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ON GRAVITATIONAL INTERACTION OF SPIN 3/2 NAMBU-GOLDSTONE FERMION

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Abstract

A new gravitational interaction of spin 3/2 Nambu-Goldstone(N-G) fermion is constructed, which gives a new framework for the consistent gravitational coupling of spin 3/2 massless field. The action is invariant under a new global supersymmetry.

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The supersymmetry(SUSY)[1][2][3] is an essential notion to unify spacetime and matter. However it is totally unrealistic symmetry so far in the observed low energy particle physics and should be broken spontaneously. It is well understood that the Nambu-Goldstone(N-G) fermion with spin 1/2 would appear in the spontaneous breakdown of SUSY and that it can be converted to the longitudunal components of the spin 3/2 field(gravitino) through the superHiggs mechanism. This is demonstrated explicitly by the introduction of the local gauge coupling of Volkov-Akulov(V-A) model[2] of a nonlinear realization of SUSY(NL SUSY)to the supergravity(SUGRA) gauge multiplet[4].

In ref.[5] and [6], we have proposed a supersymmetric composite unified model for spacetime and matter, superon-graviton model(SGM) based upon SO(10) super-Poincaré algebra, where we have regarded the spin 1/2 N-G fermions of V-A NL SUSY[2] as the fundamental objects(superon-quintet) for matter. SGM may be the most economical model that accomodates all observed particles in a single irreducible representation of a (semi)simple group. The NL SUSY may give a framework to describe the unity of nature from the compositeness viewpoint for matter. In SGM all particles participating in (super)Higgs mechanism except graviton are composites of N-G fermions, superons. In ref.[6], we have constructed the gauge invariant SGM action and clarified the systematics in the unified model building.

In thie letter we extend the framework[6] to N-G fermion with the higher spin. Following the arguments of V-A, the action of N-G fermion $\psi^\mu_\alpha(x)$ with spin 3/2 is already written down by Baaklini as a nonlinear realization of a new superalgebra containing a vector-spinor generator Q^μ_α[7]. We study in detail the gravitational interaction of Baaklini model[7]. We will see that the similar arguments to SGM can be performed and produce a new gauge invariant action, which is the straightforward generalization of SGM action. The phenomenological implications of spin 3/2 fundamental constituents are discussed briefly.

In ref.[7], a new SUSY algebra containing a spinor-vector generator Q^μ_α is introduced as follows:

$$\{Q^\mu_\alpha, Q^\nu_\beta\} = \varepsilon^{\mu\nu\lambda\rho} P_\lambda (\gamma_\rho \gamma_5 C)_{\alpha\beta}, \tag{1}$$

$$[Q^\mu_\alpha, P^\nu] = 0, \tag{2}$$

$$[Q^\mu_\alpha, J^{\lambda\rho}] = \frac{1}{2}(\sigma^{\lambda\rho} Q^\mu)_\alpha + i\eta^{\lambda\mu} Q^\rho_\alpha - i\eta^{\rho\mu} Q^\lambda_\alpha, \tag{3}$$

where Q^μ_α are vector-spinor generators satisfying Majorana condition $Q^\mu_\alpha = C_{\alpha\beta}\overline{Q}^\mu_\alpha$, C is a charge conjugation matrix and $\frac{1}{2}\{\gamma^\mu, \gamma^\nu\} = \eta^{\mu\nu} = (+,-,-,-)$. By extending the arguments of V-A model of NL SUSY[2], they obtain the following action as the nonlinear representation of the new SUSY algabra.

$$S = \frac{1}{\kappa}\int \omega_0 \wedge \omega_1 \wedge \omega_2 \wedge \omega_3 = \frac{1}{\kappa}\int \det w_{ab} d^4x, \tag{4}$$

$$w_{ab} = \delta_{ab} + t_{ab}, \quad t_{ab} = ia\varepsilon_{acde}\bar{\psi}^c\gamma^d\gamma_5\partial_b\psi^e, \tag{5}$$

where κ and a are up to now arbitrary constants with the dimension of the fourth power of length(i.e., a fundamental volume of spacetime) and ω_a is the following differential forms

$$\omega_a = dx_a + ia\varepsilon_{abcd}\bar{\psi}^b\gamma^c\gamma_5 d\psi^d, \tag{6}$$

which is invariant under the following (super)translations

$$\psi^a_\alpha \longrightarrow \psi^a_\alpha + \zeta^a_\alpha, \tag{7}$$

$$x_a \longrightarrow x_a + ia\varepsilon_{abcd}\bar{\psi}^b\gamma^c\gamma_5\zeta^d, \tag{8}$$

where ζ^a_α is a constant Majorana tensor-spinor parameter.

