A Subcarrier Allocation Algorithm for Utility Proportional Fairness in OFDM Systems

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Abstract—In this paper, a concept of utility proportional fairness is defined, which achieves fairness of utility for different applications. The optimization problem of utility proportional fairness has been formulated meanwhile the optimal solution is presented. The utility proportional fairness is applied in the orthogonal frequency division multiplexing (OFDM) wireless networks, and the optimal subcarrier allocation for utility proportional fairness is deduced. An effective and practical simplified subcarrier allocation algorithm is proposed, in which the instantaneous data rate is substituted by average rate via exponentially weighted low pass time window. Simulation results illustrate that both the optimal and simplified subcarrier allocation algorithms guarantee the utility fairness between different applications well, and the simplified algorithm achieves a perfect tradeoff between utility fairness and system throughput and provides the quality of server (QoS) for multimedia applications.

Keywords- orthogonal frequency division multiplexing (OFDM); proportional fair; subcarrier allocation; utility fairness

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM), which could overcome multipath fading effectively and achieve high frequency efficiency, is one of the main technologies in the future wireless communication systems. Since the subcarriers could be allocated dynamically among users in a scheduling interval, the radio resource management in the OFDM systems has some overwhelming feature.

The authors in [1] have pointed that the concept of utility in the economics can be introduced to formulate the gain of customer satisfaction when certain resource is assigned to a user in communication systems. Papers [2-5] propose some scheduling schemes for maximizing the sum of utilities of users, which are based on variety utility functions and with different restrictions. Those resource allocation schemes give a tradeoff between system efficiency and data rate fairness of users. However, it is ignored that the utility levels of different applications are different even if they are given the same data rates. So when users have different applications, the system optimization should take not the rate fairness but the utility fairness into account [6].

In the early papers [7, 8], the concept of utility fairness in the wireless networks has been proposed, and fairness scheme based on equal utility levels has been designed for wireless packets access networks. A bandwidth allocation scheme is presented to equalize the utility levels for multimedia applications [9]. Given absolutely equal utility levels for all users can guarantee fairness between users with different applications, but the sum of utilities which stand for total customer’s satisfaction of the system is low. For this reason, it is required a tradeoff between system efficiency and utility fairness, just as the research of rate fairness [5, 10].

The rest of this paper is organized as follows. First we give the concept of utility proportional fairness (UPF) and the optimal solution of UPF. In Section III, we describe the OFDM system model, and give the optimal subcarrier allocation algorithm for UPF. Then we simplify the subcarrier allocation by using the average rates instead of the instantaneous ones for practicality. The Section IV presents the simulation results to demonstrate the performance of the proposed schemes. We make conclusion at the end of this paper.

II. UTILITY PROPORTIONAL FAIRNESS

Utility represents the degree of satisfaction of a user when a certain radio resource is provided to this user in the wireless networks. Utility function is a curve mapping the obtained resource to perceived performance of a user. It is monotonically non-decreasing, in other words, allocation more resource to a user should not degrade the performance of his application [2]. Diverse applications have various utility function curves and different parameters. The key advantage of the utility function is that it can inherently reflect the user’s QoS requirement and quantify the adaptability of an application.

Kelly et al. [10] have defined the proportional fair (PF) allocation of rates, and used utility function to allocate bandwidth for elastic traffic in the fixed broadband networks. It is proved that maximizing the sum of logarithmic transmission rates is compatible with the rate proportional fairness among users. In the following paragraphs of this paper, the traditional PF [10] would be named rate proportional fairness (RPF) in order to distinguish the utility proportional fairness (UPF) defined in this paper.
A. Definition of UPF

Let the set of active users be $I = \{i \mid i = 1, 2, \ldots, I\}$, $u_i$ is the utility level of user $i$ when it has been allocated some resource. The vector of utility level be $\mathbf{u} = (u_1, u_2, \ldots, u_I)^T$, with the set of achievable utility vector $S$.

**Definition 1**: A utility vector $\mathbf{u}^*$ is said to be utility proportional fair, if for any other utility vector $\mathbf{u}$, the aggregate of proportional changes in the utilities is non-positive

$$\sum_{i=1}^{I} \frac{u_i - u_i^*}{u_i} \leq 0$$

(1)

Thus, an allocation is UPF if any change in the distribution of the utility would result in the sum of the proportional changes of the utilities to be non-positive. If $u_i = r_i$, the RPF could be seen as a particular case of UPF.

