}^{\text{BP}}$ being the fluxes in the solar model of Bahcall–Pinsonneault from 1992. For $\Phi_{\text{Be}} \cong (0.7 - 1.3)\Phi_{\text{Be}}^{\text{BP}}$ we find also at 95% C.L. two new (one new) $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ ($\nu_e \leftrightarrow \nu_s$) oscillation solutions: i) for $\Phi_{\text{B}} \cong (0.35 - 0.43)\Phi_{\text{B}}^{\text{BP}}$ at $\Delta m^2 \cong (4.7 - 6.5) \times 10^{-12} \text{ eV}^2$ ($(4.8 - 6.4) \times 10^{-12} \text{ eV}^2$) and $\sin^2 2\theta \gtrsim 0.71$ (0.74), and ii) for $\Phi_{\text{B}} \cong (0.45 - 0.65)\Phi_{\text{B}}^{\text{BP}}$ at $\Delta m^2 \cong (3.2 - 4.0) \times 10^{-11} \text{ eV}^2$ and $\sin^2 2\theta \gtrsim 0.59$. The physical implications of the new solutions for the future solar neutrino experiments are discussed. The data rule out at 97% – 98% (99 %) C.L. the possibility of a universal (neutrino energy independent) suppression of the different components of the solar neutrino flux, resulting from solar ν_e oscillations or transitions into active (sterile) neutrino.

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1. INTRODUCTION

In the present paper we investigate the stability of the vacuum oscillation [1–4] solution of the solar neutrino problem [5,6] with respect to variations of the total fluxes of the solar ^8B and ^7Be neutrinos. Recent studies have indicated that the current solar model predictions [7–12] for the ^8B neutrino flux, Φ_{B} , vary from model to model with rather large uncertainties [12,13]. The results for Φ_{B} derived in all solar models presently discussed in the literature except that of ref. [12], lie in the interval $(4.43\text{--}6.62)\times 10^6 \nu_e/\text{cm}^2/\text{sec}$, while the prediction of the "low" flux model of ref. [12], $\Phi_{\text{B}} = 2.77 \times 10^6 \nu_e/\text{cm}^2/\text{sec}$, differs from those of the "high" flux models of refs. [7] and [11] approximately by the factors 2.0 and 2.4. The predictions [7–12] for the total flux of ^7Be neutrinos, Φ_{Be} , vary by $\sim 20\%$, from $\Phi_{\text{Be}} = 4.34 \times 10^9 \nu_e/\text{cm}^2/\text{sec}$ in ref. [8] to $\Phi_{\text{Be}} = 5.18 \times 10^9 \nu_e/\text{cm}^2/\text{sec}$ in refs. [11]. At the same time none of the solar models developed to date provides a satisfactory description of the existing solar neutrino data [5,14–16]. In particular, the upper limits on the value of the ^7Be neutrino flux, which can be inferred from the data, are considerably lower than the values predicted by the models, as first noticed in ref. [17] and confirmed in several subsequent more detailed studies [18] utilizing different methods. The above result follows not only from joint analyses of the data from all solar neutrino experiments, Homestake [5], Kamiokande [14], GALLEX [15] and SAGE [16], but also from the Homestake and Kamiokande, or from the Kamiokande and SAGE and/or GALLEX data. Since the recent calibration of the GALLEX detector [19] leaves little room for doubts about the correctness of the GALLEX results, both the data from the Davis et al. and Kamiokande experiments have to be incorrect in order for the indicated conclusion to be not valid. The discrepancy between the value of Φ_{Be} suggested by the analyses of the available solar neutrino data and the solar model predictions for Φ_{Be} represents a major new aspect of the solar neutrino problem. No astrophysical and/or nuclear physics explanation of this discrepancy has been proposed so far.

Assuming that the ^7Be neutrino flux has a value in the interval $0.7\Phi_{\text{Be}}^{\text{BP}} \leq \Phi_{\text{Be}} \leq 1.3\Phi_{\text{Be}}^{\text{BP}}$, where $\Phi_{\text{Be}}^{\text{BP}}$ is the flux predicted in the reference solar model of Bahcall – Pinsonneault [7], we determine in the present study the range of values of the ^8B neutrino flux, for which the

results of the solar neutrino experiments can be described in terms of two-neutrino vacuum oscillations of the solar neutrinos into an active $\nu_e \leftrightarrow \nu_{\mu(\tau)}$, or sterile $\nu_e \leftrightarrow \nu_s$, neutrino. Similar analyses for the MSW solution [20] with solar ν_e transitions into an active neutrino, $\nu_e \rightarrow \nu_{\mu(\tau)}$, were performed in refs. [21,22]. Results for the case of solar $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations were obtained in ref. [23] for $\Phi_{\text{Be}} = 0.8\Phi_{\text{Be}}^{\text{BP}}$ and $0.4\Phi_{\text{B}}^{\text{BP}} \leq \Phi_{\text{B}} \leq 1.6\Phi_{\text{B}}^{\text{BP}}$, where $\Phi_{\text{B}}^{\text{BP}}$ is the ^8B neutrino flux predicted in the reference model [7]. However, we find, in particular, that at 95% C.L. a new vacuum $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ ($\nu_e \leftrightarrow \nu_s$) oscillation solution of the solar neutrino problem exists in the region $\Delta m^2 \cong (4.7 - 6.5) \times 10^{-12} \text{ eV}^2$ ($(4.8 - 6.4) \times 10^{-12} \text{ eV}^2$) and $0.71 (0.74) \lesssim \sin^2 2\theta \leq 1.0$ for $\Phi_{\text{B}} \cong (0.35 - 0.43)\Phi_{\text{B}}^{\text{BP}}$ and $\Phi_{\text{Be}} = (0.7 - 1.3)\Phi_{\text{Be}}^{\text{BP}}$, Δm^2 and $\sin^2 2\theta$ being the two parameters characterizing the oscillations (see, e.g., refs. [1–4]). A second $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillation solution is found for $0.45\Phi_{\text{B}}^{\text{BP}} \lesssim \Phi_{\text{B}} \lesssim 0.65\Phi_{\text{B}}^{\text{BP}}$ and $0.7\Phi_{\text{Be}}^{\text{BP}} \leq \Phi_{\text{Be}} \leq 1.3\Phi_{\text{Be}}^{\text{BP}}$, and for values of Δm^2 and $\sin^2 2\theta$ which lie within the intervals $\Delta m^2 \cong (3.2-4.0) \times 10^{-11} \text{ eV}^2$ and $0.59 \lesssim \sin^2 2\theta \leq 1.0$. Both these solutions were not noticed in ref. [23]. For $\Delta m^2 > 4.1 \times 10^{-11} \text{ eV}^2$ the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations allow to describe at 95% C.L. the existing solar neutrino data for any value of Φ_{Be} from the interval $(0.7-1.3)\Phi_{\text{Be}}^{\text{BP}}$ (for $\Phi_{\text{Be}} = 0.7\Phi_{\text{Be}}^{\text{BP}}$) and for $0.57 (0.51)\Phi_{\text{B}}^{\text{BP}} \lesssim \Phi_{\text{B}} \lesssim 3.4\Phi_{\text{B}}^{\text{BP}}$. The corresponding allowed regions of values of the parameters Δm^2 and $\sin^2 2\theta$ in all these cases are derived as well. Except for $0.35\Phi_{\text{B}}^{\text{BP}} \lesssim \Phi_{\text{B}} \lesssim 0.44\Phi_{\text{B}}^{\text{BP}}$ the oscillations into a sterile neutrino $\nu_e \leftrightarrow \nu_s$ are excluded at 95% C.L. if $\Phi_{\text{Be}} = (0.7 - 1.3)\Phi_{\text{Be}}^{\text{BP}}$; at 98% C.L. they are allowed by the (mean event rate) solar neutrino data for $0.32\Phi_{\text{B}}^{\text{BP}} \lesssim \Phi_{\text{B}} \lesssim 1.8\Phi_{\text{B}}^{\text{BP}}$.

We have also performed a study which shows that the data do not favour the hypothesis of neutrino energy independent suppression of the solar neutrino flux: it is excluded, depending on the value of Φ_{Be} from the interval $(0.7 - 1.3)\Phi_{\text{Be}}^{\text{BP}}$, at (97% – 98%) C.L. when the suppression is due to $\nu_e \rightarrow \nu_{\mu(\tau)}$ ($\nu_e \leftrightarrow \nu_{\mu(\tau)}$) or $\nu_e \rightarrow \bar{\nu}_{\mu(\tau)}$ transitions (oscillations), and at (99.0% – 99.7%) C.L. if it results from $\nu_e \rightarrow \nu_s$ ($\nu_e \leftrightarrow \nu_s$) transitions (oscillations).

The vacuum oscillation solution at $\Delta m^2 \gtrsim 4.4 \times 10^{-11} \text{ eV}^2$ imply a non-negligible suppression of the pp ν_e flux (approximately by a factor (0.50 – 0.70)), and a not very strong suppression of the 0.862 MeV ^7Be ν_e flux, the relevant suppression factor ranging from ap-

proximately 0.30 (possible for $\Phi_{\text{Be}} = 1.3\Phi_{\text{Be}}^{\text{BP}}$) to 0.98 (possible if $\Phi_{\text{Be}} \cong (0.7 - 1.0)\Phi_{\text{Be}}^{\text{BP}}$ and $\Phi_{\text{B}} \cong \Phi_{\text{B}}^{\text{BP}}$).

The new $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ ($\nu_e \leftrightarrow \nu_s$) oscillation solution located in the region $\Delta m^2 \cong (4.7 - 6.5) \times 10^{-12} \text{ eV}^2$, which exists for low values of Φ_{B} , is quite similar to the MSW nonadiabatic ($\nu_e \rightarrow \nu_{\mu(\tau)}$ [21] or $\nu_e \rightarrow \nu_s$ transition [26]) solutions for similar values of Φ_{B} : it corresponds to a rather strong suppression of the 0.862 MeV ${}^7\text{Be}$ ν_e flux (by a factor (0.06 – 0.26)), to a moderate suppression of the pp neutrino flux (by a factor not less than 0.7 only) and to ${}^8\text{B}$ neutrino flux practically not affected by the oscillations. The physical implications of the indicated new vacuum oscillation solutions for the future solar neutrino experiments are also briefly discussed. Our results show, in particular, that the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ vacuum oscillation solution of the solar neutrino problem is stable with respect to changes of the predictions for the ${}^8\text{B}$ and ${}^7\text{Be}$ neutrino fluxes.

We use the latest published data from all four solar neutrino experiments [5,14–16] in our analysis:

$$\bar{R}(\text{Ar}) = (2.55 \pm 0.25) \text{ SNU}, \quad (1)$$

$$\bar{\Phi}_{\text{B}}^{\text{exp}} = (2.89 \pm 0.42) \times 10^6 \text{ cm}^{-2}\text{sec}^{-1}, \quad (2)$$

$$\bar{R}_{\text{GALLEX}}(\text{Ge}) = (77.1 \pm 9.9) \text{ SNU}, \quad (3)$$

$$\bar{R}_{\text{SAGE}}(\text{Ge}) = (69 \pm 13) \text{ SNU}, \quad (4)$$

where $\bar{R}(\text{Ar})$, and $\bar{R}_{\text{GALLEX}}(\text{Ge})$ and $\bar{R}_{\text{SAGE}}(\text{Ge})$, are respectively the average rates of ${}^{37}\text{Ar}$ and ${}^{71}\text{Ge}$ production by solar neutrinos observed in the experiments of Davis et al. [5], and GALLEX [15] and SAGE [16], and $\bar{\Phi}_{\text{B}}^{\text{exp}}$ is the flux of ${}^8\text{B}$ neutrinos measured by the Kamiokande collaborations [14]. In eqs. (1) – (4) the quoted errors represent the added in quadratures statistical (1 s.d.) and systematical errors.

2. THE ${}^8\text{B}$ AND ${}^7\text{Be}$ NEUTRINO FLUXES

It is convenient to introduce the parameters

$$f_{\text{B}} \equiv \frac{\Phi_{\text{B}}}{\Phi_{\text{B}}^{\text{BP}}} \geq 0, \quad f_{\text{Be}} \equiv \frac{\Phi_{\text{Be}}}{\Phi_{\text{Be}}^{\text{BP}}} \geq 0, \quad (5)$$

in terms of which we shall describe the possible deviations of Φ_B and Φ_{Be} from their values in the reference model [7]. The fluxes Φ_B and Φ_{Be} in the models [7,8,11,12] correspond, respectively, to $f_B = 1.0; 0.78; 1.16; 0.49$, and $f_{Be} = 1.0; 0.89; 1.06; 0.89$.

