Natural Oscillation Control Experiments on a Prototype Mechanical Rectifier

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Abstract: For a typical biologically inspired multi-segmental mechanical system, the system’s locomotion behavior is rectified from the segments’ rhythmic oscillation. A central pattern generator (CPG) based controller has been developed to achieve the locomotion with a certain sense of natural oscillation. We designed and manufactured a prototype mechanical rectifier (PMR) system which captures the essential mechanism of multi-segmental systems. Experiments were conducted on the PMR system to reveal the effectiveness of the natural oscillation entrainment controller.

Keywords: Oscillation, locomotion, robotics, neuronal control, central pattern generator

1. INTRODUCTION

Animal locomotion as the act of self-propulsion by an animal has many manifestations like walking, swimming, crawling, running, and flying. A typical animal locomotion pattern is achieved by rhythmically moving its organisms interacted with environment. Physically, locomotion requires energy to overcome friction, drag, inertia, and gravity, though in many circumstances some of these factors are negligible. The research on exploiting the mechanical resonance between body and environment for an efficient locomotion can be traced back to Cavagna et al. (1977). The idea of exploiting natural rhythmic patterns has led to many efficient robotic locomotors that are robust against and adaptive to environmental changes (see, e.g., Hat-sopoulos (1996); Williamson (1998); Verdaasdonk et al. (2007); Iwasaki and Zheng (2006); Futakata and Iwasaki (2008); Blair and Iwasaki (2011).) One of the current engineering research focuses on mechanical locomotion behaviors is directed to a class of so-called mechanical rectifier systems, which are engineering analogue of various locomotion behaviors of multi-segmental animals including snake, leech, octopus, jellyfish, etc. In this model, it is shown how the change in the body shape (e.g., the segmental link angles) results from the interaction with environment (e.g., the locomotion forward velocity with respect to the inertial frame) and the body actuation. On the contrary, it is also shown how the aforementioned undulatory body shape is rectified to a forward locomotion velocity. These two effects are coupled to form control principles for animal locomotion as designed by nature and for autonomous robotic locomotors in engineering perspective. In this sense, this model is called a mechanical rectifier model. To uncover the fundamental engineering mechanism underlying a rectifier model, the idea of building a prototype mechanical rectifier (PMR) was first proposed by Iwasaki and Liu (2004). The PMR model is simple enough to allow us to extract engineering principles by theoretical analysis, simulation, and experiments. At the same time, it captures the essential dynamical mechanism of animal locomotion. The PMR system consists of two main parts: a double pendulum and a disk. The intended operation of the PMR is basically to swing the pendulum to make the disk rotate. The key is that oscillatory motion of the body (pendulum) can generate locomotion (rotation of the disk) if (and only if) the oscillations of body parts are appropriately coordinated. In this paper, we discuss the design and manufacture of a PMR structure and use it as the experimental platform to examine a kind of natural oscillation entrainment controller.

For standard mechanical systems described by symmetric positive definite mass, stiffness, and damping matrices, a natural mode of oscillation is defined to be a free response of the modified system obtained by removing all the damping effects to achieve marginal stability for a sustained oscillation. However, in typical models of body-environment interactions during locomotion including the PMR model, the environmental forces on the body appear in the equation of motion as terms containing an asymmetric stiffness matrix. In Chen and Iwasaki (2009), the definition of natural oscillation was extended to non-symmetric systems. In particular, it was revealed that a natural oscillation can be achieved by a central pattern generator (CPG) motivated controller which adjusts the damping effect by a proper amount. In fact, biological control mechanisms for animal locomotion are known to consist of CPGs (see, e.g., Brown (1911); Grillner and Dubuc (1988)). In engineering literature such as Taga et al. (1991); Lewis and Bekey (2002); Marder and Bucher...
2. A PMR MODEL

The PMR system shown in Fig. 1 consists of two rigid links with flexible joints and a rotating disk. Each link is driven by a DC motor through a harmonic gearbox with zero backlash. The primary link is coupled to the first drive by means of a flexible joint. It carries at its end the second harmonic drive which is coupled to the second rigid link via another flexible joint. Both motors and both flexible joints are instrumented with quadrature optical encoders.

