Introduction to Space-Time Codes

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I. INTRODUCTION

Optimal design and successful deployment of high-performance wireless networks present a number of technical challenges. These include regulatory limits on usable radio frequency spectrum and a complex time-varying propagation environment affected by fading and multipath. In order to meet the growing demand for higher data rates at better quality of service (QoS) with fewer dropped connections, boldly innovative techniques that improve both spectral efficiency and link reliability are called for. Use of multiple antennas at the receiver and transmitter in a wireless network is a rapidly emerging technology that promises higher data rates at longer ranges *without* consuming extra bandwidth or transmit power. This technology, popularly known as *smart antenna technology*, offers a variety of leverages which if exploited correctly can enable multiplicative gains in network performance.

Smart antenna technology provides a wide variety of options, ranging from single-input, multiple-output (SIMO) architectures that collect more energy to improve the signal to noise ratio (SNR) at the receiver, to multiple-input, multiple-output (MIMO) architectures that open up multiple data pipes over a link. The number of inputs and outputs here refers to the number of antennas used at the transmitter and receiver, respectively. Figure 1 shows a typical MIMO system with M_t transmit antennas and M_r receive antennas. The space-time (S-T) modem at the transmitter (Tx) encodes and modulates the information bits to be conveyed to the receiver and maps the signals to be transmitted across space (M_t transmit antennas) and time. The S-T modem at the receiver (Rx) processes the signals received on each of the M_r receive antennas according to the transmitter's signaling strategy and demodulates and decodes the received signal.

Different smart antenna architectures provide different benefits which can be broadly classified as *array gain*, *diversity gain*, *multiplexing gain* and *interference reduction*. The signaling strategy at the transmitter and the corresponding processing at the receiver are designed based on link requirements (data rate, range, reliability etc.). For example, in order to increase the point-to-point spectral efficiency (in bits/sec/Hz) between a transmitter and receiver, multiplexing gain is required which is provided by the MIMO architecture. The signaling strategy also depends on the availability of channel information at the transmitter. For example, MIMO does not require channel knowledge at the transmitter, although it enjoys improved performance if channel information is available. On the other hand, spatial division multiple access (SDMA) does require channel information at the transmitter which is used to increase the network throughput at the media access (MAC) layer. The advantage of point-to-multipoint SDMA over point-to-point MIMO is that SDMA deploys multiple antennas only at the cellular base station or wireless local area network (LAN) access point, thus reducing cost of the cellphone or network interface card (NIC).

The basic smart antenna architectures are summarized in Table 1 along with different algorithms that can be implemented on each architecture. Each combination of algorithm and architecture provides a key differentiating advantage and the corresponding improvement in network performance. The baseline architecture used for comparison is single-input, single-output

Metric	Max range and reliability	Max data rate/user	Max network throughput
Gain over SISO	Array gain / diversity gain	Multiplexing gain	Interference reduction
SIMO ($M_r \times 1$)			
Rx diversity	X		
CCI nulling			X
MISO ($1 \times M_t$)			
Tx diversity	X		
Beamforming	X		
SDMA (M_t users)			X
MIMO ($M_r \times M_t$)			
Tx/Rx diversity	X		
CCI nulling			X
SDMA (M_t users)			X
SM (M_t streams)		X	

TABLE I

KEY BENEFITS (X) OF DIFFERENT SMART ANTENNA ARCHITECTURES

(SISO), i.e. $M_t = M_r = 1$, where M_t is the number of transmit antennas and M_r is the number of receive antennas. The newly introduced acronyms in Table 1 are as follows : transmitter (Tx), receiver (Rx), multiple-input single-output (MISO), cochannel interference (CCI), and spatial multiplexing (SM). Note that Table 1 assumes that in order to maintain low cost analog components, the maximum constellation size per transmit antenna cannot be increased when multiple antennas are added. It also assumes that $M_r - 1$ interferers are jointly nulled for CCI reduction, M_t users are simultaneously served by SDMA, and M_t data streams are transmitted over an $M_r \times M_t$ MIMO link. Finally, the benefits listed are direct gains achievable with the smart antenna techniques listed in the leftmost column. Depending on the channel conditions and adaptation algorithms implemented at the medium access control (MAC) and physical (PHY) layers, these direct benefits may trigger indirect cumulative gains such as improved network throughput.