The Kamiokande data, evidently, imposes limits on the values Φ_B (and f_B) can possibly have. The expression for the predicted event rate in the Kamiokande detector, $R(K)$, if the 8B (electron) neutrinos undergo two-neutrino transitions into an active neutrino $\nu_{\mu(\tau)}$ (due to vacuum oscillations $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ or MSW transitions $\nu_e \rightarrow \nu_{\mu(\tau)}$), or $\bar{\nu}_{\mu(\tau)}$ (due to spin-flavour conversion $\nu_e \rightarrow \bar{\nu}_{\mu(\tau)}$) on their way to the Earth, has the form ¹:

$$R(K) = f_B \Phi_B^{\text{BP}} \int_0^{14.4 \text{ MeV}} n(E) \sigma_K(E) [P(E) + 0.16(1 - P(E))] dE, \quad (6)$$

where $n(E)$ is the normalized to 1 spectrum of 8B neutrinos, $\int_0^{14.4 \text{ MeV}} n(E) dE = 1$, $\sigma_K(E)$ is the $\nu_e - e^-$ elastic scattering cross-section for 8B neutrinos with energy E , in which the recoil e^- detection efficiency and energy resolution functions of the Kamiokande detector are included, $P(E)$ is the probability of survival of the 8B ν_e having energy E ($(1 - P(E))$ is the probability of the $\nu_e \rightarrow \nu_{\mu(\tau)}$ transition due to vacuum oscillations or the MSW effect, or of the $\nu_e \rightarrow \bar{\nu}_{\mu(\tau)}$ conversion), and we have used the fact that $\sigma_{\nu_{\mu(\tau)}e}(E)/\sigma_{\nu_e e}(E) \cong \sigma_{\bar{\nu}_{\mu(\tau)}e}(E)/\sigma_{\nu_e e}(E) \cong 0.16$ in the energy range of interest, $\sigma_{\nu_l e}(E)$ and $\sigma_{\bar{\nu}_l e}(E)$, $l=e, \mu, \tau$, being the $\nu_l - e^-$ and $\bar{\nu}_l - e^-$ elastic scattering cross-sections. In the case of $\nu_e \leftrightarrow \nu_s$ oscillations or $\nu_e \rightarrow \nu_s$ transitions the term with the coefficient 0.16 is absent from the expression in the right hand side of eq. (6).

Given $R(K)$, Φ_B^{BP} , $n(E)$ and $\sigma_K(E)$, the minimal allowed value of f_B , as it follows from (6), is determined by the maximal possible value of $[P(E) + 0.16(1 - P(E))]$, which is 1 and is reached when $P(E) = 1$. Thus, we have $f_B \geq R(K)/R_{\text{BP}}(K) = (0.51 \pm 0.07)$, where $R_{\text{BP}}(K)$ is the event rate predicted in the BP model [7], and we have used the Kamiokande

¹In the numerical calculations we have performed we have included the Kamiokande energy resolution and trigger efficiency functions in the expression under the integral in eq. (6).

result, eq. (2). At 99.73% (95%) C.L. this implies

$$f_B \gtrsim 0.30 \text{ (0.37)}. \quad (7)$$

It is trivial to convince oneself that the above lower limit on f_B holds also in the case of solar ν_e two-neutrino oscillations (transitions) into sterile neutrino ν_s , as well as for oscillations (transitions) involving more than two neutrinos (sterile and/or active). The limit (7) is universal: it does not depend on the type of possible oscillations (transitions), and on the specific mechanism responsible for them.

Similarly, the maximal allowed value of f_B by the Kamiokande data corresponds to $\min [P(E) + 0.16 (1 - P(E))] = 0.16$. We have then: $f_B \leq R(K)/(0.16 R_{BP}(K)) = (3.2 \pm 0.44)$, which gives at 99.73% (95%) C.L.

$$f_B \lesssim 4.5 \text{ (4.1)}. \quad (8)$$

Inequality (8) is universal for two-neutrino solar ν_e oscillations or transitions into an active neutrino $\nu_{\mu(\tau)}$ or $\bar{\nu}_{\mu(\tau)}$.

Contrary to the lower limit (7), the upper limit (8) is not valid for two-neutrino $\nu_e \leftrightarrow \nu_s$ ($\nu_e \rightarrow \nu_s$) oscillations (transitions) or ν_e oscillations (transitions) involving more than two neutrinos. In the first case, for instance, the maximal value of f_B would correspond to the $\min P(E)$, and the use of the general property of the probability $P(E)$, $\min P(E) = 0$, does not allow one to derive a useful upper limit on f_B from the Kamiokande data.

In our study of the stability of the results on the vacuum oscillation solutions with respect to Φ_B and Φ_{Be} variations the following approach is adopted. The fluxes of the pp, pep and the CNO neutrinos (see, e.g., refs. [6]) are kept fixed and their values were taken from ref. [7]. The fluxes of the 8B and 7Be neutrinos, and correspondingly, f_B and f_{Be} , are treated as fixed parameters, which, however, are allowed to take any values within certain intervals. In the case of Φ_{Be} the interval chosen corresponds to

$$0.7 \leq f_{Be} \leq 1.3. \quad (9)$$

It is somewhat wider than the interval formed by the current solar model predictions: 0.89 – 1.06. For Φ_B values in the intervals determined by the inequalities (7) and (8) were considered. The searches for a $\nu_e \leftrightarrow \nu_s$ oscillation solution were preformed for $0.3 \leq f_B \leq 4.0$.

The indicated approach was motivated by the fact that the contributions of the CNO neutrinos to the signals in all three types of detectors [5,14–16] are predicted to be relatively small [6–12], and that (apart from the CNO neutrinos) the spreads in the predictions for the fluxes Φ_B and Φ_{Be} are the largest. Some of the values of Φ_{Be} used in the analyses, as those corresponding to $f_{Be} = 0.7$ and 1.3, for example, are incompatible with the constraint on the solar neutrino fluxes which the data on the solar luminosity impose (see, e.g., refs. [27,28]):

$$\Phi_{pp} + 0.958\Phi_{Be} + 0.955\Phi_{CNO} + 0.910\Phi_{pep} = (6.517 \pm 0.02) \times 10^{10} \text{ cm}^{-2}\text{sec}^{-1}, \quad (10)$$

where $\Phi_{CNO} = \Phi_N + \Phi_O$, and Φ_{pp} , Φ_{pep} , Φ_N and Φ_O are the fluxes of the pp, pep and the CNO neutrinos. However, a 20% – 30% change in Φ_{Be} with respect to $\Phi_{Be}^{BP} = 4.89 \times 10^9 \nu_e/\text{cm}^2/\text{sec}$ is required by (10) to be balanced by only a few percent change of the pp neutrino flux, and the latter will have a small effect on the predictions for the signal in the Ga–Ge experiments [15,16]. Besides, the aim of the present study (as like of the analogous studies of the MSW solutions in refs. [21,22,26]) is, in particular, to determine the ranges of values of Φ_B and Φ_{Be} for which the possibility of vacuum oscillations of solar neutrinos cannot be excluded by the existing solar neutrino data. Certainly, values of Φ_B corresponding to, e.g., $f_B \cong 3$ seem at present unlikely to appear in any realistic solar model.

In the absence of "unconventional behaviour" (vacuum oscillations, MSW transitions, etc.) of solar neutrinos, the signals in the Cl–Ar and Ga–Ge experiments can be written in the following form within the above approach:

$$R(\text{Ar}) = (6.20f_B + 1.17f_{Be} + 0.40_{CNO} + 0.23_{pep}) \text{ SNU}, \quad (11)$$

$$R(\text{Ge}) = (70.8_{pp} + 3.1_{pep} + 35.8f_{Be} + 13.8f_B + 7.9_{CNO}) \text{ SNU}, \quad (12)$$

where $6.20f_B$ SNU is the contribution in $R(\text{Ar})$ due to the ^8B neutrinos, etc.

We have used the χ^2 -method in our analysis. In computing the χ^2 for a given pair of values of the parameters Δm^2 and $\sin^2 2\theta$ we have ignored the estimated uncertainties in the reference model predictions [7] for the solar neutrino fluxes as the ranges within which we have varied Φ_B and Φ_{Be} exceed by far the uncertainties. We did, however, take into account the uncertainties in the detection cross-sections for the detectors [5,14–16].

3. THE VACUUM OSCILLATION SOLUTIONS

3.1 The Case of $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ Oscillations

3.1.1 Allowed Regions of the Parameters

Searching for vacuum $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ and $\nu_e \leftrightarrow \nu_s$ oscillation solutions we have scanned the region $10^{-12} \text{ eV}^2 \leq \Delta m^2 \leq 10^{-9} \text{ eV}^2$ and $10^{-2} \leq \sin^2 2\theta \leq 1.0$. It was found that at 95% C.L. and for $0.7 \leq f_{Be} \leq 1.3$ the two-neutrino $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations of the solar ν_e allow one to describe the data (1) – (4) for rather large intervals of values of f_B . These intervals depend somewhat on the value of f_{Be} . Below we give the solution intervals for f_B (at 95% C.L.) in the three representative cases of $f_{Be} = 0.7; 1.0; 1.3$:

$$\nu_e \leftrightarrow \nu_{\mu(\tau)} : \quad f_{Be} = 0.7, \quad 0.35 \lesssim f_B \lesssim 3.4, \quad (13a)$$

$$f_{Be} = 1.0, \quad 0.35 \lesssim f_B \lesssim 3.4 \quad (13b)$$

$$f_{Be} = 1.3, \quad 0.35 \lesssim f_B \lesssim 0.43 \quad \text{and} \quad 0.46 \lesssim f_B \lesssim 3.4. \quad (13c)$$

The allowed regions of values of Δm^2 and $\sin^2 2\theta$ corresponding to the solutions (13a), (13b) and (13c) are shown in Figs. 1a, 1b and 1c, respectively.

As Figs. 1a–1c illustrate, a deviation of the ^8B and ^7Be neutrino fluxes from the values corresponding to $f_B = f_{Be} = 1$ leads to two effects: i) noticeable shift (and change in size) of the allowed $\Delta m^2 - \sin^2 2\theta$ regions in the case $f_B = f_{Be} = 1$ towards smaller ($f_B < 1$) or larger ($f_B > 1$) values of $\sin^2 2\theta$ with the allowed values of Δm^2 remaining practically within the interval of the $f_B = f_{Be} = 1$ solution, $4.4 \times 10^{-11} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 9.8 \times 10^{-11} \text{ eV}^2$, and ii) appearance of new allowed regions of values of Δm^2 , i.e. of new solutions, at $\Delta m^2 \lesssim 4.1 \times 10^{-11} \text{ eV}^2$. We find two such new solutions (see Figs. 1a – 1c):

(A) for $0.7 \leq f_{\text{Be}} \leq 1.3$ and $0.35 \lesssim f_{\text{B}} \lesssim 0.44$ with Δm^2 and $\sin^2 2\theta$ lying in the intervals

$$4.7 \times 10^{-12} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 6.5 \times 10^{-12} \text{ eV}^2, \quad 0.71 \lesssim \sin^2 2\theta \leq 1.0, \quad (14)$$

(B) for any value of f_{Be} from the interval (9) (for $f_{\text{Be}} = 0.7$) and 0.45 (0.42) $\lesssim f_{\text{B}} \lesssim 0.65$ (0.66), and for Δm^2 and $\sin^2 2\theta$ having values within the intervals ²

$$3.2 \times 10^{-11} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 4.0 \times 10^{-11} \text{ eV}^2, \quad 0.59 \lesssim \sin^2 2\theta \leq 1.0. \quad (15)$$

Both solutions (A) and (B) are stable with respect to changes of f_{Be} ³. Nevertheless the regions of values of Δm^2 and $\sin^2 2\theta$ of these solutions vary somewhat with f_{Be} : eqs. (14) and (15) represent the largest intervals and correspond practically to $f_{\text{Be}} \cong 0.7$.

Let us discuss the above results. The probability that a solar electron neutrino with energy E will not change into $\nu_{\mu(\tau)}$ (or ν_s) on its way to the Earth when $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ ($\nu_e \leftrightarrow \nu_s$) oscillations take place, can be written in the form:

$$P_{\text{osc}}(E; R(t)) = 1 - \frac{1}{2} \sin^2 2\theta \left[1 - \cos 2\pi \frac{R(t)}{L_{\text{v}}} \right], \quad (16)$$

where $L_{\text{v}} = 4\pi E/\Delta m^2$ is the oscillation length in vacuum,

$$R(t) = R_0 \left[1 - \epsilon \cos 2\pi \frac{t}{T} \right], \quad (17)$$

is the Sun–Earth distance at time t of the year ($T = 365$ days), $R_0 = 1.4966 \times 10^8$ km and $\epsilon = 0.0167$ being the mean Sun–Earth distance and the ellipticity of the Earth orbit around the Sun. The term with the ϵ factor in eq. (17), as is well known [2-4,25], is a source of seasonal

²For $f_{\text{Be}} \cong 0.7$ there are also new solutions in the region $10^{-10} \text{ eV}^2 < \Delta m^2 < 10^{-9} \text{ eV}^2$ and $0.62 \lesssim \sin^2 2\theta \lesssim 0.80$, representing three very narrow strips (almost lines) of allowed values of Δm^2 and $\sin^2 2\theta$ in the $\Delta m^2 - \sin^2 2\theta$ plane (see Fig. 1a). However, these solutions are not stable with respect to variations of f_{Be} and disappear when f_{Be} is slightly increased (they do not exist for $f_{\text{Be}} = 1.0$, for example). We shall not discuss them further.

³The possibility of a "low" ⁸B neutrino flux solution for $f_{\text{Be}} = 1.0$ at $\Delta m^2 = 6.0 \times 10^{-12} \text{ eV}^2$ and $\sin^2 2\theta = 0.8$ was suggested on the basis of qualitative arguments in ref. [29]. Our results show that at 95% C.L. the indicated point in the relevant parameter space is marginally excluded by the current solar neutrino data .

variation effects in the case of the vacuum oscillation solution of the solar neutrino problem ⁴. Since we are dealing in the analysis of interest with experimental results averaged over at least few complete years of measurements, the relevant probability is actually the probability (16) averaged over a period of 1 year, $\bar{P}_{\text{osc}}(E; R_0, \epsilon)$. If $2\pi R_0/L_\nu \lesssim 2.5\pi$, $\bar{P}_{\text{osc}}(E; R_0, \epsilon)$ practically coincides with the probability (16) in which the parameter ϵ is formally set to zero, i.e., with $P_{\text{osc}}(E; R_0)$ (see Figs. 2a and 2b). This implies that for the values of $\Delta m^2 \lesssim 10^{-10}$ eV² of interest one has $\bar{P}_{\text{osc}}(E; R_0, \epsilon) \cong P_{\text{osc}}(E; R_0)$ for all neutrinos with energy $E \gtrsim 3$ MeV, i.e., for the dominant fraction of the ⁸B neutrino flux. If, however, $2\pi R_0/L_\nu \gg 2.5\pi$, the effect of the averaging can be quite dramatic for the oscillation's amplitude ⁵ and (for a given $\sin^2 2\theta$) $\bar{P}_{\text{osc}}(E; R_0, \epsilon)$ can differ considerably from $P_{\text{osc}}(E; R_0)$, as Figs. 2a and 2b illustrate.