Each flexible joint stiffness can be reconfigured by using different pair of springs with different stiffness. All linear springs are from the Associated Spring Raymond. Four high-resolution quadrature optical encoders from US Digital are used as sensors to measure the angular displacements of two motors and two links. The end point of the second link is in touch with the disk through a finger and exerts friction force during oscillation. The disk is mounted on a base through a bearing structure. A Faulhaber DC-tachogenerator with linear speed/back-
with \( \mathbf{u} \) to be designed later. The detailed model derivation is given below.

First, let \( \mathbf{x} = [x_1 \ x_2]^T \) and \( \mathbf{y} = [y_1 \ y_2]^T \) be the \( x \)- and \( y \)-coordinates of the two link centers, that is, \( \mathbf{x} = F\mathbf{q} \), \( \mathbf{y} = F\mathbf{s}_0 \).

As a result,
\[
\ddot{\mathbf{x}} = -F\mathbf{C}_0 \ddot{\mathbf{q}}^2 - FS_0 \ddot{\mathbf{q}}^2, \quad \ddot{\mathbf{y}} = -FS_0 \ddot{\mathbf{q}}^2 + FC_0 \ddot{\mathbf{q}}.
\]

Let \( f_1 = [f_{1x} \ f_{1y}]^T \), \( f_2 = [f_{2x} \ f_{2y}]^T \) and \( g = [g_x \ g_y]^T \) be the forces on link ends, and denote \( \mathbf{f}_x = [f_{1x} \ f_{2x}]^T \) and \( \mathbf{f}_y = [f_{1y} \ f_{2y}]^T \). Then, from Newton’s law for translational motion, we have, with \( e := [0 \ 1]^T \).

\[
\mathbf{M}\ddot{\mathbf{x}} = (I - E)f_x - e g_x, \quad \mathbf{M}\ddot{\mathbf{y}} = (I - E)f_y - e g_y
\]

or
\[
f_x = (I+E)\mathbf{M}\ddot{\mathbf{x}} + (I+E)e g_x, \quad f_y = (I+E)\mathbf{M}\ddot{\mathbf{y}} + (I+E)e g_y.
\]

On the other hand, let \( \ddot{\mathbf{p}}(\vartheta) = \mu(\vartheta) - \mu_c \) be the relative position between the link tip and disk center, which becomes \(-\hat{T}(\vartheta)\ddot{\vartheta}\) with 90° rotation. As a result, we have
\[
g = \mu_c(\ddot{\mathbf{p}}(\vartheta) + \nu T\ddot{\vartheta}(\vartheta)) = 2\mu_c[-s_\vartheta \ c_\vartheta]^T L\ddot{\vartheta} + 2\mu_c \nu T[c_\vartheta - c_\varphi \ s_\vartheta - s_\varphi]^T l.
\]

Next, from Newton’s law for rotational motion, we have
\[
M L^2/3\ddot{\vartheta} = \ddot{\vartheta} - E\tau + LS_0(I + E)f_x - LC_0(I + E)f_y + L[0 \ e][s_\vartheta - c_\varphi]^T g
\]

where \( \ddot{\vartheta} = \tau - K_\vartheta(I - E)^T \vartheta - \varphi \). Substituting the \( \ddot{\mathbf{x}} \), \( \ddot{\mathbf{y}} \), \( f_x \), \( f_y \) and \( g \) as calculated before into this equation gives
\[
ML^2/3\ddot{\vartheta} = \ddot{\vartheta} - E\tau + S_\vartheta F^T M \ddot{\mathbf{x}} - C_\vartheta F^T M \ddot{\mathbf{y}} + S_\vartheta F^T e g_x - C_\vartheta F^T e g_y + L[0 \ e][s_\vartheta - c_\varphi]^T g,
\]

where we note \( L(I + E)^2 = F^T \). Then,
\[
ML^2/3\ddot{\vartheta} = \ddot{\vartheta} - E\tau + S_\vartheta F^T M \ddot{\mathbf{x}} - C_\vartheta F^T M \ddot{\mathbf{y}} + 2L[s_\vartheta - c_\varphi]^T g
\]

and
\[
ML^2/3\ddot{\vartheta} = (I - E)\tau - K_\vartheta(\ddot{\mathbf{q}}(I - E)^T \vartheta - \varphi) + S_\vartheta F^T M[-F\mathbf{C}_0 \ddot{\mathbf{q}}^2 - FS_0 \ddot{\mathbf{q}}^2] - C_\vartheta F^T M[-FS_0 \ddot{\mathbf{q}}^2 + FC_0 \ddot{\mathbf{q}}] + 2L[s_\vartheta - c_\varphi][2\mu_c[-s_\vartheta, c_\vartheta]^T L\ddot{\vartheta} + 2\mu_c \nu T[c_\vartheta - c_\vartheta \ s_\vartheta - s_\varphi]^T l].
\]