Note that while array gain, diversity gain and interference reduction are all provided by simple SIMO and MISO systems, multiplexing gain which is required to increase point-to-point throughput is only provided by MIMO systems. In fact SIMO architectures can increase the network throughput only if the base station uses SDMA technology. In the next few sections we will explore the subtleties of smart antenna gains in greater depth. Starting with a simple signal model, the basic smart antenna benefits namely array gain, diversity gain, multiplexing gain and interference reduction will then be discussed in greater detail.

II. Multiple antenna channel model

Consider a MIMO system with M_t transmit antennas and M_r receive antennas as shown in Figure 2. For simplicity we consider only flat fading; i.e., the fading is not frequency selective. When a continuous wave (CW) probing signal, s, is launched from the j^{th} transmit antenna, each of the M_r receive antennas sees a complex-weighted version of the transmitted signal. We denote the signal received at the i^{th} receive antenna by $h_{ij}s$, where h_{ij} is the channel response between the j^{th} transmit antenna and the i^{th} receive antenna. The vector $[h_{1j}\ h_{2j}\ \cdots\ h_{M_Rj}]^T$ [1] is the signature induced by the j^{th} transmit antenna across the receive antenna array. It is

[1] The superscript ^{T} stands for matrix transpose.

convenient to denote the MIMO channel ($\mathbf{H}$) in matrix notation as shown below.

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M_T} \\ h_{21} & h_{22} & \cdots & h_{2M_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M_R1} & h_{M_R2} & \cdots & h_{M_RM_T} \end{bmatrix} \tag{1}$$

The channel matrix $\mathbf{H}$ defines the input-output relation of the MIMO system and is also known as the channel transfer function. If a signal vector $\mathbf{x} = [x_1 \ x_2 \ \cdots \ x_{M_t}]^T$ is launched from the transmit antenna array (x_j is launched from the j^{th} transmit antenna) then the signal received at the receive antenna array, $\mathbf{y} = [y_1 \ y_2 \ \cdots \ y_{M_r}]^T$ can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{2}$$

where $\mathbf{v}$ is the $M_r \times 1$ noise vector consisting of independent complex-gaussian distributed elements with zero mean and variance σ_v^2 (white noise). Note that the above channel matrix can be interpreted as a snapshot of the wireless channel at a particular frequency and at a specific instant of time. When there is rich multipath with a large delay spread, $\mathbf{H}$ varies as a function of frequency. Likewise, when the scatterers are mobile and there is a large doppler spread, $\mathbf{H}$ varies as a function of time. With sufficient antenna separation at the transmit and receive arrays, the elements of the channel matrix $\mathbf{H}$ can be assumed to be independent, zero-mean, complex-gaussian random variables (Rayleigh fading) with unit variance in sufficiently rich multipath. This model is popularly referred to as the i.i.d Gaussian MIMO channel. In general if antennas are separated by more than half the carrier wavelength ($\frac{\lambda}{2}$) [1], the channel fades can be modeled as independent Gaussian random variables.

This point-to-point model can be extended to multiple users by indexing $\mathbf{H}$ as $\mathbf{H}_u$ where $\mathbf{H}_u$ is the $M_r \times M_t$ channel from the u^{th} user to the receiver, as shown below

$$\mathbf{y} = [\mathbf{H}_1 \cdots \mathbf{H}_U] \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_U \end{bmatrix} + \mathbf{v} \tag{3}$$

where $\mathbf{x}_u$ is the $M_t \times 1$ signal transmitted from the u^{th} user. This system model can be easily generalized to unequal numbers of transmit antennas at different users. In this chapter we will focus on the single user case.

III. Benefits of smart antenna technology

Equipped with the mathematical system description via (2) and (3), we will now describe different smart antenna gains in detail.

