Simulation for Pedestrian Dynamics by Real-Coded Cellular Automata (RCA)

Kazuhiro Yamamoto1*, Satoshi Kokubo1, Katsuhiro Nishinari2

1 Dep. Mechanical Science and Engineering, Nagoya University, Japan
* kazuhiro@mech.nagoya-u.ac.jp
2 Dep. Aerospace Engineering, University of Tokyo, Japan

Abstract
In this paper, we propose a new approach for pedestrian dynamics. We call it a Real-coded Cellular Automata (RCA). The scheme is based on the Real-coded Lattice Gas (RLG), which has been developed for fluid simulation. Similar to RLG, the position and velocity can be freely given, independent of grid points. Our strategy including the procedure for updating the position of each pedestrian is explained. It is shown that the movement of pedestrians in an oblique direction to the grid is successfully simulated by RCA, which was not taken into account in the previous CA models. Moreover, from simulations of evacuation from a room with an exit of various widths, we obtain the critical number of people beyond which the clogging appears at the exit.

PACS: 45.70.Vn; 89.65.Lm; 02.70.Ns
Keywords: Pedestrian dynamics, real-coded cellular automata, crowd

1. Introduction
Since Cellular Automata (CA) have been proposed by von Neumann in the late 1940s, CA have been applied in a variety of scientific researches on complex system, including traffic models and biological fields. It is an idealization of a physical system in which space and time are all discrete. As one of the examples achieving most remarkable progress is the CA model for pedestrian dynamics. Since the pedestrian flows are caused by collective crowd behavior, it is difficult to handle directly each pedestrian by solving coupled differential equations, although the social force model has been proposed and could reproduce some basic features of pedestrian behavior [1]. CA approach could be more appropriate to describe pedestrian dynamics in complex situations because of its simplicity, flexibility and efficiency.

The floor field CA model has been developed for pedestrian dynamics [2-4], where two kinds of floor fields, a static and a dynamic one, are introduced to translate a long-ranged spatial interaction into an attractive local interaction. Different CA models have also been proposed so far to simulate bi-directional flow and clogging at the exit [5-8]. It is important for the models to reproduce known collective behaviors of
pedestrians such as lane formation in a corridor, oscillations of the direction at bottle-
necks and the so-called faster-is-slower effect in evacuation [9]. It has been con-
firmed that the social force model and floor field model successfully show all of these collective dynamics.

In our previous study, the extended floor field CA model has been presented to consider the complex room of arbitrary geometry [10]. To describe the evacuation dynamics, the static floor field is given according to the minimum path based on the visibility graph and Dijkstra’s algorithm. As seen in Fig. 1, the von Neumann neighborhood was adopted. For each pedestrian, the transition probability, \( P_{x,y} \), where \( x \) and \( y \) is a move in \( x \) and \( y \) directions, respectively. The pedestrian moved to the nearest four cells at next time step or remained at the same cell, but he could only move in four directions: forward, backward, left, and right. That is, the direction of each pedestrian movement was limited. This might be a problem if we discuss the evacuation time in detail.

Figure 2 shows the example of evacuation toward the exit. We consider two paths of A and B. Needless to say, the distance of path B is much shorter than that of path A in real situation (see left figure), because there are no grids and people can take any paths. Since there are grids in the CA simulation (see right figure), both are the same distance. Therefore, if we count the evacuation time in CA model, the oblique four directions in Fig. 1 may be needed as well. However, it should be noted that, because of the longer movement within one time step, the allowance of movement toward the oblique neighbor cells corresponds to the faster motion of the pedestrian, which may also give unrealistic solution. This is one of the serious common problems in all CA models proposed so far. To improve the model, it is better to consider any direction and any velocity of pedestrian movement.

![Fig. 1 Target cells for a person at the next time step.](image-url)
In the present paper, we propose the real-coded cellular automata (RCA) as a new numerical model for pedestrian dynamics. The position and the velocity can be freely given, independent of grid points. The procedure of updating rule for each pedestrian is explained in the next section.

