A Comparison Between the Coefficients of the Kubelka-Munk and DORT2002 Models

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Contents Page

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Abstract

This paper gives the exact translation $k = 2\sigma_a$ and $s = \sigma_s$ between the scattering and absorption coefficients of the Kubelka-Munk and DORT2002 models. It is shown that the conditions, under which this exact translation is valid, are perfectly diffuse light, perfectly isotropic scattering, and only two channels in the DORT2002 model. Reasons for non-validity in other cases are given.

It is also shown that it is possible to use grammage instead of thickness for DORT2002, as has previously been shown for Kubelka-Munk.

Since the s and k values of the Kubelka-Munk model are so well known and in such widespread use, a translation to and from the DORT2002 coefficients will make it easy to use the models together, to compare results, and to change from one model to the other.

1. Introduction

Kubelka and Munk [1] in 1931 presented a now well-known model for light scattering, which was further refined by Kubelka [2, 3]. Despite several limitations, the Kubelka-Munk model is in widespread use for multiple scattering calculations in paper, coatings, printed paper, paint, plastic and textile, probably due to its explicit form and ease of use.

DORT2002 is a fast and accurate numerical implementation of a discrete ordinate solution method for the radiative transfer problem in vertically inhomogeneous turbid media. The model is based on papers by Edström [4] and Edström and Lehto [5, 6], which extensively treat a radiative transfer problem formulation and a solution method. The DORT2002 software is implemented in MATLAB, and is adapted to light scattering simulations in paper and print.

DORT2002 is a natural generalization of the Kubelka-Munk model, which can be seen as a simple special case of DORT2002. DORT2002 has been shown to be more accurate, and to have a considerably wider range of applicability than Kubelka-Munk [7].

The aim of this paper is to make a comparison of the scattering and absorption coefficients of the Kubelka-Munk model and the DORT2002 model, and to specify under what conditions an exact translation is valid. Since the s and k values of the Kubelka-Munk model are so well known and in such widespread use, a translation to and from the DORT2002 coefficients is valuable for the future use of both models.

Section 2 and 3 give the relevant Kubelka-Munk and DORT2002 equations respectively, rewritten in a form to make comparisons easy. In section 4 the comparison of the coefficients is made, in section 5 it is shown that it is possible to use grammage instead of thickness also for DORT2002, and section 6 contains a short discussion.

2. Kubelka-Munk Equations

The Kubelka-Munk equations can be written

$$\begin{cases} -di = -(s+k)idx + sjdx \\ dj = -(s+k)jdx + sidx \end{cases}, \tag{1}$$

where *i* is the intensity in the downward direction and *j* is the intensity in the upward direction. *s* and *k* are the light scattering and absorption coefficients, and *x* is the distance measured from the background and upwards.

Introducing I^+ and I^- as the intensities in the upward and downward directions, the Kubelka-Munk equations can be rewritten as

$$\begin{cases} \dfrac{dI^+}{dx} = -kI^+ - sI^+ + sI^- \\ -\dfrac{dI^-}{dx} = -kI^- - sI^- + sI^+ \end{cases}. \tag{2}$$

The first term on the right hand side is absorption, i.e. the intensity "disappears", the second term is intensity scattered into the opposite direction, and the third term is a contribution to the intensity, scattered from the opposite direction.

Among the conditions behind the derivation of the Kubelka-Munk equations are that the light is assumed to be perfectly diffuse. The fact that light incident at an angle has longer optical path to a certain depth in the medium is not explicitly considered (or it can be assumed to be built into the coefficients *s* and *k*).

Hence, *k* is the part of the intensity that is *absorbed* and "disappears" upon passage of the infinitesimal thickness *dx* of the medium. *s* is the part of the intensity that is *scattered to the opposite direction* upon passage of the infinitesimal thickness *dx* of the medium.

3. DORT2002 Equations

The DORT2002 model is a more general model than Kubelka-Munk. Kubelka-Munk is exactly the special case of DORT2002, where the illumination and scattering is completely isotropic, and where only two channels are considered. The DORT2002 equations for two channels are

$$\begin{cases} \mu\dfrac{dI^+(\tau,\mu)}{d\tau} = I^+(\tau,\mu) - \dfrac{1}{2}\dfrac{\sigma_s}{\sigma_a+\sigma_s}\displaystyle\int_0^1 I^+(\tau,\mu')d\mu' - \dfrac{1}{2}\dfrac{\sigma_s}{\sigma_a+\sigma_s}\displaystyle\int_0^1 I^-(\tau,\mu')d\mu' \\ -\mu\dfrac{dI^-(\tau,\mu)}{d\tau} = I^-(\tau,\mu) - \dfrac{1}{2}\dfrac{\sigma_s}{\sigma_a+\sigma_s}\displaystyle\int_0^1 I^+(\tau,\mu')d\mu' - \dfrac{1}{2}\dfrac{\sigma_s}{\sigma_a+\sigma_s}\displaystyle\int_0^1 I^-(\tau,\mu')d\mu' \end{cases}, \tag{3}$$

where I^+ and I^- are the intensities in the upward and downward directions, respectively. σ_a and σ_s are the DORT2002 absorption and scattering coefficients, μ

is the cosine of polar angle, and τ is the optical depth measured from the top of the medium and downwards.

