Route Guidance: Bridging System and User Optimization in Traffic Assignment

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Overview

- Motivation
- User optimum issues
- System optimum
- Proposed model and coordination methods
- Experiment results
- Future work
Motivation

- Present route guidance systems (RGS):
  - Unsustainable assumptions on low percentage of vehicles following the routes
  - Not considering the effects of their own route recommendation
  - Based on user optimization, generally resulting in Wardrop equilibrium (user optimum) which can be arbitrarily more costly than the system optimum
  - Fair for the users of the same O-D pair, generally unfair for different O-D pairs

- Objective: bridge the system optimization which assumes collaborative road infrastructure users and user optimization which assumes selfish users in traffic assignment considering additional fairness and Social Welfare aspects.
User Optimum

- Selfish noncooperative network users
- Traffic routed on minimum-latency paths
- The outcome of selfish routing: Nash equilibrium
  - Traffic assignment is at Nash Equilibrium if no network user has an incentive to switch paths; this occurs when all traffic travels on minimum – latency paths
  - Nash equilibrium does not in general minimize the total latency
  - Price of anarchy – the worst possible ratio between the total latency of a Nash equilibrium and of an optimal routing of the traffic
  - The cost of anarchy can be generally arbitrarily large

- Can it work better?
System Optimum

- Minimize the overall system’s travel time
- Desirable from the traffic authority’s point of view
- Drawbacks:
  - Routing some of the drivers on unacceptably long paths in order to use shorter paths for many other drivers – unacceptable for many self-concerned drivers
  - Possible lack of fairness on the same and different O-D pairs
  - Constrained system optimum:
    - Fairness issues previously tackled by acceptable paths (within fixed factors of the optimal ones)
Proposed architecture

At the upper layer, we use a 4-level decomposition method to reach a subproblem which can be optimized individually locally by every O-D pair independently of other O-D pairs.

At the lower layer, we use Bertsekas’ auction algorithm with similar objects guaranteeing the achievement of an optimal solution.
Proposed: Nash social welfare optimization with the constraints on envy-free and fair O-D paths

we introduce a normalized mean path duration cost of agent $w \in W$,

$$
\gamma_w(x_w, \{x_l\}_{l \in M(W)}) = \left| P_w \right| \sqrt{\prod_{k \in P_w} f^k \cdot x^k}
$$

Furthermore, we propose the following envy criterion for O-D agents. An allocation is $\alpha$-envy-free, where $\alpha$ is a maximum tolerance factor for non-enviousness such that $0 < \alpha < 1$ if:

$$
\gamma_w \geq \gamma_{w^\prime}^\alpha, \forall w, w^\prime \in W | w \neq w^\prime
$$
The balance between egalitarian and utilitarian social welfare is given by the maximization of the Nash product which is the product of the agents’ individual utilities.

\[
\max N(x_w) = \prod_{w \in W} 1/\gamma_w = -\sum_{w \in W} \log \gamma_w
\]

The non-easily decomposable mathematical programming model with included envy-freeness and fairness parameters is then:
Mathematical programming model

Objective function

\[ \min z(x_w) = \sum_{w \in W} \log \gamma_w = \sum_{w \in W} \log \left[ \frac{|P_w|}{\prod_{k \in P_w} \sum_{a \in A} f_a(x_a) \cdot \phi_{ak} x_k} \right] \]

subject to:

\[ \sum_{w \in W} \sum_{k \in P_w} \phi_{ak} \cdot x_k \leq u_a, \forall a \in A \]

\[ \gamma_w \geq \gamma_{w^*}^\alpha, \forall w, w^* \in W | w \neq w^* \]

\[ \sum_{k \in P_w} \psi_{wk} \cdot x^k = R_w, \forall w \in W \]

\[ x_k \geq 0, \forall k \in P_w, w \in W \]
Figure 1. Distributed dual decomposition structure of our network utility maximization problem
Lower layer path assignment auction algorithm

- Each vehicle agent \( a \) is described by the tuple
  \[ a = \{ w_a, p_a, S_a, c_a \} \]

- Vehicle Satisfaction
  \[ S_a = \gamma \cdot S_a(t) + (1 - \gamma) \cdot S_a(t-1), \gamma \in [0; 1] \]
  \[ S_a(t) = 1 - \frac{f_a(t)}{f_{sp}^{w}(t)} \]

- The auction algorithm applied to vehicles:
  - A modification of Bertsekas auction algorithm for similar objects,
  - runs in iterations,
  - each iteration composed of a bidding and the assignment phase,
  - the bids of vehicles are modified by the vehicles' satisfaction values,
  - The bids are compared in respect to the O-D pair total flow,
  - performed in parallel for all O-D pairs.
Simulation experiments

- 10 graphs with 50 nodes with the total demand of 2500 O-D pairs.
Experiment results

![Graph showing experiment results](image)

- **Mean path duration cost vs. Origin agents**
- Blue line: User optimization
- Red line: System Optimization
- Green line: Social Welfare concerned Optimization

**Axes:**
- Y-axis: Mean path duration cost $C_p$ [h]
- X-axis: Origin agents (ordered in nondecreasing order of path costs)

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Future work

- Efficient incentives to the drivers on the worst-off O-D pairs
- Application to the air and rail transport
- Integration with signaling infrastructure on road networks,
- Simulation on real road networks with available historical data
Thank you on your attention!