The linearized system at \( \vartheta = \vartheta_o \) is
\[
ML^2/3\ddot{\vartheta} = (I - E)\tau - K_\vartheta(\ddot{\mathbf{q}}(I - E)^T \vartheta - \varphi) + K_\vartheta \vartheta - S_\vartheta F^T MF_\mathbf{S}_0 \ddot{\mathbf{q}} - C_\vartheta F^T MF_\mathbf{C}_0 \ddot{\mathbf{q}} - 4\mu_c G_\vartheta^T G_\vartheta \ddot{\vartheta} - 4\mu_c G_\vartheta^T TG_\vartheta \ddot{\vartheta}
\]

where we note \([c_\vartheta - c_\varphi \ s_\vartheta - s_\varphi]^T l \approx -G_\vartheta \ddot{\vartheta} \).

The dynamics of the disk is governed by Newton’s law for rotational motion is given as follows:
\[
m_c \ddot{\vartheta} = -\mu_c v - [\hat{T}(\vartheta(\vartheta)) + \nu \ddot{\vartheta}(\vartheta)] T g = -\mu_c v + 2\mu_c [-c_\vartheta - c_\varphi \ s_\vartheta - s_\varphi] \ddot{\vartheta} + 2\mu_c \nu T[c_\vartheta - c_\vartheta \ s_\vartheta - s_\varphi]^T l.\]

Then, the linearized model is
\[
m_c \ddot{\vartheta} = -\mu_c v + 4\mu_c \theta^T G_\vartheta^T G_\vartheta \ddot{\vartheta} - 4\mu_c v \theta^T G_\vartheta^T G_\vartheta \theta.
\]

Now it is ready to see the equations (1) and (2) where the torque input \( \mathbf{u} \) or \( \mathbf{r} \) generates periodic motion of two links that interact with the disk to yield its rotational velocity \( v \).

### 3. CONTROLLER DESIGN

As developed in the previous section, the locomotion system under investigation is composed of the two equations (1) and (2). The major objective is to find an effective controller \( u \) such that the closed-loop system can achieve two properties: (i) the double-link (\( \theta \)) displays a natural oscillation; and (ii) a desired rotation velocity \( v \) is generated by the undulatory motion of the links in natural oscillation. Corresponding to the two properties, the controller design and analysis are given below.

#### 3.1 Natural Oscillation of Links

We will first consider the system (1) and define its natural oscillation in terms of the state \( \vartheta \). Let us re-visit the definition of natural oscillation given in Chen and Iwasaki (2009) with a slight modification.

**Definition 1.** Consider the system (1) with the damping effect adjusted by a parameter \( \epsilon \in \mathbb{R} \) and define the modified system with no input:
\[
J \ddot{\vartheta} - \epsilon J \ddot{\vartheta} + K \dot{\vartheta} = 0. \tag{3}
\]

If this system has a nonzero characteristic root on the imaginary axis \( \lambda = \pm j\omega \) with associated mode shape \( z \) for a specific value \( \epsilon = \rho \), then the corresponding natural motion of (3) is called a natural oscillation \( (\omega, z) \) of the original system (1) with damping factor \( \rho \), where \( \omega \) and \( z \) are referred to as the natural frequency and mode shape of the natural oscillation.

The natural oscillation can be explicitly calculated using the following lemma given in Chen and Iwasaki (2009).

**Lemma 1.** Consider the system (1). Let \( \omega, \rho \in \mathbb{R} \) and \( z \in \mathbb{C}^2 \) be given. Then, \( (\omega, z) \) is a natural oscillation of (1) with damping factor \( \rho \) if and only if \( (\omega, z, \rho) \in \mathbb{N} \) where
\[
\mathbf{N} := \{ (\omega, z, \rho) \in \mathbb{R} \times \mathbb{C}^2 \times \mathbb{R} : \omega = \sqrt{\mathbb{R}(z)} , \rho = \frac{\mathbb{R}(z)}{\sqrt{\mathbb{R}(z)}} , (\omega, z) \in \mathbb{M} \}
\]

and \( \mathbb{M} \) is the set of eigenvalue/eigenvector pairs of \( M := J^{-1}K \):
\[
\mathbb{M} := \{ (\omega, z) \in \mathbb{C} \times \mathbb{C}^2 : (\omega - M) z = 0 \}.
\]