In the discrete ordinate approximation (using Gaussian numerical quadrature) for two channels, the $I^{\pm}(\tau,\mu)$ are replaced with their hemispherical averages, $I^{\pm}(\tau)$. The average of μ becomes

$$\int_0^1 \mu d\mu = \frac{1}{2}. \tag{4}$$

It should be noted here, that the distance *x* used in Kubelka-Munk, and the optical depth τ used in DORT2002 are in opposite directions. Furthermore, it holds that

$$\frac{\text{optical depth}}{\text{real depth}} = \sigma_e, \tag{5}$$

where σ_e is the DORT2002 extinction coefficient, defined as $\sigma_e = \sigma_a + \sigma_s$. Therefore, $d\tau = -\sigma_e dx = -(\sigma_a + \sigma_s)dx$, and the DORT2002 equations become

$$\begin{cases} \frac{dI^+}{dx} = -2\sigma_a I^+ - 2\sigma_s I^+ + \sigma_s I^+ + \sigma_s I^- \\ -\frac{dI^-}{dx} = -2\sigma_a I^- - 2\sigma_s I^- + \sigma_s I^- + \sigma_s I^+ \end{cases}. \tag{6}$$

The equation has been written in this form here for the comparison with Kubelka-Munk. The first term on the right hand side is absorption, i.e. the intensity "disappears", the second term is totally scattered intensity, the third term is a contribution to the intensity, "scattered" from the same direction, and the fourth term is a contribution to the intensity, scattered from the opposite direction. The second and third terms together form the net scattered intensity to the opposite direction.

The fact that light incident at an angle has longer optical path to a certain depth in the medium is explicitly considered, and the effect is the factor ½ from the average of μ.

Hence, $2\sigma_a$ is the part of the intensity that is *absorbed* and "disappears" upon passage of the infinitesimal thickness *dx* of the medium. σ_s is the part of the intensity that is *scattered to the opposite direction* upon passage of the infinitesimal thickness *dx* of the medium.

4. Comparison of Coefficients

The conditions, under which an exact translation between the coefficients of the Kubelka-Munk and the DORT2002 models is valid, is perfectly diffuse light, perfectly isotropic scattering, and only two channels in the DORT2002 model.

Under these conditions it is obvious from the calculations above that the following relations hold:

Kubelka-Munk	DORT2002
k	$2\sigma_a$
s	σ_s
$\left(\frac{1}{2}(k+2s)\right)$	(σ_e)

(7)

The three validity conditions deserve a short discussion each. First, if the light has another angular distribution than perfectly diffuse, the average of μ is changed. Thus (4) is not valid, and (3) does not yield (6). Second, if the scattering is anisotropic, the phase function is no longer $\equiv 1$, and the integrals in (3), where the phase function is implicitly present, are changed so that they no longer are the hemispherical average $I^{\pm}(\tau)$. Thus, (3) does not yield (6).

The third validity condition deserves a longer discussion, the contents of which is derived and verified in [7]. It is tempting to think that if illumination and scattering were perfectly diffuse, the resulting light distribution would be perfectly diffuse. This is however wrong. The light distribution deviates from the perfectly diffuse due to the finite thickness of the medium. With more channels DORT2002 detects and quantifies this. In this case (3) becomes far more complicated, and a simple translation to (6) is not possible.

It is important to realize that scattering is a local phenomenon, and even if every scattering event is perfectly diffuse, the total scattered light distribution need not be perfectly diffuse due to edge effects. Kubelka and Munk did not recognize this fact in the first paper on their model. They assume infinite horizontal extension to avoid edge effects, but do not consider edge effects due to finite thickness. Even the follow-up paper of Kubelka that aims to theoretically derive a range of validity for that model does not recognize this fact, but rather assumes the opposite in the line of reasoning: “Practically it will be so when the illumination is a perfectly diffuse one and when the material forming specimens of different thickness reflects and transmits always perfectly diffused light only”. It is intuitively tempting to believe so, but this is not true when the medium has finite thickness since some light escapes through the lower boundary, and the light distribution becomes slightly changed.

The difference between the erroneous total reflectance given by Kubelka-Munk and the true value given by DORT2002 can be up to 15% and more, depending on the properties of the medium, and this *even under the theoretically ideal conditions that Kubelka-Munk was created for*.

5. Using Grammage Instead of Thickness

Van den Akker [8] showed that the Kubelka-Munk differential equations remain unchanged if the original scattering and absorption coefficients S and K (unit m^{-1}) are replaced with the specific scattering and absorption coefficients s and k (unit $m^2 \cdot kg^{-1}$), and the thickness X is replaced with weight per unit area (grammage) W. He proposed, based on practical application of the Kubelka-Munk model, that these should be used instead, which is indeed often the case. All relationships are unaffected by this.

This same convention is readily applied to the DORT2002 model. Although derived for scattering and absorption coefficients with unit m^{-1} and optical depth τ (changed to thickness X in the derivation above), the equations remain unchanged if specific scattering and absorption coefficients with unit $m^2 \cdot kg^{-1}$ are used instead, together with the grammage W. As for the Kubelka-Munk model, all relationships are unaffected by this.

It should be noted that, for both Kubelka-Munk and DORT2002, this can be done only if the scattering and absorption coefficients as well as the density of the medium are constant. Otherwise the specific scattering and absorption coefficients as well as the grammage will not be constants, which is assumed by the theory.

6. Discussion

An exact translation between the scattering and absorption coefficients of the Kubelka-Munk and DORT2002 models is given, together with the conditions under which it is valid. Reasons for non-validity in other cases are given.

Since the s and k values of the Kubelka-Munk model are so well known and in such widespread use, a translation to and from the DORT2002 coefficients will make it easy to use the models together, to compare results, and to change from one model to the other. This latter point is especially interesting, since it has been shown [5, 7] that it in several cases would be relevant to change from Kubelka-Munk to the more general DORT2002.

7. Acknowledgements

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