Now we consider the gravitational interaction of Baaklini model(4). We show that the arguments performed in SGM[6] of spin 1/2 N-G field $\psi_\alpha(x)$ can be extended straightforwardly to spin 3/2 Majorana N-G field ψ^a_α. In the present case, as seen in (7) and (8) NL SUSY SL(2C) degrees of freedom (i.e. the coset space coordinates ψ^a_α representing N-G fermions) in addition to Lorentz SO(3,1) coordinates are embedded at every curved spacetime point with GL(4R) invariance.
Following the arguments of SGM[6], it is natural to introduce formally a new vierbein field $w^a{}_\mu(x)$ through the NL SUSY invariant differential forms ω_a in (6) as follows:

$$\omega^a = w^a{}_\mu dx^\mu, \tag{9}$$

$$w^a{}_\mu(x) = e^a{}_\mu(x) + t^a{}_\mu(x), \quad t^a{}_\mu(x) = ia\varepsilon^{abcd}\bar{\psi}_b\gamma_c\gamma_5\partial_\mu\psi_d, \tag{10}$$

where $e^a{}_\mu(x)$ is the vierbein of Einstein Genaral Relativity Theory(EGRT) and Latin $(a, b, ..)$ and Greek $(\mu, \nu, ..)$ are the indices for local Lorentz and general coordinates, respectively. By noting $(\psi^\mu_\alpha(x))^2 = 0$, we can easily obtain the inverse of the new vierbein, $w_a{}^\mu(x)$, in the power series of $t^a{}_\mu$ which terminates with $(t)^4$:

$$w_a{}^\mu = e_a{}^\mu - t^\mu{}_a + t^\rho{}_a t^\mu{}_\rho - \cdots. \tag{11}$$

Similarly we introduce formally a new metric tensor $s^{\mu\nu}(x)$ in the abovementioned curved spacetime as follows:

$$s^{\mu\nu}(x) \equiv w_a{}^\mu(x) w^{a\nu}(x). \tag{12}$$

It is easy to show $w_a{}^\mu w_{b\mu} = \eta_{ab}$, $s_{\mu\nu}w_a{}^\mu w_b{}^\mu = \eta_{ab}$, ..etc. In order to obtain simply the action in the abovementioned curved spacetime, which is invariant at least under GL(4R), NL SUSY and local Lorentz transformations, we follow formally EGRT as performed in SGM[6]. That is, we require that the (mimic) vierbein $w^a{}_\mu(x)$ and the metric $s^{\mu\nu}(x)$ should have formally a general coordinate transformations under the supertranslations:

$$\delta x_\mu = -\xi_\mu, \quad \delta\psi^a = \zeta^a, \tag{13}$$

wher where $\xi^\mu = -ia\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\nu\gamma_\rho\gamma_5\zeta_\sigma$.
Remarkably we find that the following nonlinear transformations

$$\delta\psi^a(x) = \zeta^a - ia(\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\nu\gamma_\rho\gamma_5\zeta_\sigma)\partial_\mu\psi^a \tag{14}$$

$$\delta e^a{}_\mu(x) = ia(\varepsilon^{\rho\nu\sigma\lambda}\bar{\psi}_\nu\gamma_\sigma\gamma_5\zeta_\lambda)D_{[\mu}e^a{}_{\rho]} \tag{15}$$

induce the desirable transformations on $w^a{}_\mu(x)$ and $s^{\mu\nu}(x)$ as follows:

$$\delta_{\zeta_1} w^a{}_\mu = \xi_1^\nu\partial_\nu w^a{}_\mu + \partial_\mu\xi_1^\nu w^a{}_\nu, \tag{16}$$

$$\delta_{\zeta_1} s_{\mu\nu} = \xi_1^\kappa\partial_\kappa s_{\mu\nu} + \partial_\mu\xi_1^\kappa s_{\kappa\nu} + \partial_\nu\xi_1^\kappa s_{\mu\kappa}, \tag{17}$$

where also throughout the paper D_μ is the covariant derivative of GL(4,R) with the symmetric affine connection and $\xi_{\zeta_1}^\rho = -ia\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\nu\gamma_\rho\gamma_5\zeta_{1\sigma}$. That is, $w^a{}_\mu(x)$ and $s^{\mu\nu}(x)$ have general coordinate transformations under the new supertransformations (14) and (15).