**Definition 2**: If the optimal solution for an optimization problem $P$ satisfies the definition of UPF, then $P$ is said to be the optimization problem of UPF.

**Theorem 1**: The expression of the optimization problem $P$ which will get the optimal utility vector $\mathbf{u}^*$ for UPF is

$$\max \sum_{i=1}^{I} (\ln u_i + a_i)$$

(2)

In which, $a_i$ is any constant.

**Proof**: Set utility vector $\mathbf{u}^*$ satisfied the definition of UPF in (1), for any utility vector $\mathbf{u}$, it could be derived from (1) that

$$\sum_{i=1}^{I} \frac{u_i - u_i^*}{u_i} = \sum_{i=1}^{I} \left[ (\ln u_i^* + a_i) \cdot (u_i - u_i^*) \right] \leq 0$$

(3)

Let $f(\mathbf{u}) = \sum_{i=1}^{I} (\ln u_i + a_i)$, since $\ln u_i$ is a concave function, $f(\mathbf{u})$ is a concave function obviously. From the Criterion Theorem of concave function, $\forall \mathbf{u} \in S$, have

$$f(\mathbf{u}^*) - f(\mathbf{u}) \geq \nabla f(\mathbf{u}^*)^T \cdot (\mathbf{u}^* - \mathbf{u})$$

(4)

It can be concluded from (3) and (4) that

$$\nabla f(\mathbf{u}^*)^T \cdot (\mathbf{u}^* - \mathbf{u}) = \sum_{i=1}^{I} \left[ (\ln u_i^* + a_i) \cdot (u_i^* - u_i) \right] \geq 0$$

so $f(\mathbf{u}^*) \geq f(\mathbf{u})$.

B. Optimal Solution of UPF

According to the property of the concave function, a local optimal solution is global optimal solution, so given any utility vector $\mathbf{u}$, it is true that $f(\mathbf{u}^*) \geq f(\mathbf{u})$. It means that $\mathbf{u}^*$ is the optimal solution of $P$, i.e., form (2), we can get the utility vector $\mathbf{u}^*$ satisfied the definition of UPF.

Since $a_i$ denotes any constant, without lost of generality, we assume that $a_i = 0$, the optimization problem $P$ can be simplified as $P'$ which will also obtain $\mathbf{u}^*$. The optimization problem $P'$ is

$$\max \sum_{i=1}^{I} \ln u_i$$

(5)

III. UPF OF OFDM SYSTEMS

A. OFDM System Model

In OFDM cellular systems, the total bandwidth is $B$, the total number of available subcarriers is $K$, the bandwidth occupied by a subcarrier is $B_k$ (i.e. $B_k = B/K$). Suppose that the channel state information which is measured by user equipment is feedback to its base station through control.
channel without delay and error. $g_{ik}$ denotes the channel gain of the $i$th user on the $k$th subcarrier. The maximum bit rate to be transmitted for the user $i$ on the $k$th subcarrier is

$$r_{ik} = B_k \log_2 \left(1 + \frac{g_{ik} p_i}{\Gamma (I_{\text{inter}} + N_0)} \right)$$  \hspace{1cm} (8)$$

$\Gamma$ is a constant related to a targeted bit error rate (BER), $\Gamma = -\ln (5BER)/1.5$. $I_{\text{inter}}$ and $N_0$ denote the inter-cell interference and the thermal noise.

### B. Subcarrier Allocation for UPF

We assume that the transmit power on each subcarrier is fixed with adaptive modulation and coding (AMC), so resource allocation problem is simplified to dynamic subcarrier assignment according to the utilities of users. If $c_{ik} = 1$ indicates that subcarrier $k$ is assigned to user $i$, otherwise $c_{ik} = 0$. The data rate obtained by the user $i$ in a scheduling interval is $r_i = \sum_{k=1}^{K} c_{ik} r_{ik}$. Derived from (6), the optimization problem of UPF in OFDM system can be expressed as

$$\max \sum_{i=1}^{I} \ln u_i \left( \sum_{k=1}^{K} c_{ik} r_{ik} \right)$$

$$s.I. \sum_{k=1}^{K} c_{ik} = 1$$  \hspace{1cm} (9)$$

The restriction implies that multiple users can’t share one subcarrier at the same time. From (7), we have

$$\nabla f (r^*)^T (r^* - r) = \frac{u_i'(r_i^*)}{u_i'(r_i^*)} r_{ik} - \frac{u_j'(r_j^*)}{u_j'(r_j^*)} r_{jk} \geq 0$$  \hspace{1cm} (10)$$