Consider first the solutions at $\Delta m^2 \gtrsim 4.4 \times 10^{-11}$ eV², i.e., in the region in which the $f_B = f_{Be} = 1$ solution lies. The allowed regions corresponding to these solutions converge continuously (changing their shape and dimensions) to the allowed region in the case $f_B = f_{Be} = 1$ when f_B and f_{Be} are varied continuously from the values they have for a given solution to 1. The new solutions (A) and (B) identified above are "disconnected" from the $f_B = f_{Be} = 1$ solution and disappear when f_B and f_{Be} change continuously from 0.35 to 1.

For $\Delta m^2 \gtrsim 4.4 \times 10^{-11}$ eV² and for the energies of the pp neutrinos, $E \leq 0.42$ MeV, the cosine term in the expression for the probability (16) is a fastly oscillating function of E . Therefore the integration over the neutrino energy E in the contribution of the pp neutrinos to the signal in the Ga–Ge experiments suppresses the part of the contribution containing the cosine term and one has effectively in this case [2,3] $P_{\text{osc}}^{\text{pp}}(E; R_0) \cong 1 - \frac{1}{2} \sin^2 2\theta$. This implies that depending on the value of $\sin^2 2\theta$ the pp ν_e flux is suppressed approximately by

⁴Detailed predictions for the seasonal variation effects in the present and future solar neutrino experiments in the case of the $f_B \cong 1$, $f_{Be} \cong 1$ vacuum oscillation solution are given in ref. [25].

⁵It is not difficult to convince oneself that up to corrections which do not exceed 5×10^{-3} the periods of $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations implied by the probabilities $\bar{P}_{\text{osc}}(E; R_0, \epsilon)$ and $P_{\text{osc}}(E; R_0)$ coincide.

a factor (0.50 – 0.70). The values of Δm^2 for which the minimal values of $\sin^2 2\theta$ allowed by the data occur (see Figs. 1a – 1c) are determined by the condition $\cos 2\pi R_0/L_\nu = -1$ for $E = 0.862$ MeV – the energy of the dominant ${}^7\text{Be}$ neutrino component of the solar neutrino flux; they correspond for a given $\sin^2 2\theta$ to a maximal possible suppression of the flux of the 0.862 MeV ${}^7\text{Be}$ electron neutrinos at the Earth surface due to the oscillations $\nu_e \leftrightarrow \nu_{\mu(\tau)}$. Actually, the suppression of the 0.862 MeV ${}^7\text{Be}$ ν_e flux due to the vacuum oscillations is not very strong and can practically be absent in the case of the solution under discussion, the relevant suppression factor ranging from approximately 0.30 (possible when $f_{\text{Be}} = 1.3$) to 0.98 (possible for $f_{\text{Be}} \cong (0.7 - 1.0)$ and $f_{\text{B}} \cong 1$).

The maximal value of f_{B} , $\max f_{\text{B}} \cong 3.4$, for which the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations provide a description of the solar neutrino data is determined primarily by the Kamiokande result (2) (as its independence on f_{Be} indicates) and by the specific dependence of $P_{\text{osc}}(E; R_0)$ (see eq. (16)), on the solar neutrino energy E . It can be understood qualitatively by considering the constraints the Kamiokande data imply in this particular case. For any fixed $\Delta m^2 \lesssim 10^{-10}$ eV², the probability $P_{\text{osc}}(E; R_0)$ has at most one minimum in the interval of ${}^8\text{B}$ neutrino energies $7.5 \text{ MeV} \lesssim E \leq 14.4 \text{ MeV}$ relevant to the Kamiokande experiments (see Figs. 2a and 2b). The suppression of the integral in the expression (6) for $R(\text{K})$ is maximal when the minimum of $P_{\text{osc}}(E; R_0)$ occurs at $E = E_{\text{min}}$, $7.5 \text{ MeV} < E_{\text{min}} < 14.4 \text{ MeV}$. In this case $P_{\text{osc}}(E; R_0)$ increases monotonically with the change of E in the indicated interval both for $E < E_{\text{min}}$ and $E > E_{\text{min}}$. This implies that there exists a maximal possible suppression of the flux of ${}^8\text{B}$ electron neutrinos with $E \gtrsim 7.5 \text{ MeV}$ (i.e., of the integral in the right hand side of eq. (6)) due to $P_{\text{osc}}(E; R_0)$, and hence a maximal possible value of f_{B} for which the vacuum $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations can provide an explanation of the Kamiokande result (2). The value one obtains numerically, $\max f_{\text{B}} \cong 3.4$, is somewhat smaller than the upper bound (8) and is reached for, e.g., $f_{\text{B}} = 1$ at $\Delta m^2 \cong 8.2 \times 10^{-11}$ eV² and $\sin^2 2\theta = 1.0$. For these values of Δm^2 and $\sin^2 2\theta$ one has $\min P_{\text{osc}}(E_{\text{min}}; R_0) = 0$ and $E_{\text{min}} \cong 9.5 \text{ MeV}$, and the suppression of the integral in eq. (6) corresponds effectively to a constant factor $[P_{\text{osc}}(E; R_0) + 0.16(1 - P_{\text{osc}}(E; R_0))] \cong 0.2$. The value of Δm^2 for

which the solution for $f_B \cong 3.4$ exists depends, although weakly, on the value of f_{Be} chosen within the interval (9) because $\bar{P}_{osc}(E; R_0, \epsilon)$ for $E=0.862$ MeV is very sensitive to even small changes of Δm^2 in the vicinity of 8.2×10^{-11} eV², as Fig. 2b (after the necessary rescaling of the values of E on the horizontal axis by the ratio $(10^{-10}/(8.2 \times 10^{-11})) \cong 1.2$) illustrates (we have: $2\pi R_0/L_v = 37.9018(\Delta m^2/10^{-10}\text{eV}^2)(1\text{MeV}/E)$ and in this case, for instance, $\bar{P}_{osc}(E = 0.862 \text{ MeV}; R_0, \epsilon) = P_{osc}(E = 0.862 \text{ MeV}; R_0) \cong 0.50$ (0.70) for $\Delta m^2 = 8.2$ (8.3) $\times 10^{-11}$ eV² and $\sin^2 2\theta = 1.0$). Therefore the necessary suppression of the ⁷Be contribution to the signals in the Cl–Ar and the Ga–Ge experiments for the different values of f_{Be} considered is achieved by small changes of the value of Δm^2 around the value 8.2×10^{-11} eV². These changes do not affect the suppression of the contributions of the ⁸B neutrinos in $R(\text{Ar})$ and $R(\text{K})$, required by the Cl–Ar and Kamiokande data.

In a similar way one can understand the minimal values of f_B , $\min f_B \cong 0.51$; 0.54; 0.57 corresponding to $f_{Be} = 0.7$; 1.0; 1.3, for which there exists at 95% C.L. a $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillation solution in the region $\Delta m^2 \gtrsim 4.4 \times 10^{-11}$ eV². Depending on f_{Be} these solutions occur for values of $\Delta m^2 \cong (4.3-5.1) \times 10^{-11}$ eV² and for $\sin^2 2\theta \cong (0.57-0.75)$ (see Figs. 1a – 1c). The minimal values of f_B allowed by the data are again determined by the Kamiokande result, and by the specific energy dependence of $P_{osc}(E; R_0)$ for values of Δm^2 in the vicinity of $\Delta m^2 \cong 4.9 \times 10^{-11}$ eV². Indeed, for $\Delta m^2 = (4.3-5.1) \times 10^{-11}$ eV² and for the energies of ⁸B neutrinos $7.5 \text{ MeV} \lesssim E \leq 14.4 \text{ MeV}$ detected by the Kamiokande experiments, $P_{osc}(E; R_0)$ is a monotonically (rather steeply) increasing function of E and for $\sin^2 2\theta = 0.60$ (0.75) one has 0.5 (0.4) $\lesssim P_{osc}(E; R_0) \lesssim 0.8$ (0.7) (see Figs. 2a and 2b). In this case and, e.g., for $f_{Be} = 0.7$, the maximal suppression due to $P_{osc}(E; R_0)$ of the integral in the expression for $R(\text{K})$, eq. (6), corresponds effectively to a constant factor $[P_{osc}(E; R_0) + 0.16(1 - P_{osc}(E; R_0))] \cong 0.7$. The Kamiokande data then imply (95% C.L.) $f_B \gtrsim 0.52$. The minimal value of f_B one obtains depends somewhat on the value of f_{Be} because the requisite suppression of the ⁷Be 0.862 MeV electron neutrino flux (and of the contributions of the 0.862 MeV ⁷Be neutrinos to $R(\text{K})$ and $R(\text{Ge})$) is achieved now by an adjustment of the value of $\sin^2 2\theta$ within the interval $0.57 - 0.75$ (rather than by changing Δm^2), which in turn leads to a non-negligible change

of $P_{\text{osc}}(E; R_0)$ for $7.5 \text{ MeV} \lesssim E \leq 14.4 \text{ MeV}$. In the case of the solution with $f_{\text{Be}} = 0.7$ and $f_{\text{B}} = 0.53$, for instance, the pp and $0.862 \text{ MeV } ^7\text{Be}$ electron neutrino fluxes are suppressed due to the oscillations by the factors 0.70 and 0.46, respectively, while the pep (CNO) neutrino flux (fluxes) is not (are mildly) suppressed. The predictions for the signals in the Cl–Ar and Ga–Ge detectors read in this case $R(\text{Ar}) \cong 2.7 \text{ SNU}$ and $R(\text{Ge}) \cong 74 \text{ SNU}$.

It should be noted that the allowed regions found at 95% C.L. for $f_{\text{Be}} \cong (0.7 - 1.3)$ and $f_{\text{B}} \cong (0.8 - 1.2)$ in the present study lie practically all within the allowed regions one obtains at 95% C.L. in the reference model [7] when the estimated uncertainties in the theoretical predictions are included in the analysis.

Let us add finally that the minimal values of the χ^2 –function for the solution under discussion in the three cases $f_{\text{Be}} = 0.7; 1.0; 1.3$ respectively read 0.67; 0.51; 0.53 (for 2 d.f.) and take place at $f_{\text{B}} = 2.2; 2.4; 2.4$ for $\Delta m^2 = (7.2; 7.4; 7.5) \times 10^{-11} \text{ eV}^2$ and $\sin^2 2\theta = 1$. For $f_{\text{B}} \leq 1$, $\min \chi^2 = 2.9$ ⁶ and is reached for $f_{\text{B}} = 1$, $\Delta m^2 = 6.1 \times 10^{-11} \text{ eV}^2$ and $\sin^2 2\theta = 0.86$. In the case of solution (A), eq.(14), and for $f_{\text{Be}} = 0.7$ (1.0), $\min \chi^2 = 4.4$ (4.5) and corresponds to $f_{\text{B}} = 0.39$, $\Delta m^2 = 5.4$ (5.6) $\times 10^{-12} \text{ eV}^2$ and $\sin^2 2\theta = 1.0$. The $\min \chi^2$ value is somewhat larger for solution (B): for $f_{\text{Be}} = 1.0$, for instance, one has $\min \chi^2 = 5.3$.

3.1.2 Physical Implications of the New Low Φ_{B} Solution

The physical implications of the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillation solution for $f_{\text{Be}} \cong 1$ and $f_{\text{B}} \cong 1$ and values of Δm^2 in the interval $4.4 \times 10^{-11} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 10^{-10} \text{ eV}^2$ have been extensively discussed in the literature (see refs. [2,25], [23] and the articles by A. Acker et al. and by V. Barger et al. quoted in ref. [3]). The solutions we have found in the same Δm^2 region for $f_{\text{Be}} \neq 1$ and $f_{\text{B}} \neq 1$ lead to generically similar implications and we shall not consider them here.

Of the two new solutions (A), eq. (14), and (B), eq. (15), solution (A) is more interesting phenomenologically, has a lower χ^2 –value, and therefore we shall discuss only it,

⁶In contrast, $\min \chi^2 = 0.25$ in the case of the MSW small mixing-angle $\nu_e \rightarrow \nu_{\mu(\tau)}$ solution.

although rather briefly. For the values of Δm^2 from the interval given in (14) one has: i) $\bar{P}_{\text{osc}}(E; R_0, \epsilon) \gtrsim 0.97$ (0.94) for $E \geq 7.5$ (5.0) MeV, ii) for neutrino energies in the vicinity of 0.862 MeV $\bar{P}_{\text{osc}}(E; R_0, \epsilon)$ has its first local minimum when E decreases from values for which $\bar{P}_{\text{osc}}(E; R_0, \epsilon) \cong 1$ (see Figs. 2), and iii) the first local maximum of $\bar{P}_{\text{osc}}(E; R_0, \epsilon)$ as E decreases below 0.862 MeV occurs in the interval $0.23 \text{ MeV} \lesssim E \lesssim 0.42 \text{ MeV}$ ⁷. Correspondingly, if solution (A) is realized, the signals due to the ⁸B neutrinos in the present (and the future SNO [30] and Super-Kamiokande [31]) detectors will not practically be affected by the vacuum $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations (remember that for solution (A) $f_B \cong (0.35 - 0.44)$), the 0.862 MeV ⁷Be ν_e flux will be suppressed by the suppression factor (0.06 - 0.26), while the signal due to the pp electron neutrinos in the Ga-Ge detectors will be mildly reduced by a suppression factor not smaller than 0.7. Thus, from the point of view of how the different components of the solar neutrino flux are affected, the vacuum oscillation solution (A) is very similar to the low f_B MSW $\nu_e \rightarrow \nu_{\mu(\tau)}$ [21] or $\nu_e \rightarrow \nu_s$ [26] transition nonadiabatic solution. However, some of the physical implications of the two solutions differ considerably. In particular, i) the predicted distortion of the spectrum of the pp neutrinos is much stronger in the case of the vacuum oscillation solution (A)⁸ (Fig. 3a) than for the corresponding MSW nonadiabatic solution, and ii) if solution (A) is valid, the ⁷Be and pp electron neutrino fluxes at the Earth surface will exhibit seasonal variations which cannot take place in the case of the MSW solutions (see ref. [32] and the first article quoted in ref. [24]). In what follows we shall discuss briefly the seasonal variation effects predicted in the case of solution (A).