Let \( \varrho \) be the smallest damping factor to which there corresponds a natural oscillation:
\[
\varrho := \min_{(\omega, z) \in \mathbb{M}} \frac{\mathbb{R}(z)}{\sqrt{\mathbb{R}(z)}}. \tag{4}
\]

Suppose the minimizer is unique. We focus on the natural oscillation with the smallest damping factor \( \varrho \), and will refer to it simply as the natural oscillation. The main task is to design a feedback controller for the following natural entrainment problem.
Natural Entrainment Problem: Consider the system (1). Let the orbit of the natural oscillation \((\omega, z)\) be defined by \(^1\)

\[
\mathcal{O} := \{ (\psi(t), \dot{\psi}(t)) \in \mathbb{R}^2 \times \mathbb{R}^2 \mid t \in \mathbb{R} \},
\]

\[
\dot{\psi}(t) := Z \sin(\omega t + \gamma).
\]

A controller is said to achieve exact entrainment to the natural oscillation \((\omega, z)\) if the following property holds: When the initial condition \((\theta(0), \dot{\theta}(0))\) is sufficiently close to the orbit \(\mathcal{O}\), i.e., when \(\min_{(\omega, \theta) \in \mathcal{O}} \| \theta(0) - \theta_o \| + \| \dot{\theta}(0) - \dot{\theta}_o \| \) is sufficiently small, the trajectory of the closed-loop system \(\bar{\theta}(t)\) converges to the orbit \(\mathcal{O}\), i.e., there exists \(t_o\), dependent upon the initial condition, such that \(\lim_{t \to \infty} \| \theta(t) - \bar{\theta}(t + t_o) \| = 0\).

Inspired by the CPG control mechanism in animal locomotion, some nonlinear controllers have been constructed for the problem of entrainment to natural oscillation. For example, a slightly modified positive derivative feedback controller is borrowed from Chen and Iwasaki (2009) as follows.

**Theorem 1.** Consider the system (1). Let \((\omega, z)\) be the natural oscillation with damping factor \(\varphi\) as described in (4). Let \(\epsilon, \eta \in \mathbb{R}\) be such that \(\eta > 0, \epsilon < \varphi\). Then, the positive parameter \(r = (\varphi - \epsilon)/(\kappa(\eta \omega)\eta)\) satisfies \(r > \omega(\varphi - \epsilon)\) where \(\kappa\) is the describing function of \(\psi\). And the controller

\[
u = D\dot{\theta} + \epsilon J\dot{\theta} + rJZ\psi(\eta Z^{-1}\dot{\theta}). \tag{5}
\]

is expected to achieve the entrainment to the natural oscillation \((\omega, z)\).

**Remark 1.** In (5), the term \(w = rJZ\psi(\eta Z^{-1}\dot{\theta})\) takes the following form:

\[
w = G\psi(q), \quad q = f(s)H\theta \tag{6}
\]

where \(G\) and \(H\) are \(2 \times 2\) real matrices, \(f(s)\) is a scalar transfer function (in particular, \(f(s) = s\) for the positive derivative feedback controller), and \(\psi\) is a static nonlinearity satisfying the following properties:

- \(\psi\) is odd, bounded, and strictly increasing.
- \(\psi(x)\) is strictly concave on \(x > 0\), and \(\psi(0) = 1\).

The structure in (6) is motivated by the biological CPG control mechanisms. In particular, the simplest input-output model of neuronal dynamics is given by \(v_{post} = \psi(f(s)v_{pre})\) from the presynaptic potential \(v_{pre}\) to the postsynaptic potential \(v_{post}\) where \(\psi\) and \(f(s)\) represent the threshold nonlinearity and dynamics (time lag, adaptation, etc.) associated with synaptic and cell membrane processes, respectively. The controller in (6) is a network of multiple neurons (CPG) with the interconnections specified by \(G\) and \(H\).