Therefore, if subcarrier $k$ assigned to user $i$ is the optimal solution for UPF, it should satisfy the following condition

$$\frac{u_i'(r_i^*)}{u_i'(r_i^*)} r_{ik} \geq \frac{u_j'(r_j^*)}{u_j'(r_j^*)} r_{jk} \quad \forall j \neq i \in I$$  \hspace{1cm} (11)$$

According to (11), the optimal subcarrier assignment has the following rule for UPF

$$\hat{i}(k) = \arg \max_{i \in I} \left\{ \frac{u_i'(r_i^*)}{u_i'(r_i^*)} r_{ik} \right\}$$  \hspace{1cm} (12)$$

Where $\hat{i}(k)$ represents that subcarrier $k$ should be assigned to the user $i$. It implies that, in the OFDM systems, when equal transmit power allocation is considered, the optimal subcarrier allocation algorithm (SAA) is assigned the $k$th subcarrier to the user $i$ who has the maximal value of $\frac{u_i'(r^*)}{u_i'(r^*)} r_{ik}$. In all the expression mentioned above, $r_i^* = \sum_{k=1}^{K} c_{ik} r_{ik}$.

Although the optimal subcarrier allocation rule has been formulated in (12), the rate expression for each user $r_i = \sum_{k=1}^{K} c_{ik} r_{ik}$ makes it can’t be solved, as it is a NP-hard problem. Some heuristic algorithm such as genetic algorithm could be used to solve this problem. Even so, the computational complexity is unendurable in the scheduling interval, and the SAA should be simplified further.

### C. Simplified SAA for UPF

A simple and intuitionist simplified method is that the instantaneous data rate is averaged by the exponentially weighted low pass time window, and the optimization problem of UPF is rewritten as

$$\max \sum_{i=1}^{I} \ln u_i (\bar{r}_i(t))$$

$$s.I. \sum_{k=1}^{K} c_{ik} = 1$$  \hspace{1cm} (13)$$

In the scheduling interval $t$, the exponentially weighted averaged rate is expressed as

$$\bar{r}_i(t) = (1 - \rho) \bar{r}_i(t-1) + \rho r_i(t)$$  \hspace{1cm} (14)$$

$r_i(t) = \sum_{k=1}^{K} c_{ik} r_{ik}(t)$ is the instant rate of user $i$ in the scheduling interval $t$, and $r_{ik}(t)$ is the instant rate of user $i$ on the subcarrier $k$ in the scheduling interval $t$. $\rho = T_s/T_w$, in which $T_s$ is the length of scheduling timeslot, $T_w$ is the length of the time window.

Let $x = \bar{r}_i(t)$, $x_0 = \bar{r}_i(t-1)$ and from (14) we have $x - x_0 = \rho [r_i(t) - \bar{r}_i(t-1)]$. Using one order Taylor formula, it follows that

$$\ln u_i (\bar{r}_i(t)) = \ln u_i (\bar{r}_i(t-1)) + \frac{u_i'(\bar{r}_i(t-1))}{u_i'(\bar{r}_i(t-1))} (\rho [r_i(t) - \bar{r}_i(t-1)])$$
Since $\bar{r}_i(t - 1)$ is irrelevant to the scheduling interval $t$, setting $u_i'(\bar{r}_i(t - 1)) = \alpha_i$, the problem (13) becomes the following linear objective problem

$$\max \sum_{i=1}^{l} \alpha_i r_i(t)$$  \hspace{1cm} (15)

It means that (15) maximizes the sum of weighted rates. The weights are adaptively controlled by the proportion of marginal utility and utility both with respect to the average rates. The simplified SAA by the method of average data rate for UPF is

$$\hat{i}(k, t) = \arg \max_{i \in I} \alpha_i r_{ik}(t)$$  \hspace{1cm} (16)

From (16), we know that, simplified by average rate, the subcarrier allocation algorithm meeting UPF is that assigning $k$th subcarrier to the user $i$ who has the maximal $\alpha_i r_{ik}(t)$ on this subcarrier in the scheduling interval $t$. The linear objective problem simplifies the allocation algorithm dramatically, and the computational complexity is lowered to $IK$. Using the average data rate during a certain period time instead of the instantaneous one could be explained as that since the users are sensitive with the experience of longer time, the average data rate during the time longer than the scheduling timeslot is reasonable. On the other hand, it will exploit the time diversity though the time window, which will improve the system capability as well [5].