2. Numerical procedure of RCA

Here, we explain our new approach for arbitrary velocity and directions for pedestrian dynamics. It is based on the Real-coded Lattice Gas (RLG), which has been developed for fluid simulation [11,12]. In RLG model, similar to the Lattice Gas Automata [13,14], the particles are used for modeling fluid as a fully discrete molecular dynamics. The main difference is that the particles have continuous velocity distributions to show Maxwell-Boltzmann distribution in the equilibrium state. Furthermore, collision and streaming schemes do not depend on the explicit lattice structure in the discrete space. That is, the particle of lattice gas has real number in the velocity, and travel to any direction. We apply this scheme to the CA model for pedestrian dynamics. We call it Real-coded Cellular Automata (RCA). The numerical procedure is explained briefly.

The update rule of RCA consists of 3 steps, and the position of the pedestrian is renewed. The update rules are applied to each pedestrian randomly. The unit discrete time step of $\Delta t$ is used, and the space is discretized with grids. The grid is square and its length is $\Delta$. Here, it is assumed that the pedestrian moves toward the target, for example, the exit in Fig.2.

1) First, the streaming process is performed to move the pedestrian position by its moving velocity. It can be described simply as the sum of position and velocity vectors of pedestrian $i$,

$$\mathbf{x}'_i = \mathbf{x}_i + \mathbf{v}_i \quad (1)$$
where \(x'_i\) and \(x_i\) are the post- and pre-streaming position for the pedestrian \(i\), and \(v_i\) is its moving velocity. In this method, \(v_i\) can be arbitrary velocity and \(x'_j\) is not on at the grid at this stage. Then, as shown in Eq.2, the velocity components in \(x\)- and \(y\)-directions are divided into two parts of \([v_i]\) and \(\{v_i\}\); the former is the integer part corresponding to grid number and the latter is the decimal part less than the grid length.

\[
\begin{align*}
v_{x,i} &= [v_{x,i}] \Delta_x / \Delta_i + \{v_{x,i}\} \Delta_x / \Delta_i \\
v_{y,i} &= [v_{y,i}] \Delta_y / \Delta_i + \{v_{y,i}\} \Delta_y / \Delta_i
\end{align*}
\]  
\tag{2}

2) To keep the pedestrian position right on the grid point, the pedestrian is repositioned on the grid point. This procedure is shown in Fig. 3. There are four candidates, points A, B, C, and D. Which one is selected is stochastically determined by each probability. As shown in Eqs.3-6, the probability of movement to each point is \(p_A\), \(p_B\), \(p_C\), \(p_D\), respectively.

\[
\begin{align*}
p_A &= \{v_{x,i}\} \{v_{y,i}\} \\
p_B &= (1 - \{v_{x,i}\}) \{v_{y,i}\} \\
p_C &= \{v_{x,i}\} (1 - \{v_{y,i}\}) \\
p_D &= (1 - \{v_{x,i}\}) (1 - \{v_{y,i}\})
\end{align*}
\]  
\tag{3-6}

Needless to say, the sum of these values is 1. However, this is not the final position. The next third step is needed to avoid the collision between pedestrians.

![Fig. 3 The position and movement of the pedestrian at the step 2](image)

3) The third step is needed only when the pedestrian attempt to move to the grid point where someone already exists. In this case, he remains at the pre-streaming position.
Instead, he changes the angle of $+45^\circ$ or $-45^\circ$. The choice of $+45^\circ$ or $-45^\circ$ is determined to make the pedestrian face the grid where nobody stays, which corresponds to our natural behavior when we try to avoid instantly the collision during walking or running. If the pedestrian may hit the wall, he also changes the direction. It could be a corner in the corridor when people evacuate in the building [4].

In the above rule of RCA model, the pedestrian only change the direction and always keeps the same magnitude of velocity. In the next section, some results are shown to demonstrate the capability of RCA.