### 3.2 Rotation of Disk

Now, the double link pendulum in natural oscillation has been achieved by an entrainment controller inspired by a CPG based structure. In the formulation of natural oscillation and the control design procedure, the rotation speed \(v\) is assumed to be externally imposed. Actually, a more interesting question of the locomotion system is that the introduced natural oscillation in turn generates the velocity \(v\). Next, we will analyze how the rotation speed \(v\) is induced by the link oscillation. An assumption is first imposed to facilitate the analysis:

**Assumption 1.** For the CPG controller based on a given velocity \(v_0\), the closed-loop system is sufficiently fast to achieve the natural oscillation associated with any \(v\) in the neighborhood of \(v_0\).

For a natural oscillation \((\omega, z)\), depending on the velocity \(v\), we can assume \(\theta(t) = \Re(z\omega^t)\), and hence,

\[
\hat{\theta}(t) = \Re(j\omega z e^{j\omega t}) + \delta(z, \omega)
\]

for some term \(\delta\) vanishing at \((z, \omega) = 0\). Then, some direct calculation is given below:

\[
\begin{align*}
\theta^T \theta &= |z|^2 |\hat{P}| |z| + |z|^2 |\hat{P}| |z| \\
\theta^T \hat{Q} \theta &= -\omega |z|^2 |\hat{Q}| |z| - \omega |z|^2 |\hat{Q}| |z| + \theta^T \hat{Q} \delta \\
\end{align*}
\]

and

\[
\begin{align*}
\hat{P}_{ij} &= P_{ij} \cos(\gamma_i - \gamma_j)/2 \\
\hat{Q}_{ij} &= -Q_{ij} \sin(\gamma_i - \gamma_j)/2,
\end{align*}
\]

where \(|z|\) is the column vector with entries \(|z_i|\). By using this calculation, we can rewrite (2) as

\[
\dot{v}/\|z\|^2 + [\xi(v) + \hat{\xi}(v, t)]v + \zeta(v) = 0 \tag{7}
\]

where \(c_c = \mu_c/m_c\) and \(c_v = \mu_v/m_v\), and

\[
\begin{align*}
\xi(v) &= c_c/\|z\|^2 + 4c_v |z_o|^2 |\hat{P}| |z_o| \\
\hat{\xi}(v, t) &= 4c_v |z_o|^2 |\hat{P}(t)| |z_o| \\
\zeta(v) &= 4c_v \omega |z_o|^2 |\hat{Q}| |z_o| \\
\end{align*}
\]

Here, we use \(z_o := z/\|z\|\) with \(\|z_o\| = 1\) which is the normalized eigenvector representing the oscillation phases and relative amplitudes. The quantities, in particular, \(\omega, z_o,\) and \(Q\) in the above development, depend on the velocity \(v\), and will be explicitly expressed as \(\omega(v), z_o(v)\), and \(Q(v)\) when the dependence is important.

Clearly, for a given \(v, \xi(v)\) and \(\hat{\xi}(v)\) are two constants, but \(\xi(v, t)\) and \(\zeta(v, t)\) are sinusoidal-like time varying functions of frequency \(2\omega\). With the decomposition of the functions \(\xi, \zeta,\) we can decompose the velocity \(v = \bar{v} + \tilde{v}\) in a corresponding manner. Let \(\bar{v}\) be governed by the dynamics:

\[
\dot{\bar{v}}/\|z\|^2 + \xi(v)\bar{v} + \zeta(v) = 0, \tag{8}
\]

then \(\tilde{v}\) must be governed by

\[
\begin{align*}
\dot{\tilde{v}}/\|z\|^2 &= -\xi(v)\bar{v} + \xi(v)\tilde{v} - \xi(v)\bar{v} \\
+ [\zeta(v) - \zeta(v)] - \tilde{v}(v, t)v - \zeta(v, t). \tag{9}
\end{align*}
\]

We call \(\bar{v}\) and \(\tilde{v}\) the average velocity and ripple, respectively. It has been shown in Chen et al. (2010) that the
magnitude of the ripple $\tilde{v}$ is expected to be small if the oscillation amplitude is small. Now, it is ready to reach a conclusion on the average velocity $\overline{v}$ in the following theorem.