For $\Delta m^2 \leq 6.5 \times 10^{-12} \text{ eV}^2$ and $E \geq 0.233 \text{ MeV}$ (0.217 MeV) one has: $2\pi\epsilon R_0/L_\nu \leq$

⁷One can explain the minimal (maximal) value of f_B for which solution (A) exists, the reason for the difference between the maximal allowed values of Δm^2 (and the values themselves) for a given f_{Be} in the cases $f_B = 0.35$ and $f_B = 0.38$ etc., in a similar way we did it earlier, e.g., for the maximal value of f_B allowed by the data and the corresponding values of Δm^2 .

⁸It is also very different from the distortion of the pp neutrino spectrum in the case of the solutions with $4.4 \times 10^{-11} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 10^{-10} \text{ eV}^2$ (see the third article quoted in ref. [2]).

0.18 (0.19) $\ll 1$. Thus, in the case of solution (A) the probability $P_{\text{osc}}(E; R(t))$, eq. (16), can be represented as a power series in the small parameter $(2\pi\epsilon(R_0/L_\nu) \cos 2\pi(t/T))$. Neglecting all the terms smaller than 10^{-3} in this series we obtain:

$$P_{\text{osc}}(E; R(t)) \cong P_{\text{osc}}(E; R_0) + P_{\text{seas}}(E; R_0, \epsilon, t), \quad (18)$$

where the term

$$P_{\text{seas}}(E; R_0, \epsilon, t) = \frac{1}{2} \sin^2 2\theta \left[2\pi\epsilon \frac{R_0}{L_\nu} \sin 2\pi \frac{R_0}{L_\nu} \right] \cos 2\pi \frac{t}{T} - \frac{1}{4} \sin^2 2\theta \cos 2\pi \frac{R_0}{L_\nu} \left[2\pi\epsilon \frac{R_0}{L_\nu} \cos 2\pi \frac{t}{T} \right]^2 \quad (19)$$

is responsible for the seasonal variation effects of interest. Obviously, for fixed values of the parameters Δm^2 and $\sin^2 2\theta$, the difference between the values of $P_{\text{seas}}(E; R_0, \epsilon, t)$ in December/January ($t \cong 0$) and June/July ($t \cong 0.5T$) is the largest.

For ^8B neutrinos with $E \geq 5$ MeV (6.44 MeV) we have $2\pi\epsilon R_0/L_\nu \leq 8.2 \times 10^{-3}$ (6.3×10^{-3}) and it follows from eqs. (18) and (19) that the seasonal variation effect in the signal of the Super-Kamiokande (SNO) detector will be too small to be observable. The effect can be much larger for the signals due to the ^7Be and/or pp neutrinos in the Ga-Ge, BOREXINO [33] and HELLAZ [34] detectors.

In the case of solution (A) one has for the predicted average rate of Ge production per year in the Ga-Ge experiments for $f_{\text{Be}} = 0.7; 1.0; 1.3$: $\bar{R}(\text{Ge}) \lesssim 80; 84; 88$ SNU. The difference between the rates of Ge production in December/January ($t \cong 0$) and June/July ($t \cong 0.5T$), $\Delta R_{\text{seas}}(\text{Ge})$, due to i) the term (19) in the vacuum oscillation probability (18), and ii) the change of the neutrino fluxes with the change of the Sun-Earth distance due to the standard geometrical effect, as can be shown, satisfies: $4.0 \text{ SNU} \lesssim \Delta R_{\text{seas}}(\text{Ge}) \lesssim 8.7$ (8.1) SNU, the maximal value corresponding to $f_{\text{Be}} = 1.3$ (0.7). A convenient relative measure of the predicted seasonal effect is the seasonal (December/January – June/July) asymmetry:

$$A_{\text{seas}}(\text{Ge}) = \frac{R_0^2}{R(\text{Ge})} \left[\frac{R(\text{Ge}; t)}{R^2(t)} \Big|_{t \cong 0} - \frac{R(\text{Ge}; t)}{R^2(t)} \Big|_{t \cong 0.5T} \right], \quad (20)$$

where $R(\text{Ge}; t) [R_0/R(t)]^2$ is the rate of Ge production at time t of the year and $R(t)$ is given by eq. (17). For solution (A) we have: $0.072 \lesssim A_{\text{seas}}(\text{Ge}) \lesssim 0.13$, the contribution due

purely to the geometrical factor $R^{-2}(t)$ being $4\epsilon = 0.0668$. For given Δm^2 and $\sin^2 2\theta$ the change of $A_{\text{seas}}(\text{Ge})$ with the change of f_{Be} is negligibly small.

The corresponding seasonal (December/January – June/July) asymmetry in the signal due to the ${}^7\text{Be}$ (0.862 MeV) neutrinos in the BOREXINO detector is given (up to corrections smaller than 10^{-3}) by

$$A_{\text{seas}}^{\text{a}}(\text{BOR}) = 0.0668 + \frac{0.79}{0.21 + 0.79P_{\text{osc}}(E; R_0)} \frac{2\pi\epsilon R_0/L_{\nu}}{L_{\nu}} \sin^2 2\theta \sin 2\pi \frac{R_0}{L_{\nu}}, \quad (21)$$

where 0.0668 is the asymmetry in the absence of oscillations, and we have used the fact that $\sigma_{\nu_{\mu(\tau)}e}(E)/\sigma_{\nu_e e}(E) \cong 0.21$ for $E = 0.862$ MeV. Note that the asymmetry $A_{\text{seas}}^{\text{a}}(\text{BOR})$ does not depend on the total flux of 0.862 MeV ${}^7\text{Be}$ neutrinos. Note also that in the case of solution (A) for $E = 0.862$ MeV we have $\sin 2\pi R_0/L_{\nu} > 0$ and the second term in eq. (21) is always positive. As can be shown, the asymmetry $A_{\text{seas}}^{\text{a}}(\text{BOR})$ changes very little with the variation of Δm^2 and $\sin^2 2\theta$ within the allowed regions of values for the solution (A) and $A_{\text{seas}}^{\text{a}}(\text{BOR}) \cong (0.11 - 0.13)$.

The HELLAZ experiment [34] is envisaged to detect pp neutrinos having energy $E \geq 0.217$ MeV and to measure their spectrum. The experiment will be based on the $\nu - e^-$ elastic scattering reaction. Since the energy of the incident pp neutrino in each event will be reconstructed, one can define a seasonal asymmetry in the signal of HELLAZ, generated by neutrinos having energy within a given interval $E_1 \leq E \leq E_2$ ($E_1 \geq 0.217$ MeV, $E_2 \leq 0.42$ MeV): $A_{\text{seas}}(\text{H}; E_1, E_2)$. The expression for $A_{\text{seas}}(\text{H}; E_1, E_2)$ can be obtained formally from eq. (20) by replacing $R(\text{Ge}; t)$ and $\bar{R}(\text{Ge})$ with the corresponding quantities – event rate at time t of the year, $R(\text{H}; E_1, E_2, t)$, and mean event rate per year, $\bar{R}(\text{H}; E_1, E_2)$, for HELLAZ.

For solution (A) of interest and $E \geq 0.217$ MeV one has $2\pi\epsilon R_0/L_{\nu} \lesssim 0.19$, and the seasonal asymmetry in the total (neutrino energy integrated) signal of the HELLAZ detector, $A_{\text{seas}}^{\text{a}}(\text{H})$, as numerical calculations show, satisfies $0.018 \lesssim A_{\text{seas}}^{\text{a}}(\text{H}) \lesssim 0.067$, the smallest (the largest) value being reached for $\Delta m^2 \cong 5 \times 10^{-12}$ eV² (6×10^{-12} eV²). Note that for $\Delta m^2 \cong 5 \times 10^{-12}$ eV² the asymmetry due to the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations compensates to a large

extent the geometrical one, rendering the net asymmetry hardly observable. The absence of any seasonal variations in the signal of SNO, or Super-Kamiokande, or BOREXINO, or HELLAZ detector (constant in time event rate) is known [2,25] to be one of the distinctive signatures of the vacuum oscillation solutions of the solar neutrino problem.

The comparatively small values of the asymmetry $A_{\text{seas}}^a(\text{H})$ are a consequence of two circumstances. First, the contribution of the $\nu_{\mu(\tau)}$ neutrinos (present in the pp (ν_e) flux as a result of the oscillations) reduces the asymmetry of interest. For a given energy E the expression for the latter contains the factor $(1 - \sigma_{\nu_{\mu(\tau)}e}(\text{E}) / \sigma_{\nu_e e}(\text{E}))$ which changes from 0.58 to 0.72 when E increases from 0.217 MeV to 0.42 MeV ⁹. More importantly, for values of Δm^2 from the interval (14), $\sin 2\pi R_0/L_\nu$ changes sign passing through zero in the interval $0.28 \text{ MeV} \leq E \leq 0.39 \text{ MeV}$ when E varies from 0.217 MeV to 0.42 MeV ¹⁰. As a result the two contributions to the asymmetry in the energy integrated event rate, generated by the oscillations of pp neutrinos having energy in the two intervals $0.217 \text{ MeV} \leq E \leq E_0$ and $E_0 \leq E \leq 0.42 \text{ MeV}$, $E_0 \geq 0.28 \text{ MeV}$ being the energy at which $\sin 2\pi R_0/L_\nu = 0$ ¹¹, have opposite signs and compensate partially or completely each other. A complete cancelation between the indicated two contributions takes place, for instance, for $\Delta m^2 \cong 6 \times 10^{-12} \text{ eV}^2$, for which $E_0 \cong 0.36 \text{ MeV}$.

It should be evident from the above discussion that for solution (A) the asymmetry $A_{\text{seas}}^a(\text{H}; 0.217 \text{ MeV}, E_0) \equiv A_{\text{seas}}^a(\text{H}; E \leq E_0)$ or $A_{\text{seas}}^a(\text{H}; E_0, 0.42 \text{ MeV}) \equiv A_{\text{seas}}^a(\text{H}; E \geq E_0)$ can be larger than $A_{\text{seas}}^a(\text{H})$. Indeed, it can be easily shown that either $|A_{\text{seas}}^a(\text{H}; E \leq E_0)| > A_{\text{seas}}^a(\text{H})$, or $|A_{\text{seas}}^a(\text{H}; E \geq E_0)| > A_{\text{seas}}^a(\text{H})$. One can have also $|A_{\text{seas}}^a(\text{H}; E \leq E_0) -$

⁹The same term gives rise to the factor 0.79 in the expression for $A_{\text{seas}}^a(\text{BOR})$, eq. (21).

¹⁰For $5.4 \times 10^{-12} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 6.5 \times 10^{-12} \text{ eV}^2$, $\sin 2\pi R_0/L_\nu$ changes sign two times in the interval (0.217 MeV – 0.42 MeV), passing through a second zero located at $0.217 \text{ MeV} \leq E \leq 0.26 \text{ MeV}$. However, the effect of the presence of this second zero of $\sin 2\pi R_0/L_\nu$ on $A_{\text{seas}}^a(\text{H})$ is less important than the effect of the first zero located at $E \geq 0.28 \text{ MeV}$.

¹¹Obviously, the value of E_0 depends on the value chosen of Δm^2 .

$A_{\text{seas}}^{\text{a}}(\text{H}; E \geq E_0) > A_{\text{seas}}^{\text{a}}(\text{H})$. Note that the difference $[A_{\text{seas}}^{\text{a}}(\text{H}; E \leq E_0) - A_{\text{seas}}^{\text{a}}(\text{H}; E \geq E_0)]$ is free from the geometrical term and can be nonzero only if the pp neutrinos take part in vacuum oscillations. Let us illustrate the above remarks with two examples. For $\Delta m^2 = 6 \times 10^{-12} \text{ eV}^2$ ($E_0 = 0.36 \text{ MeV}$) and $\sin^2 2\theta = 1$ one has: $A_{\text{seas}}^{\text{a}}(\text{H}) \cong 0.0668$ and $A_{\text{seas}}^{\text{a}}(\text{H}; E \leq 0.36 \text{ MeV}) \cong 0.11$. If $\Delta m^2 = 5 \times 10^{-12} \text{ eV}^2$, then $E_0 = 0.30 \text{ MeV}$ and in this case $A_{\text{seas}}^{\text{a}}(\text{H}) \cong 0.018$, $A_{\text{seas}}^{\text{a}}(\text{H}; E \leq 0.30 \text{ MeV}) \cong 0.10$, $A_{\text{seas}}^{\text{a}}(\text{H}; E \geq 0.30 \text{ MeV}) \lesssim 10^{-3}$, and $[A_{\text{seas}}^{\text{a}}(\text{H}; E \leq 0.30 \text{ MeV}) - A_{\text{seas}}^{\text{a}}(\text{H}; E \geq 0.30 \text{ MeV})] \cong 0.10$.