3. Results

3.1 Movement along straight line

First, to demonstrate the pedestrian motion by RCA model, benchmark simulation is conducted. Here, the simple motion of the pedestrian along the line is simulated. Figure 4 shows the calculation domain. In each test run, one pedestrian starts to leave from the corner grid. He moves towards to the point P along the straight line inclined to the $x$- or $y$-axis. The calculation domain is $16\text{m} \times 20\text{m}$, and the length of the line pedestrian walks, $L$, is 20 m. The grid size is 0.4 m and the time step is 0.5 s. These values are referred to Ref. 2. He keeps walking at the speed of $V = 1.3\text{ m/s}$, and his inclined angle of $\theta$ does not change. When one arrives at the end, the next test is conducted, so that the number of pedestrian in the calculation domain is always unity and no collisions between pedestrians occur. We conduct 10,000 test runs and record the time when each pedestrian arrives at the end.

Figure 5 shows the arrival time in the test. The number of people is counted to obtain the histogram. It is found that the profile is similar to the normal distribution, because the position of the pedestrian is stochastically determined. The averaged value is 15.6 s. This value is very close to the estimated time of 15.4 ($= L / V$). Therefore we have successfully solved the inclined-path problem by our RCA.

![Fig. 4 Calculation domain.](image-url)
3.2 Evacuation in a large room

Next, we conduct the evacuation simulation. Figure 6 shows the typical snapshot of evacuation in a large room. The initial number of people is 300. The calculation domain is $16\text{m} \times 16\text{m}$, and the grid length of $\Delta$ is 0.4 m. The time step of $\Delta t$ is 0.5 s. Their initial positions are randomly given, and they start to evacuate towards the exit (e.g. in case of fire). The velocity is 1.3 m/s, and exit width is 0.8 m. As seen in Ref. 3, three stages are observed: (a) beginning ($t = 0.5$ s), (b) middle ($t = 25$ s), and (c) final stages ($t = 52.5$ s). Initially, the pedestrian can pass the exit smoothly. In the middle stage, the bottleneck is thoroughly formed around the exit. This situation is automatically formed since the balance of inflow and outflow of pedestrians at the exit breaks.

When the number of people is relatively small, the evacuation process is smooth and there are no jams all over the calculation domain. However, as seen in Fig. 6, when relatively large number of people evacuate, people becomes less flexible. To examine further, we obtain the correlation between the number of people in the room and the total evacuation time. Figure 7 shows the total evacuation time as functions of the initial number of people. From this figure, two regions are observed. In the region 1, the total evacuation time is constant even if the number of people in the room is increased. By checking the time-dependent evacuation dynamics, no bottlenecks are formed.

Fig. 6 Evacuation simulation in a large room with one exit. Three typical stages are shown; (a) $t = 0.5$ s, (b) $t = 25$ s, and (c) $t = 52.5$ s.
On the other hand, in the region 2, as the initial number of people is increased, more time is needed to evacuate all people in the room. That is, the evacuation time depends on the initial number of people in this region. Expectedly, the bottleneck is observed. Thus, depending on the initial number of people, the drastic change of the pedestrian dynamics appears through the formation of bottleneck. By changing the exit door width, $W$, we examine this critical number of people where the bottleneck process appears, defined as $N_c$.

Figure 8 shows the critical number of people in the above room size as functions of exit door width. As seen in this figure, as the exit door width is larger, $N_c$ is larger. That is, more people in the room are needed to observe the bottleneck process. The curve in Fig. 8 could be changed when the room size is different. Although more benchmark studies are needed, our proposed RCA model could be a good tool to examine the evacuation dynamics, especially to count the evacuation time in the crowds.

4. Conclusions
We have presented the real-coded cellular automata (RCA) as a new numerical model for pedestrian dynamics. The approach is originally based on the real-coded lattice gas (RLG). The procedure for updating the position of each pedestrian is explained. As the benchmark study, the movement along the straight line is simulated. This situation is rather simple, but in the previous CA models, the correct evacuation time is hard to be obtained, because the movement in oblique direction is not considered. In our model, the pedestrian movement at any direction is given, and the reasonable evacuation time is calculated. In the simulation of evacuation in a large room, the so-called bottleneck is observed at the exit. We examine the critical number of people causing the bottleneck process, $N_c$. It is found that when the exit door width is larger, more people in the room are needed. To predict $N_c$ by changing the exit door width as well as the room size, this simulation is needed to construct the safety standards. We conclude that our proposed RCA can be a good tool to examine the pedestrian dynamics.

References