**Theorem 2.** Under Assumption 1, the average velocity $\overline{v}$ governed by (8) locally asymptotically converges to $\overline{v} = v_0$ if $\alpha(v_o) = v_o$ and $\alpha'(v_o) < 1$, where

$$
\alpha(v) := \frac{-4c\omega(v)|z_o(v)|^2 Q(v)|z_o(v)|}{c_e\|z(v)\|^2 + 4c_e|z_0(v)|^2 P(v)|z_0(v)|}.
$$

4. SIMULATION AND EXPERIMENT

Consider the PMR system with the following parameters. The two links have masses of $m_1 = 2.42$kg and $m_2 = 1.95$kg, and lengths of $l_1 = 0.35$m and $l_2 = 0.45$m. The disk rotational inertia is $J_c = 2$kgm$^2$. The nominal link angles are $\vartheta_0 = [25^\circ, -25^\circ]$. The friction force constants are $\mu_1 = 18$kg/s and $\mu_2 = 18$Ns/m. We consider two sets of joint stiffness $k = [3.8, 3]^T$kgm$^2$/s$^2$rad and $k = [6.8, 4]^T$kgm$^2$/s$^2$rad to examine the effect of stiffness. For different $k$'s, we can find the solution to $\alpha(x) = x$ in Fig. 3.

The result shows that, to expect a higher velocity, the stiffness of the body must be strengthened. From Fig. 3, it is easy to see that $\alpha'(v_o) < 1$ is true for both cases. In the other words, both equilibrium points $v_o = 4.6$rad/s and $v_o = 5.8$rad/s corresponding to $k = [3.8, 3]^T$ and $k = [6.8, 4]^T$ are asymptotically stable. Moreover, the natural oscillation patterns for these two cases are listed in Table 1. Also we can see in Fig. 4 and Fig. 6 that the controller very closely achieves theoretical velocity in numerical simulation. Moreover, the experimental results are shown in Fig. 5 and Fig. 7. All the simulated and experimental results are cross checked with respect to theoretical values as shown in Table 1 in terms of disk speed, link oscillation amplitudes, period, and phase difference. All results shown in Table 1 are reasonably consistent.

Table 1. Theoretical/simulated/experimental locomotion profiles with different stiffness

| $k$ = $[3.8, 3]^T$ | $v$ (rad/sec) | $|z|$ | $|z_1|$ | Period(s) | Phase |
|-------------------|---------------|-------|---------|-----------|-------|
| Theo.             | 4.6           | 9.0°  | 10.3°   | 1.21      | 130°  |
| Simu.             | 4.5           | 9.3°  | 10.6°   | 1.25      | 130°  |
| Exp.              | 5.0           | 7.9°  | 10.0°   | 1.21      | 136°  |

| $k$ = $[6.8, 4]^T$ | $v$ (rad/sec) | $|z|$ | $|z_1|$ | Period(s) | Phase |
|-------------------|---------------|-------|---------|-----------|-------|
| Theo.             | 5.8           | 9.3°  | 10.1°   | 1.00      | 129°  |
| Simu.             | 5.7           | 9.2°  | 9.9°    | 1.02      | 129°  |
| Exp.              | 6.0           | 8.7°  | 11.4°   | 1.01      | 136°  |

5. CONCLUSION

In this paper, we have discussed the design of a PMR structure and a CPG based controller for the generation of a stable disk rotation with links in natural oscillation. The phenomena is based on the simple PRM structure but it is expected to capture the fundamental principles underlying the locomotion behaviors of natural systems or complicated engineering systems. The CPG based controller has been examined by the cross check among theoretical analysis, numerical simulation, and experiments.

REFERENCES


Fig. 3. Solution to $\alpha(x) = x$. The solid curves represent $y = \alpha(x)$ for $k = [3.8, 3]^T$ and $k = [6.8, 4]^T$, and the dashed line is $y = x$. The intersection $\alpha(x) = x$ occurs at $x = 4.5$ and $x = 5.8$, respectively.

Fig. 4. Simulation profile with $k = [3.8, 3]^T$. Above: self-generated disk velocity $v$. Below: undulatory trajectories $\theta_1$ and $\theta_2$ of two links.

Fig. 5. Experimental profile with $k = [3.8, 3]^T$. Above: self-generated disk velocity $v$. Below: undulatory trajectories $\theta_1$ and $\theta_2$ of two links.

Fig. 6. Simulation profile with $k = [6.8, 4]^T$. Above: self-generated disk velocity $v$. Below: undulatory trajectories $\theta_1$ and $\theta_2$ of two links.

Fig. 7. Experimental profile with $k = [6.8, 4]^T$. Above: self-generated disk velocity $v$. Below: undulatory trajectories $\theta_1$ and $\theta_2$ of two links.