In the case of the solution with $4.4 \times 10^{-11} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 10^{-10} \text{ eV}^2$ the seasonal asymmetry in the signals of the Ga-Ge, Super-Kamiokande (SNO) and BOREXINO detectors due purely to the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations can be as large as 30%, 14% and 80%, respectively, and is predicted to be negligible for the energy integrated signal of the HELLAZ detector [2,25].

3.2 Oscillations into Sterile Neutrino $\nu_e \leftrightarrow \nu_s$

The solar neutrino oscillations into sterile neutrino, $\nu_e \leftrightarrow \nu_s$, were excluded in the case $f_{\text{B}} \cong 1$ and $f_{\text{Be}} \cong 1$ at 99% C.L. as a possible solution of the solar neutrino problem by the (mean event rate) solar neutrino data which were available by March 1994 [4]. Since then updated results have been published by all operating solar neutrino experiments. The current status of the hypothesis of solar $\nu_e \leftrightarrow \nu_s$ oscillations, including the cases $f_{\text{B}} \neq 1$ and $f_{\text{Be}} \neq 1$, $0.7 \leq f_{\text{Be}} \leq 1.3$, is summarized graphically in Figs. 4a - 4c. At 95% C.L. and for $0.7 \leq f_{\text{Be}} \leq 1.3$ ($f_{\text{Be}} = 0.7$) this possibility is not excluded by the current solar neutrino data only for

$$0.35 \lesssim f_{\text{B}} \lesssim 0.43 \quad (0.44) \quad (22)$$

and values of Δm^2 and $\sin^2 2\theta$ in the intervals

$$4.8 \times 10^{-12} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 6.2 \times 10^{-12} \text{ eV}^2, \quad 0.74 \lesssim \sin^2 2\theta \leq 1.0. \quad (23)$$

This solution is stable with respect to variations of f_{Be} within the interval (9). Obviously, it is a $\nu_e \leftrightarrow \nu_s$ oscillation analog of the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillation solution (A) (compare eqs.

(22), (23) with eq. (14)). At 95% C.L. and for $f_{Be} \cong 0.7$ there exist also two other allowed regions at larger values of Δm^2 for $f_B \cong 0.42$ and $f_B \cong 0.50$ (see Fig. 4a), but they are very small and disappear when $f_{Be} > 0.7$. If one increases the required C.L. of the description of the data to 98%, solution exists for $0.32 \lesssim f_B \lesssim 1.8$ if $f_{Be} = (0.7 - 1.3)$. The corresponding new allowed regions are scattered over the area $4.0 \times 10^{-12} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 2.0 \times 10^{-10} \text{ eV}^2$, $0.50 \lesssim \sin^2 2\theta \leq 1.0$. These regions diminish in size considerably or completely disappear as f_{Be} changes from 0.7 to 1.3: for $f_{Be} = 1.3$ most of the remaining new regions are in the form of narrow strips (see Fig. 4c).

Let us consider briefly the physical implications of the solution (22) – (23) for the future solar neutrino experiments. The deformation of the pp neutrino spectrum (Fig. 3b) is quite strong and differs somewhat from the deformation in the case of the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillation solution (A) (compare Figs. 3a and 3b). In what regards the seasonal variation effects in the signals of the Super-Kamiokande, SNO and the Ga-Ge experiments, they coincide with those for the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillation solution (A), considered in Section 3.1.2. The predicted seasonal variation effect in the signal due to the ${}^7\text{Be}$ neutrinos in the BOREXINO detector, however, is larger than in the corresponding case of $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations. The seasonal (December/January – June/July) asymmetry for the BOREXINO detector is given now by the expression

$$A_{\text{seas}}^s(\text{BOR}) = 0.0668 + \frac{2\pi\epsilon R_0/L_\nu}{P_{\text{osc}}(E; R_0)} \sin^2 2\theta \sin 2\pi \frac{R_0}{L_\nu}, \quad (24)$$

and can be as large as 42%: one has $0.18 \lesssim A_{\text{seas}}^s(\text{BOR}) \lesssim 0.42$. We find also that $A_{\text{seas}}^s(\text{BOR}) \gtrsim 1.5A_{\text{seas}}^a(\text{BOR})$. The difference between the values of $A_{\text{seas}}^s(\text{BOR})$ and $A_{\text{seas}}^a(\text{BOR})$, in particular, can be used to distinguish between the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillation solution (A) and its $\nu_e \leftrightarrow \nu_s$ oscillation analog in a solar model independent way.

The seasonal asymmetries $A_{\text{seas}}^s(\text{H}; E \leq E_0)$ and $A_{\text{seas}}^s(\text{H}; E \geq E_0)$ in the signal of the HELLAZ detector tend also to be larger than the corresponding asymmetries in the case of the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillation solution (A). For $\Delta m^2 = 5.0 \times 10^{-12} \text{ eV}^2$ ($E_0 = 0.30 \text{ MeV}$) and $\sin^2 2\theta = 1.0$, for instance, we have: $A_{\text{seas}}^s(\text{H}) \cong -0.9 \times 10^{-2}$, $A_{\text{seas}}^s(\text{H}; E \leq 0.30 \text{ MeV}) \cong 0.12$,

$A_{\text{seas}}^s(\text{H}; E \geq 0.30 \text{ MeV}) \cong -0.04$, and $[A_{\text{seas}}^s(\text{H}; E \leq 0.30 \text{ MeV}) - A_{\text{seas}}^s(\text{H}; E \geq 0.30 \text{ MeV})] \cong 0.16$. In the case of $\Delta m^2 = 6.0 \times 10^{-12} \text{ eV}^2$ ($E_0 = 0.36 \text{ MeV}$) one obtains: $A_{\text{seas}}^s(\text{H}) \cong 0.0668$, and $A_{\text{seas}}^s(\text{H}; E \leq 0.36 \text{ MeV}) \cong 0.15$.

4. ENERGY INDEPENDENT SUPPRESSION OF THE SOLAR NEUTRINO FLUX

The possibility of universal (energy independent) suppression of the pp, ${}^7\text{Be}$, pep, ${}^8\text{B}$ and CNO neutrino fluxes can be realized if solar neutrinos take part in $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ or $\nu_e \leftrightarrow \nu_s$ oscillations characterized by $\Delta m^2 \gg 10^{-4} \text{ eV}^2$. The solar matter effects for $\Delta m^2 \gg 10^{-4} \text{ eV}^2$ are negligible and neutrinos propagate in the Sun as in vacuum. The averaging over the region of neutrino production, etc. in the indicated case renders the oscillating term in the expression for the oscillation probability, eq. (16), negligible and one effectively has $P_{\text{osc}} = 1 - 1/2 \sin^2 2\theta$ for all components of the solar neutrino flux. The Voloshin, Vysotsky, Okun [35] solar ν_e spin precession scenario also leads to the indicated type of reduction of the solar ν_e flux.

In general one has to consider two possibilities: transitions (or oscillations) into active neutrino, $\nu_e \rightarrow \nu_{\mu(\tau)}$ or $\nu_e \rightarrow \bar{\nu}_{\mu(\tau)}$, and into sterile neutrino $\nu_e \rightarrow \nu_s$. From the point of view of the analysis of the solar neutrino data currently available, there is no difference between the cases of $\nu_e \rightarrow \nu_{\mu(\tau)}$ and $\nu_e \rightarrow \bar{\nu}_{\mu(\tau)}$ transitions (or oscillations). This follows from the fact that for $E \gtrsim 7.5 \text{ MeV}$ the cross-sections $\sigma_{\nu_{\mu(\tau)}e}(E)$ and $\sigma_{\bar{\nu}_{\mu(\tau)}e}(E)$ practically coincide.

We have investigated the possibility that the solar neutrino deficit is due to a suppression of the different components of the solar neutrino flux by one and the same energy independent factor R resulting from $\nu_e \rightarrow \nu_{\mu(\tau)}$ ($\nu_e \leftrightarrow \nu_{\mu(\tau)}$) or $\nu_e \rightarrow \bar{\nu}_{\mu(\tau)}$, or from $\nu_e \rightarrow \nu_s$ ($\nu_e \leftrightarrow \nu_s$) transitions (oscillations). There are two parameters in the corresponding χ^2 -analysis: R and f_{B} . They were varied within the intervals: (0.0 – 1.0) and (0.0 – 5.0), respectively. The parameter f_{Be} was assumed to have a fixed value within the interval (9).

Our analysis showed that for $f_{\text{Be}} = 0.7; 1.0; 1.3$ a neutrino energy independent suppression of the solar neutrino flux resulting from $\nu_e \rightarrow \nu_{\mu(\tau)}$ ($\nu_e \leftrightarrow \nu_{\mu(\tau)}$) or $\nu_e \rightarrow \bar{\nu}_{\mu(\tau)}$ transitions

(oscillations) is excluded by the current solar neutrino data at 97%; 98%; 98% C.L. The regions in the $R - f_B$ plane allowed in this case at 99% C.L. ($\chi^2 \leq 9.21$) are shown in Figs. 5a – 5c. Finally, for the indicated values of f_{Be} the solar neutrino data rule out the hypothesis of constant suppression of the solar neutrino flux due to $\nu_e \rightarrow \nu_s$ ($\nu_e \leftrightarrow \nu_s$) transitions (oscillations) at 99.0%; 99.5%; 99.7% C.L.

5. CONCLUSIONS

We have shown that the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ vacuum oscillation solution of the solar neutrino problem is stable with respect to changes in the predictions for the fluxes of 8B and 7Be neutrinos. For low values of Φ_B ($f_B \cong 0.35 - 0.43$) new $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ and $\nu_e \leftrightarrow \nu_s$ oscillation solutions exist. We have discussed the physical implications of these new solutions for the future solar neutrino experiments. A second new $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillation solution has been found for values of f_B which lie within the interval $f_B \cong 0.45 - 0.65$. The current solar neutrino data exclude at 99 % C.L. the possibility of universal (energy independent) suppression of the different components of the solar neutrino flux, caused by $\nu_e \leftrightarrow \nu_s$ oscillations or $\nu_e \rightarrow \nu_s$ transitions. A similar suppression resulting from solar ν_e oscillations or transitions into an active neutrino ($\nu_e \leftrightarrow \nu_{\mu(\tau)}$, $\nu_e \rightarrow \bar{\nu}_{\mu(\tau)}$) is strongly disfavoured by the data: depending on the value of f_{Be} it is excluded at 97%–98% C.L.

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REFERENCES

- [1] B. Pontecorvo, *Zh. Eksp. Teor. Fiz.* 33 (1957) 549, 34 (1958) 247, and 53 (1967) 1717; Z. Maki, M. Nakagawa and S. Sakata, *Prog. Theor. Phys.* 28 (1962) 870.
- [2] P.I. Krastev and S.T. Petcov, *Phys. Lett.* 285B (1992) 85, *ibid.* 299B (1993) 99; reported by S.T. Petcov in *Proc. of the 5th Int. Workshop on Neutrino Telescopes*, March 2–4, 1993, Venice (ed. M. Baldo Ceolin), p. 23.
- [3] L.M. Krauss, E. Gates and M. White, *Phys. Lett.* 299B (1993) 95; V. Barger, R.J.N. Phillips, and K. Whisnant, *Phys. Rev. Lett.* 69 (1992) 3135, and *Phys. Rev. D* 43 (1991) 1110; A. Acker, S. Pakvasa and J. Pantaleone, *Phys. Rev. D* 43 (1991) 1754; see also: S.M. Bilenky and B. Pontecorvo, *Phys. Rep.* 41 (1978) 225, and S.L. Glashow and L.M. Krauss, *Phys. Lett.* 190B (1987) 199.
- [4] P.I. Krastev and S.T. Petcov, *Phys. Rev. Lett.* 72 (1994) 1960, and *Proc. of the 6th Int. Workshop “Neutrino Telescopes”*, Venice, February 22–24, 1994 (ed. M. Baldo Ceolin), p. 277.
- [5] R. Davis, *Prog. Part. Nucl. Phys.* 32 (1994) 13; *Proc. of the 6th Int. Workshop “Neutrino Telescopes”*, February 22–24, 1994, Venice (ed. M. Baldo Ceolin), p. 165; B.T. Cleveland et al., *Nucl. Phys. B (Proc. Suppl.)* 38 (1995) 47.
- [6] J.N. Bahcall et al., *Rev. Mod. Phys.* 54 (1982) 767; J.N. Bahcall and R.K. Ulrich, *Rev. Mod. Phys.* 60 (1982) 297; see also: J.N. Bahcall, “*Neutrino Astrophysics*”, Cambridge University Press, Cambridge, 1989.
- [7] J.N. Bahcall and M. Pinsonneault, *Rev. Mod. Phys.* 64 (1992) 85.
- [8] S. Turck-Chièze and I. Lopes, *Ap. J.* 408 (1993) 347.
- [9] V. Castellani, S. Degl’Innocenti and G. Fiorentini, *Astron. Astrophys.* 271 (1993) 601.
- [10] G. Berthomieu et al., *Astron. Astrophys.* 268 (1993) 775.