Therefore replacing $e^a{}_\mu(x)$ in Einstein-Hilbert Lagrangian of general relativity by the new vierbein $w^a{}_\mu(x)$ we obtain the following Lagrangian which is invariant under (14) and (15):

$$L = -\frac{c^3}{16\pi G}|w|(\Omega + \Lambda), \tag{18}$$

$$|w| = det w^a{}_\mu = det(e^a{}_\mu + t^a{}_\mu), \tag{19}$$

where the overall factor is now fixed uniquely to $\frac{-c^3}{16\pi G}$, $e_a{}^\mu(x)$ is the vierbein of EGRT and Λ is a probable cosmological constant. Ω is a mimic new scalar curvature analogous to the Ricci scalar curvature R of EGRT. The explicit expression of Ω is obtained by just replacing $e_a{}^\mu(x)$ in Ricci scalar R of EGRT by $w_a{}^\mu(x) = e^a{}_\mu + t^a{}_\mu$, which gives the gravitational interaction of $\psi^a_\alpha(x)$. The lowest order term of a in the action (11) gives the Einstein-Hilbert action of general relativity. And in flat spacetime, i.e. $e_a{}^\mu(x) \to \delta_a{}^\mu$, the action (11) reduces to V-A model[2] with $\kappa^{-1} = \frac{c^3}{16\pi G}\Lambda$. Therefore our model predicts a non-zero (small) cosmological constant.
The commutators of two new supersymmetry transformations on $\psi^a_\alpha(x)$ and $e_a{}^\mu(x)$ are

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi^a = \{2ia(\varepsilon^{\mu bcd}\bar{\zeta}_{2b}\gamma_c\gamma_5\zeta_{1d}) - \xi_1^\rho\xi_2^\sigma e_a{}^\mu(D_{[\rho}e^a{}_{\sigma]})\}\partial_\mu\psi^a, \tag{20}$$

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = \{2ia(\varepsilon^{\rho bcd}\bar{\zeta}_{2b}\gamma_c\gamma_5\zeta_{1d}) - \xi_1^\sigma\xi_2^\lambda e_c{}^\rho(D_{[\sigma}e^c{}_{\lambda]})\}D_{[\rho}e^a{}_{\mu]} - \partial_\mu(\xi_1^\rho\xi_2^\sigma D_{[\rho}e^a{}_{\sigma]}). \tag{21}$$

The equations (14),(15),(20) and (21) may reveal N-G fermion (NL SUSY) nature of $\psi^a_\alpha(x)$, non-N-G nature of ${e_a}^\mu(x)$ and a generalized general coordinate- and local Lorentz-transformations, which form a closed algebra[8].
Although the discussions are displayed in parallel with spin 1/2 SGM case[6] for simplicity and generality for N-G fields, it is remarkable that the *massless* spin 3/2 field realized as a N-G fermion of NL SUSY can have a new consistent gravitational coupling without using SUGRA framework in which the local gauge invariance is respected for *massless* spin 3/2 gravitino field.
It is interesting to expand tentatively SGM action explicitly in terms of ${e^a}_\mu(x)$ and ${t^a}_\mu(x)$ in order to clarify the differences of the gravitational interaction of *massless* spin 3/2 field in between SGM and SUGRA. We obtain

$$\begin{aligned} L_{SGM} = & -\frac{c^3\Lambda}{16\pi G} e|w| - \frac{c^3}{16\pi G} eR \\ & -\frac{c^3}{16\pi G} e\{t^{\mu\nu} R_{\mu\nu} + t^{\mu\rho} {t^\nu}_\rho R_{\mu\nu} + O(t^2) + O(t^3) + \ldots + O(t^{16})\}, \quad (22) \end{aligned}$$

where $|w| = det{w^a}_\mu$, $e = det{e^a}_\mu$ and R and $R_{\mu\nu}$ are the Ricci curvature tensors of GR. The first term reduces to Baaklini action[6] with $\kappa^{-1} = \frac{c^3}{16\pi G}\Lambda$ in the Riemann-flat ${e_a}^\mu(x) \to {\delta_a}^\mu$ spacetime and the second term is the familiar E-H action of GR. We can easily see the complementary relation of graviton and superon(in the form of the stress energy tensor) in SGM. These situations are apparently rather different from SUGRA and remain to be studied in detail.
Finally we just mention the phenomenological implications of our model. As read off from the above discussions it is easy to introduce (global) SO(N) internal symmeytry in our model by replacing $\psi^a_\alpha(x) \to {\psi^{ia}}_\alpha(x), (i = 1, 2, \ldots, N)$, which may enable us to consider SGM[5] with spin 3/2 superon. However the fundamental internal symmetry for superons may be rather different from spin 1/2 SGM. For the generator of a new algebra shifts spin by 3/2 and one-superon states correspond to spin 1/2 states, although the adjoint multiplet $\frac{N(N-1)}{2}$ appear at the vector state.
Also apart from the composite SGM scenario, it is worthwhile to consider SGM with extra spacetime dimensions and its compactifications.

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