IV. SIMULATION AND ANALYSIS

In this section, we present simulation results to illustrate the performance of the optimal and simplified subcarrier allocation algorithm and the simplified algorithm when the utility proportional fairness is required.

A. System Model Parameters

Cell radius is set to 500m, and the users are uniformly distributed. It is assumed that there’re no inter-subcarrier or inter-symbol interferences, and the scheduler has the perfect knowledge of channel state information (CSI). In simulation, 12-ray Rayleigh channel is used, and shadow and fast fading are considered. The transmit power on each subcarrier is equal and fixed with adaptive modulation and coding.

In this paper, we consider two kinds of applications, one is best effort traffic and another is real-time video. The ratio of this two applications among users is 1:1. We apply the utility functions of best effort and real-time video given in [9]. In fact, we don’t care about the particular form of utility function, which can be obtained by sophisticated subjective surveys or based on the habits of the traffic.

B. Simulation Results

We compare the performance of three algorithms in the OFDM systems, which is the optimal SAA for UPF via genetic algorithm (named as UPF-GA), the simplified SAA for UPF (called UPF-S), and the RPF.

In order to assess the utility fairness of different algorithms, we define utility fairness index as

$$F = \left( \frac{\sum_{i=1}^{l} u_i}{K \sum_{i=1}^{l} u_i^2} \right)^2 .$$

Fig. 1 shows the utility fairness index of the three algorithms. It is clear that the fairness index of UPF-GA is highest, and the RPF is lower than UPF-GA and UPF-S. We also notice that the fairness index will decrease as the number of users in each sector increases.

Since the UPF (UPF-GA and UPF-S) can guarantee the utility fairness well, we analyses the system throughput next. It can be seen from the Fig. 2 that the RPF the gain of throughput is larger than UPF. However, we know that RPF is a tradeoff between rate fairness and throughput without considering the utility fairness, so the user whose utility is small with large data rate is to be treated unfair. The spectral efficiency of UPF-GA is lowest and falls as the traffic load rises, because it makes
best effort to keep fair but neglects the throughput improvement. The UPF-S trades off utility fairness and spectral efficiency best.

In the UPF-GA and UPF-S algorithms, the utility function of different applications will generate different priority, so the QoS will be guaranteed. The RPF can’t provide QoS for multimedia applications. Fig. 3 and Fig. 4 demonstrate the total packet delay of real-time video during the simulation time. The UPF-GA keeps utility fairness but decreases the system throughput, so if the traffic load is light the QoS can be guaranteed (e.g. when there are 4 users in each sector the total packet delay is smallest in the Fig. 3), but when the traffic load is heavy, the throughput decreases consequently and the packet delay rises sharply. We also give the comparisons of total packet delay of UPF-S and RPF algorithms in Fig. 4. The total packet delay of UPF-S is lower than that of RPF, which means that the UPF-S provides the QoS for real-time video by decreasing the throughput of best effort traffic. In fact, the tunable parameters of utility function can be used to change the priority of the applications.

From the results above, we derive that the UPF-S algorithm can make a perfect tradeoff between the utility fairness of and the spectral efficiency, and provide QoS for multimedia applications. Furthermore, the algorithm complexity of UPF-S is as same as the RPF, which is linear function of the number of subcarrier.

V. Conclusion

Utility theory is an available method to research resource allocation problem. Utility proportional fairness (UPF) can provide fairness among different applications. The optimal solution deduced in this paper is a general solution of UPF. In the OFDM systems, the optimal allocation of subcarriers is NP-hard problem, so we propose a simplified subcarrier allocation algorithm which uses the average data rate to lower the computational complexity efficiently and gain the time diversity effect. According to the simulation results, the optimal and simplified subcarrier allocation algorithms both have a good performance on the utility fairness, moreover, the simplified subcarrier allocation algorithm also has gain on spectral efficiency and fulfill the QoS requirement of different applications. We will study the optimal joint subcarrier and power allocation, and give a general expression of different utility fairness criterion further.

REFERENCES