- [11] C. Proffitt, *Ap. J.* 425 (1994) 849; J.N. Bahcall and M. Pinsonneault, preprint IASSNS-AST-95/24, May 1995.
- [12] A. Dar and G. Shaviv, *Nucl. Phys. B (Proc. Suppl.)* 38 (1995) 81, and private communication by A. Dar.
- [13] U. De Angelis, *Proc. of the 6th Int. Workshop "Neutrino Telescopes"*, Venice, February 22–24, 1994 (ed. M. Baldo Ceolin), p. 311; V.N. Tsytovich, Talk given at the Int. Workshop "Solar Neutrino Problem: Astrophysics or Oscillations?", 28 February – 1 March, 1994, Gran Sasso Nat. Lab., Italy.
- [14] K. Hirata et al. (Kamiokande II Collaboration), *Phys. Rev. D*41 (1991) 2222; Y. Suzuki, *Proc. of the 6th Int. Workshop "Neutrino Telescopes"*, Venice, February 22–24, 1994 (ed. M. Baldo Ceolin), p. 197.
- [15] P. Anselmann et al. (GALLEX Collaboration), *Phys. Lett.* 357B (1995) 237.
- [16] J.N. Abdurashitov et al. (SAGE Collaboration), *Phys. Lett.* 328B (1994) 234; G. Nico et al., Talk given at the XXVII Int. Conf. on High Energy Physics, July 20 – 27, 1994, Glasgow.
- [17] J.N. Bahcall and H.A. Bethe, *Phys. Rev. D*47 (1993) 1298; see also *Phys. Rev. Lett.* 65 (1990) 2233.
- [18] V. Castellani et al., *Astron. Astrophys.* 271 (1993) 601, *Phys. Lett.* 324B (1994) 425, and 329B (1994) 525; N. Hata, S.A. Bludman and P. Langacker, *Phys. Rev. D*49 (1994) 3622; A.Yu. Smirnov, Inst. for Nuclear Theory preprint INT 94–13–01, Seattle, April 1994; V.S. Berezinsky, *Comm. Nucl. Part. Phys.* 21 (1994) 249; X. Shi, D.N. Schramm and D.S.P. Dearborn, *Phys. Rev. D*50 (1994) 2414; W. Kwong and S.P. Rosen, *Phys. Rev. Lett.* 73 (1994) 369; J.N. Bahcall, *Phys. Lett.* 338B (1994) 276; S. Degl’Innocenti, G. Fiorentini and M. Lissia, report INFNFE–10–94, August 1994.
- [19] P. Anselmann et al. (GALLEX Collaboration), *Phys. Lett.* 342B (1995) 440.

- [20] S.P. Mikheyev, A.Yu. Smirnov, *Sov. J. Nucl. Phys.* 42 (1985) 913; L. Wolfenstein, *Phys. Rev. D* 17 (1978) 2369.
- [21] P.I. Krastev and A.Yu. Smirnov, *Phys. Lett.* 338B (1994) 282.
- [22] V. Berezinsky, G. Fiorentini and M. Lissia, *Phys. Lett.* 341B (1995) 38; N. Hata and P. Langacker, *Phys. Rev. D* 52 (1995) 420.
- [23] Z.G. Berezhiani and A. Rossi, *Phys. Rev. D* 51 (1995) 5229.
- [24] P.I. Krastev and S.T. Petcov, *Phys. Lett.* 299B (1993) 99; X. Shi, D.N. Schramm, and J.N. Bahcall, *Phys. Rev. Lett.* 69 (1992) 717; S.A. Bludman et al., *Phys. Rev. D* 47 (1993) 2220; L.M. Krauss, E. Gates and M. White, *Phys. Lett.* 299B (1993) 95; N. Hata and P. Langacker, *Phys. Rev. D* 48 (1993) 2937; G. Fiorentini et al., Report INFNFE-10/93; see also S.M. Bilenky and C. Giunti, *Phys. Lett.* 311B (1993) 179 and 320B (1994) 323.
- [25] P.I. Krastev and S.T. Petcov, *Nucl. Phys.* B449 (1995) 605.
- [26] P.I. Krastev and S.T. Petcov, preprint SISSA 89/95/EP.
- [27] M. Spiro and D. Vignaud, *Phys. Lett.* 242B (1990) 279.
- [28] S.T. Petcov, *Nucl. Phys. B (Proc. Suppl.)* 43 (1995) 12.
- [29] R.S Raghavan, *Science* 267 (1995) 45.
- [30] G. Ewan et al., Sudbury Neutrino Observatory Proposal, SNO-87-12, 1987.
- [31] T. Kajita, *Physics with the Super Kamiokande Detector*, ICRR Report 185-89-2, 1989.
- [32] S.T. Petcov and J. Rich, *Phys. Lett.* 224B (1989) 426.
- [33] C. Arpesella et al., BOREXINO proposal (eds. G. Bellini, R. Raghavan et al., Univ. of Milano, Milano 1992), vols. 1 and 2.
- [34] G. Laurenti et al., "HELLAZ, a Solar Neutrino Experiment with Neutrino Energy Res-

olution”, Proc. of the 5th Int. Workshop on Neutrino Telescopes, March 2–4, 1993, Venice (ed. M. Baldo Ceolin), p. 161.

[35] M.B. Voloshin and M.I. Vysotsky, Sov. J. Nucl. Phys. 44 (1986) 845; M.B. Voloshin, M.I. Vysotsky and L.B. Okun, Sov. Phys. JETP 64 (1988) 1368.

Figure Captions

Figs. 1a – 1c. Regions of values of the parameters Δm^2 and $\sin^2 2\theta$ for which the solar neutrino data can be described at 95% C.L. in terms of $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations of the solar ν_e for values of $f_{Be} = 0.7$ (a), 1.0 (b), 1.3 (c), and for values of f_B from the interval (0.35 – 2.5).

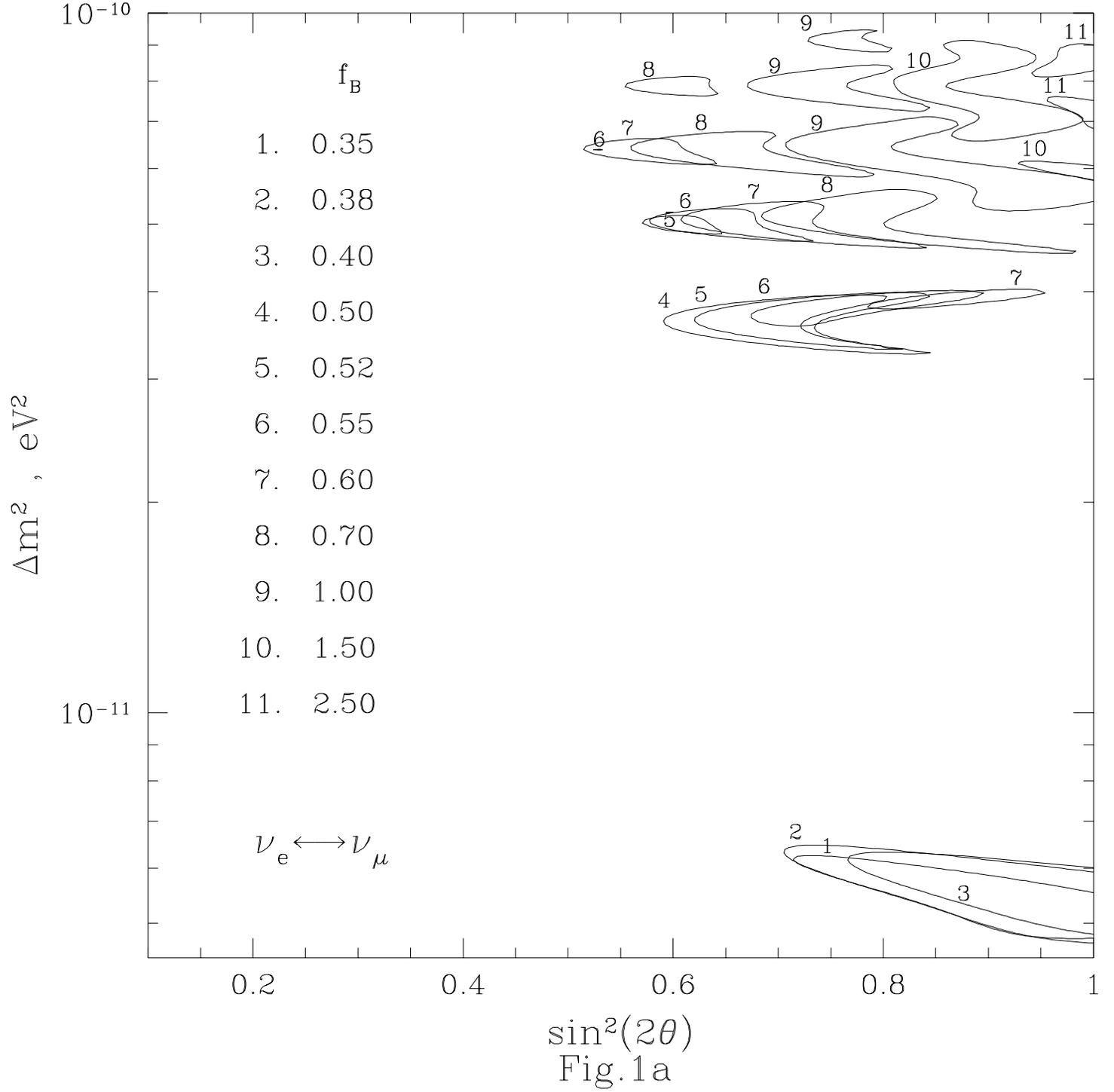
Figs. 2a – 2b. The vacuum oscillation probability for the mean distance between the Sun and the Earth, $P_{osc}(E; R_0)$ (a), and the probability (16) averaged over a period of 1 year, $\bar{P}_{osc}(E; R_0, \epsilon)$ (b), as function of the neutrino energy E for $\Delta m^2 = 10^{-10}$ eV² and $\sin^2 2\theta = 0.8$.

Figs. 3a – 3b. The deformation of the normalized to one spectrum of pp neutrinos in the cases of a) $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ and b) $\nu_e \leftrightarrow \nu_s$ oscillation solutions (14) and (23), respectively. The dotted, dashed, long-dashed, dash-dotted and long-dash-dotted lines correspond to $\Delta m^2 = (5.2; 5.4; 5.6; 5.8; 6.0) \times 10^{-12}$ eV² and $\sin^2 2\theta = 1$.

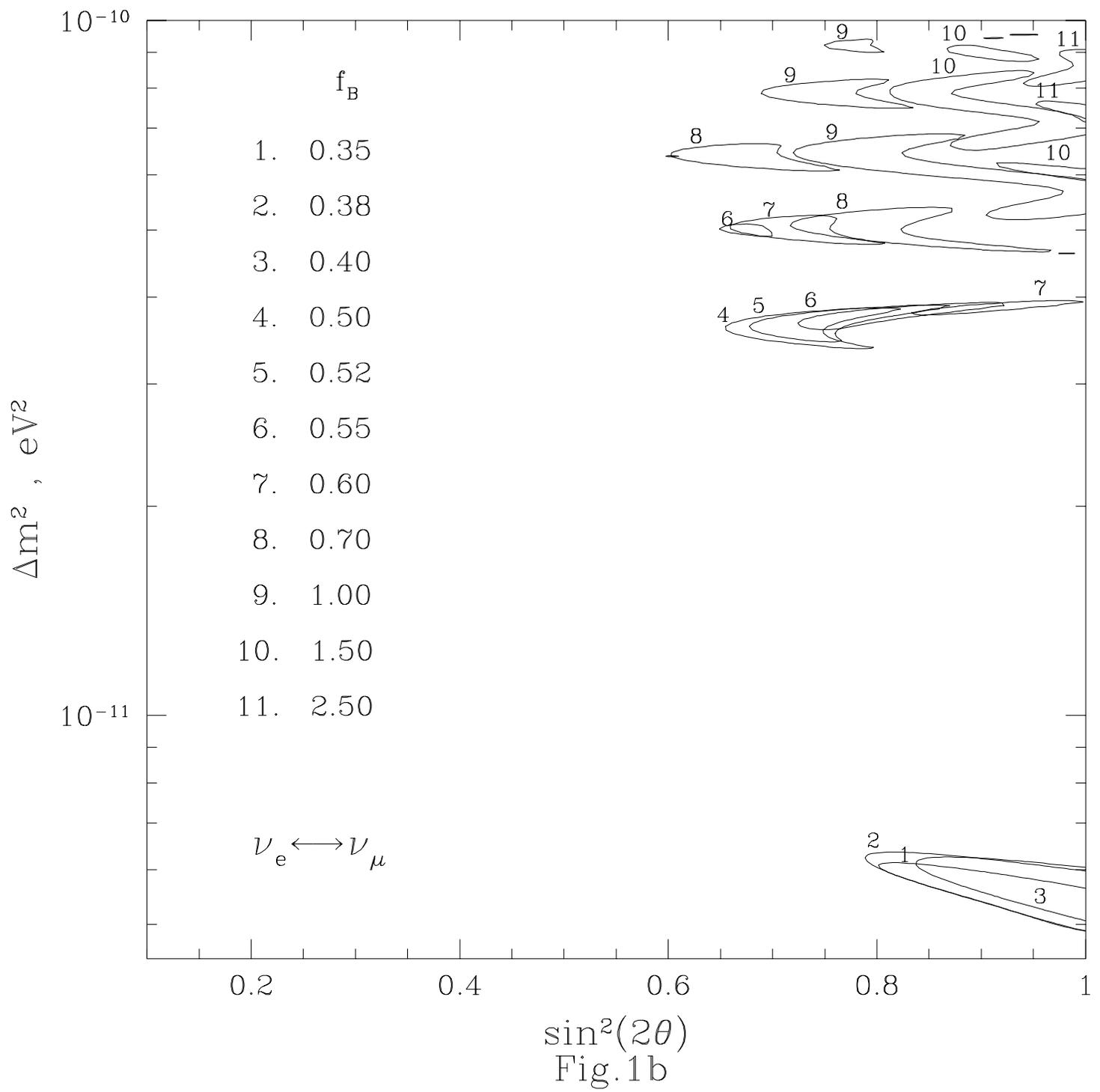
Figs. 4a – 4c. The regions of values of the parameters Δm^2 and $\sin^2 2\theta$ for which the solar neutrino data can be described at 95% C.L. (dashed lines) and 98% C.L. (solid lines) in terms of $\nu_e \leftrightarrow \nu_s$ oscillations of the solar ν_e for $f_{Be} = 0.7$ (a); 1.0 (b); 1.3 (c), and for values of f_B from the interval (0.35 - 1.5).

Figs. 5a – 5c. The regions of values of the parameters R and f_B allowed at 99% C.L. ($\chi^2 \leq 9.21$) by the solar neutrino data in the case of universal (energy independent) suppression of the different components of the solar neutrino flux by one and the same factor R , caused by vacuum oscillations or transitions of the solar neutrinos into an active neutrino ($\nu_{\mu(\tau)}$ or $\bar{\nu}_{\mu(\tau)}$) in the three cases $f_{Be} = 0.7$ (a); 1.0 (b); 1.3 (c).

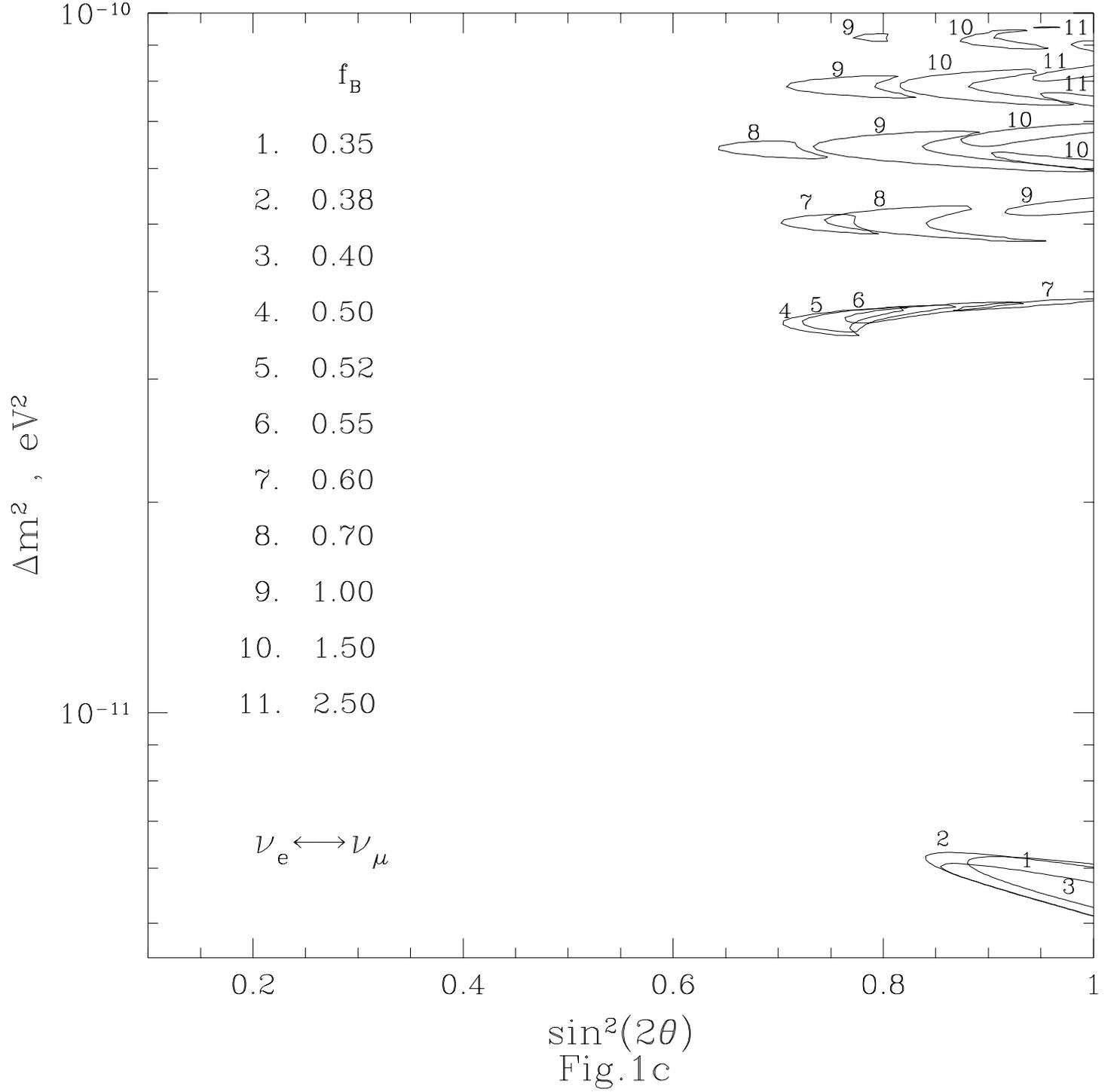
$f_{Be} = 0.7$



$f_{Be} = 1.0$



$f_{Be} = 1.3$



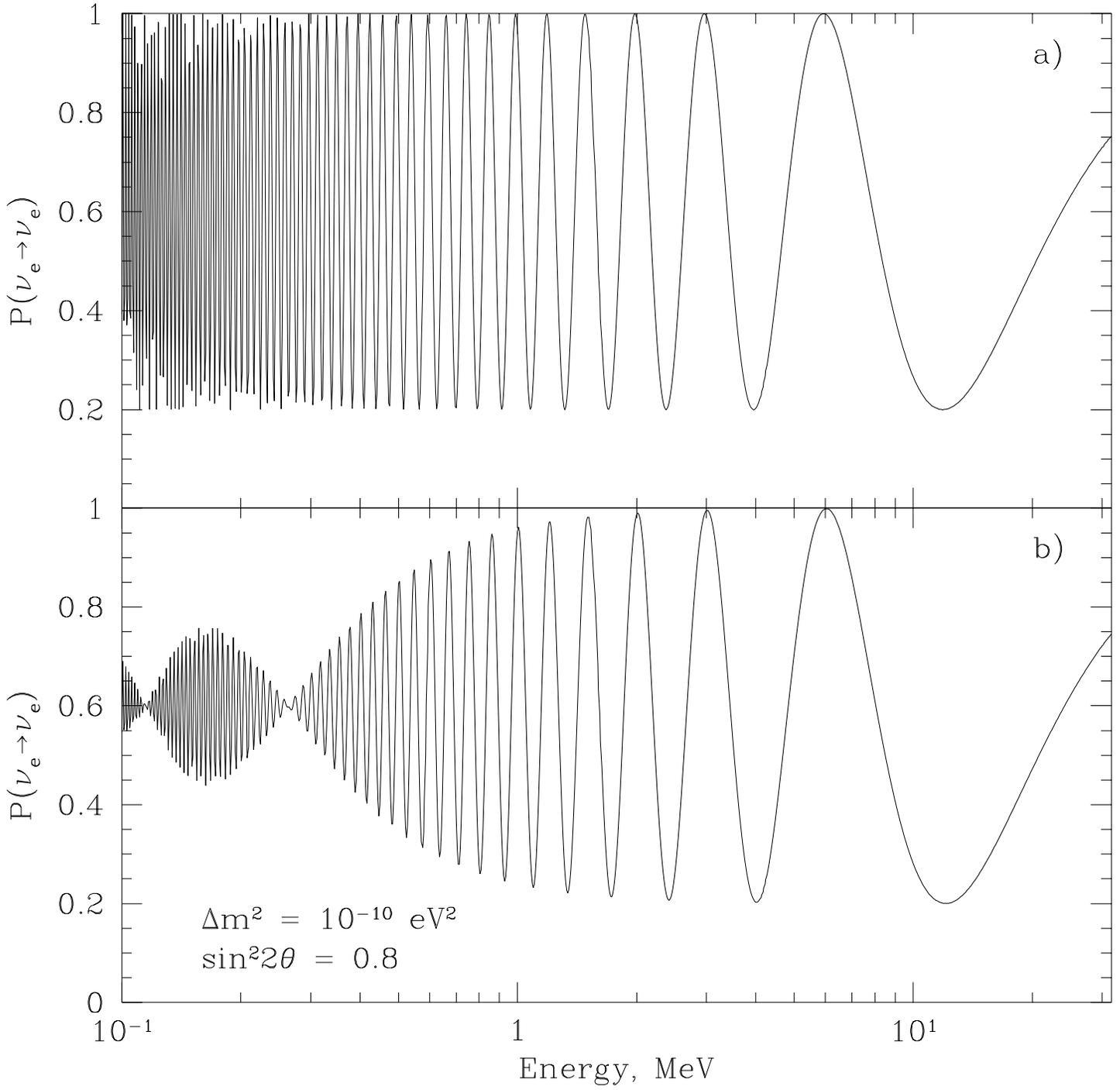


Fig.2

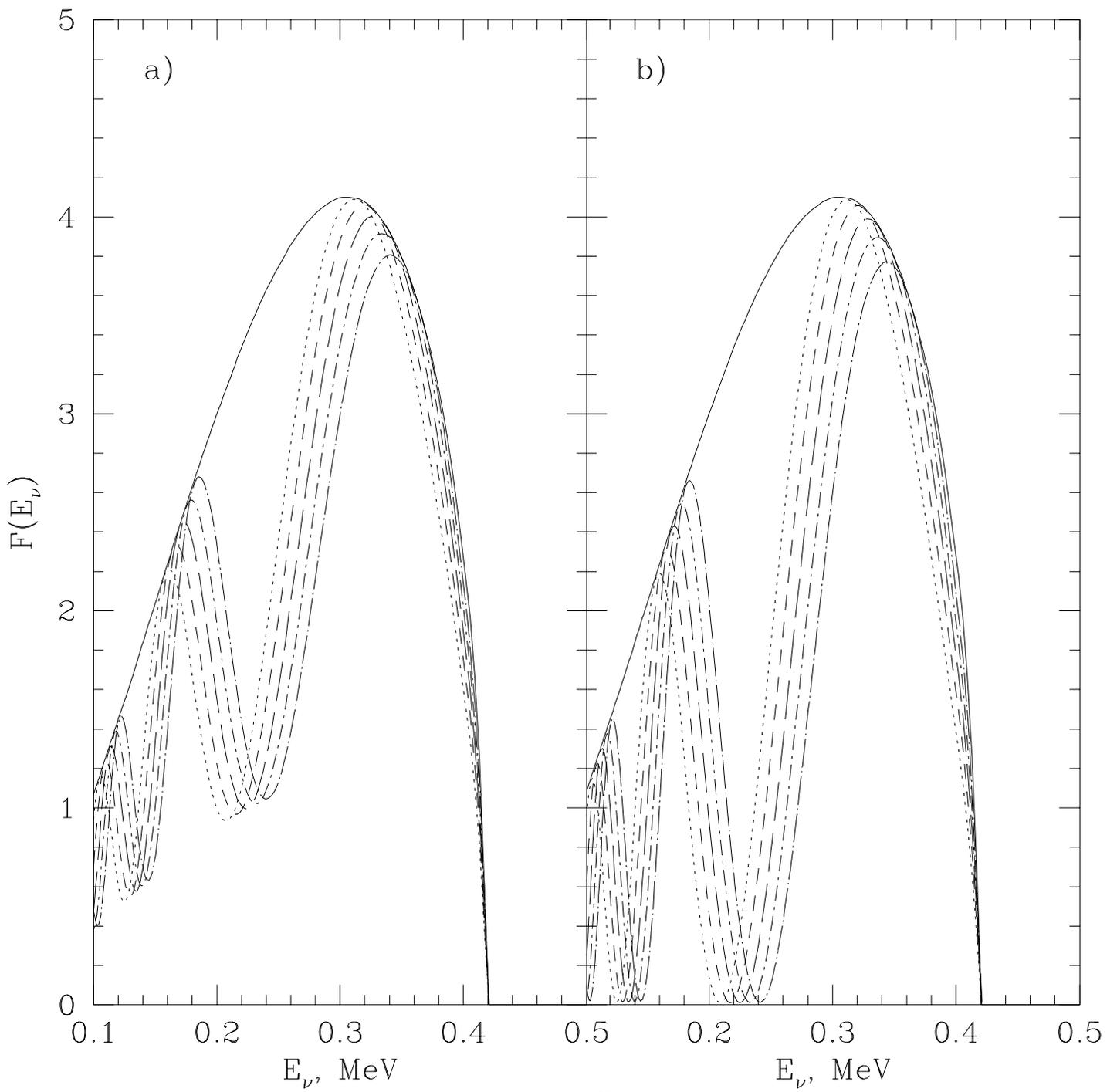
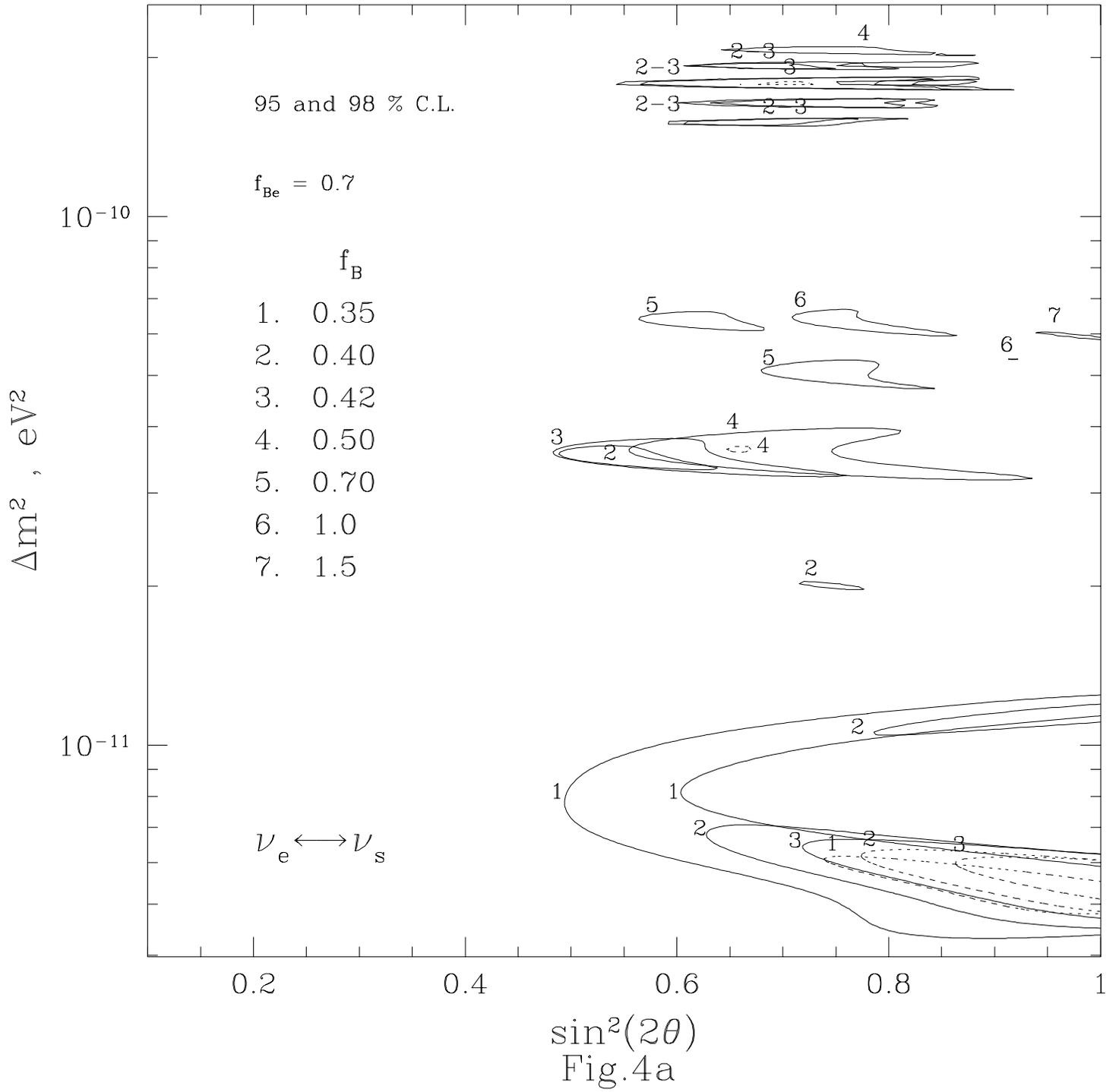
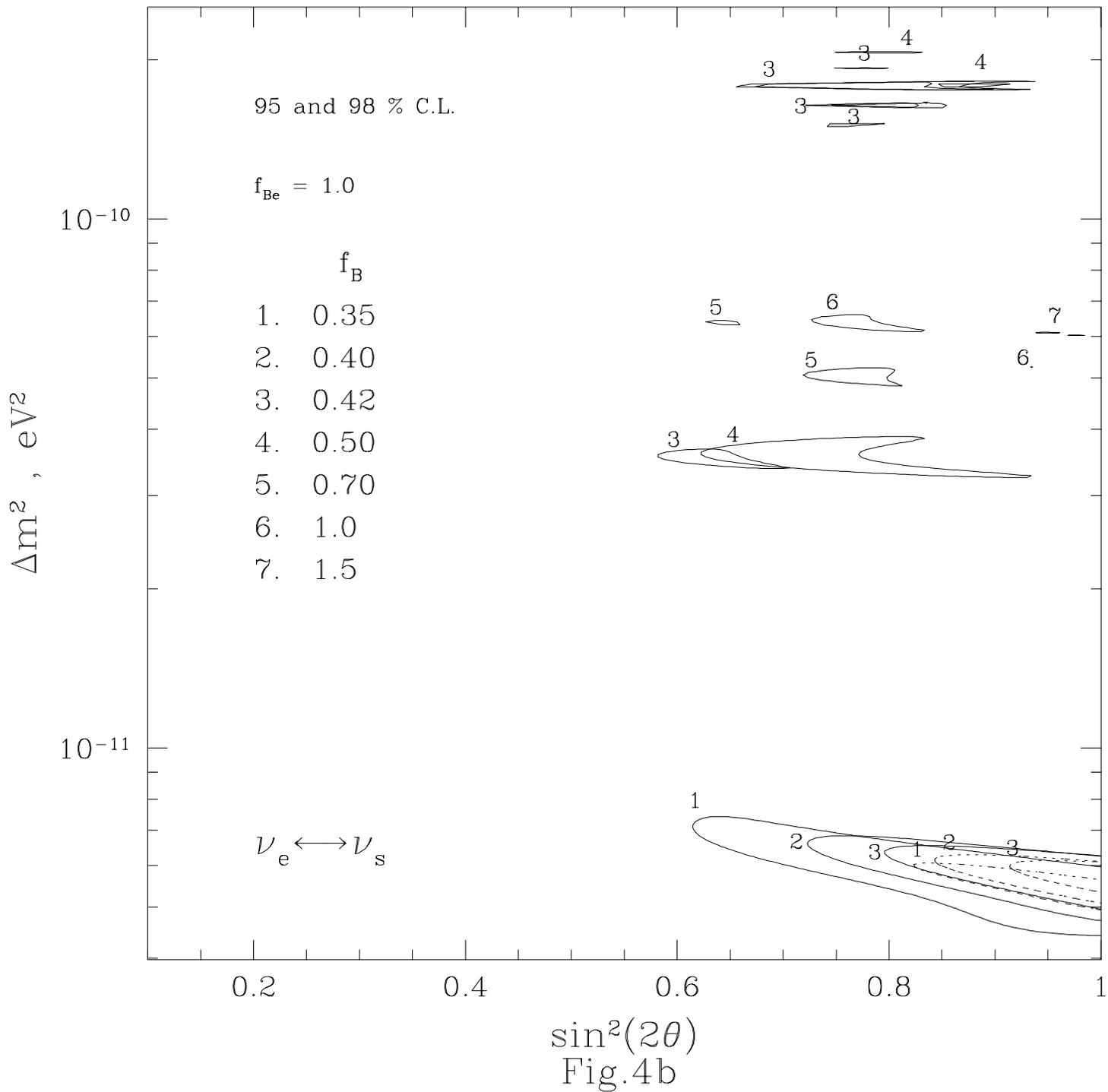
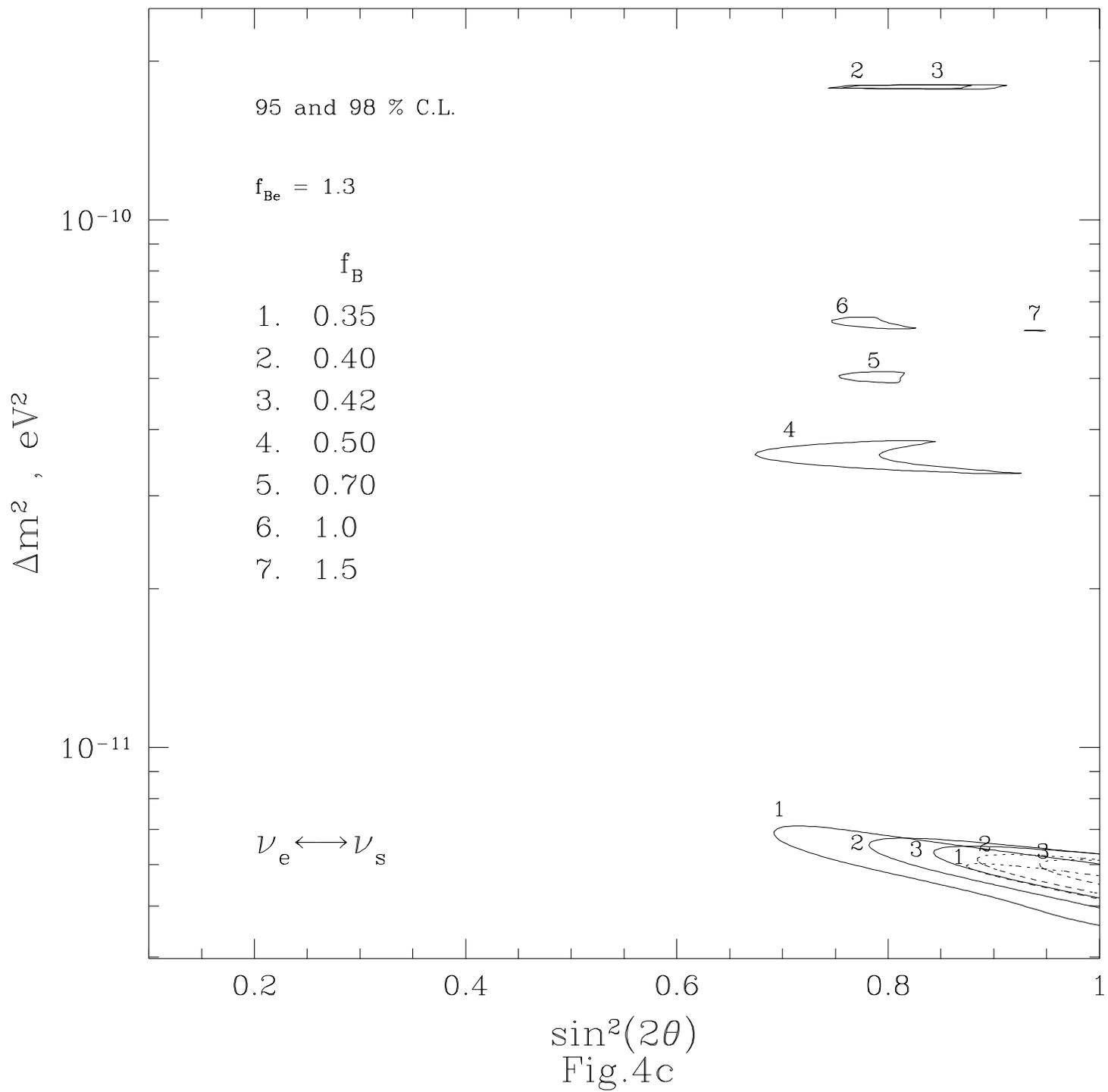


Fig.3







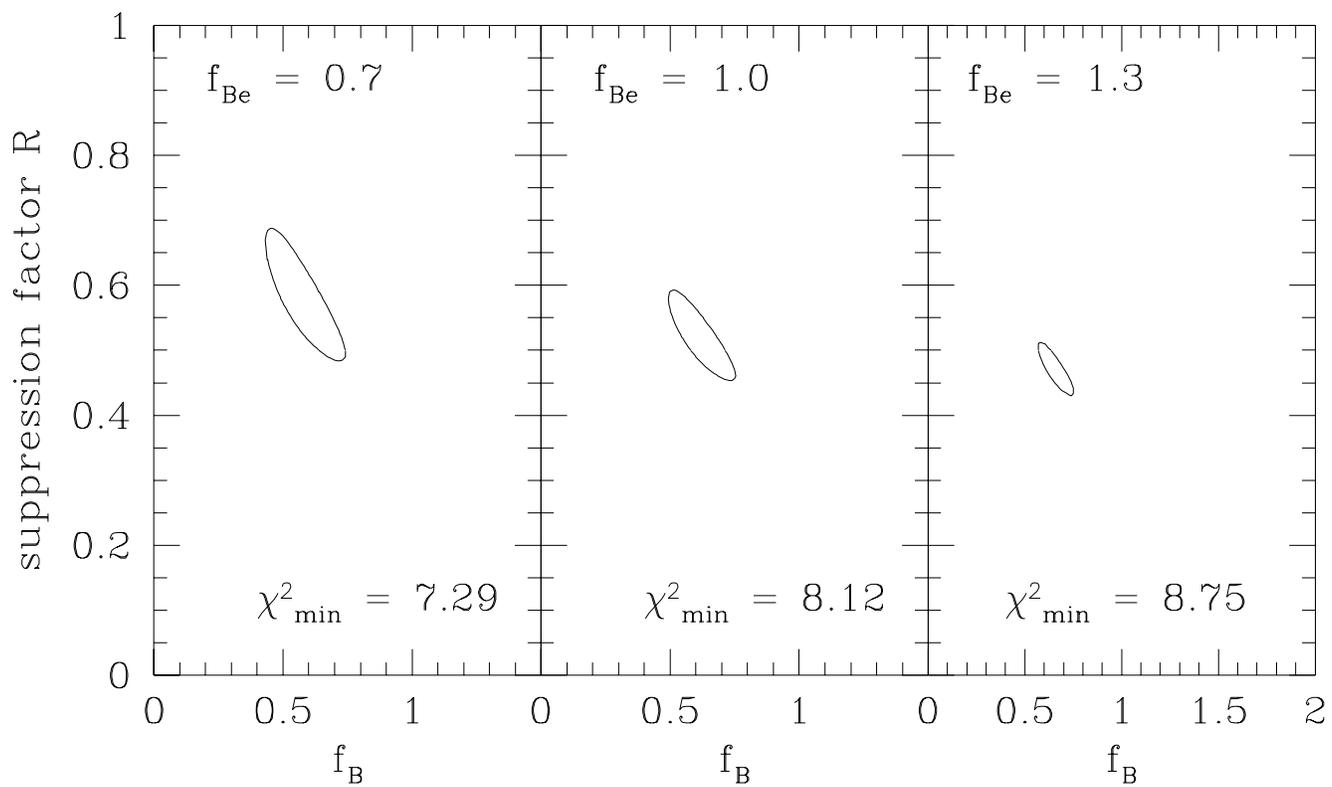


Fig.5