A. Array gain

Consider a SIMO system with one transmit antenna and two receive antennas as shown in Figure 3. The two receive antennas see different versions, y_1 and y_2, of the same transmitted signal, x. The signals y_1 and y_2 have different amplitudes and phases as determined by the propagation conditions. If the channel is known to the receiver, appropriate signal processing techniques can be applied to combine the signals y_1 and y_2 coherently so that the resultant power of the signal at the receiver is enhanced, leading to an improvement in signal quality. More specifically, the SNR at the output is equal to the sum of the SNR on the individual links. This result can be extended to systems with one transmit antenna and more than two receive antennas as follows [2]

$$\mathbf{w}^*\mathbf{y} = \mathbf{w}^*\mathbf{h}x + \mathbf{w}^*\mathbf{v} \tag{4}$$

where the optimal $M_r \times 1$ linear receive filter is $\mathbf{w} = \mathbf{h}$, and the maximum SNR is proportional to the channel norm $||\mathbf{h}||^2 = \sum_{m=1}^{M_r} |\mathbf{h}_m|^2$, where $||\mathbf{h}||^2$ is the Frobenius norm [3]. The average increase in receive signal power at the receiver = $\mathrm{E}||\mathbf{h}||^2$ is defined as *array gain* and is proportional to the number of receive antennas.

Array gain can also be exploited in systems with multiple antennas at the transmitter by using beamforming. Extracting the maximum possible array gain in such systems requires channel knowledge at the transmitter, so that the signals may be optimally processed before transmission. An example of transmit beamforming for $1 \times M_t$ MISO systems is shown below

$$y = \mathbf{h}^*(\mathbf{w}x) + v \tag{5}$$

The optimal normalized $M_t \times 1$ transmit filter is $\mathbf{w} = \mathbf{h}/||\mathbf{h}||$. Analogous to the SIMO case, the array gain in MISO systems with channel knowledge at the transmitter is equal to $\mathrm{E}||\mathbf{h}||^2$ and is proportional to the number of transmit antennas. The array gain in MIMO systems depends on

[2]The superscript * stands for conjugate transpose.

[3]The Frobenius norm of a matrix $\mathbf{A}$ is defined to be $||\mathbf{A}||^2 = \sum_{i,j} |\mathbf{a}_{i,j}|^2$

the number of transmit and receive antennas and is a function of the dominant singular value of the channel.

B. Diversity gain

Signal power in a wireless channel fluctuates (or fades) with time/frequency/space. When the signal power drops dramatically, the channel is said to be in a fade. Diversity is used in wireless systems to combat fading. The basic principle behind diversity is to provide the receiver with several looks at the transmitted signal over independently fading links (or diversity branches). As the number of diversity branches increases, the probability that at any instant of time one or more branch is not in a fade increases. Thus diversity helps stabilize a wireless link.

Diversity is available in SISO links in the form of time or frequency diversity. The use of time or frequency diversity in SISO systems often incurs a penalty in data rate due to the utilization of time or bandwidth to introduce redundancy. The introduction of multiple antennas at the transmitter and/or receiver provides spatial diversity, the use of which does not incur a penalty in data rate while providing the array gain advantage discussed earlier. In this chapter we are concerned with this form of diversity. There are two forms of spatial diversity – receive and transmit diversity.

Receive diversity applies to systems with multiple antennas only at the receiver (SIMO systems) [2]. Figure 3 illustrates a system with receive diversity. Signal x is transmitted from a single antenna at the transmitter. The two receive antennas see independently faded versions, y_1 and y_2, of the transmitted signal, x. The receiver combines these signals using appropriate signal processing techniques so that the resultant signal exhibits much reduced amplitude variability (fading) as compared to either y_1 or y_2. The amplitude variability can be further reduced by adding more antennas to the receiver. The diversity in a system is characterized by the number of independently fading diversity branches, also known as the diversity order. The diversity order of the system in Figure 3 is two and in general is equal to the number of receive antennas, M_r, in a SIMO system.

Transmit diversity is applicable when multiple antennas are used at the transmitter and has become an active area for research in the past few years [3], [16], [4]. Extracting diversity in such systems does not necessarily require channel knowledge at the transmitter. However, suitable design of the transmitted signal is required to extract diversity. Space-time coding

[5], [6] is a powerful transmit diversity technique that relies on coding across space (transmit antennas) and time to extract diversity. Figure 4 shows a generic transmit diversity scheme for a system with two transmit antennas and one receive antenna. At the transmitter, signals x_1 and x_2 are derived from the original signal to be transmitted, x, such that the signal x can be recovered from either of the received signals y_{11} or y_{21}. The receiver combines the received signals in such a manner that the resultant output exhibits reduced fading when compared to y_{11} or y_{21}. The diversity order of this system is two and in general is equal to the number of transmit antennas, M_t, in a MISO system.

Utilization of diversity in MIMO systems requires a combination of receive and transmit diversity described above. A MIMO system consists of $M_t \times M_r$ SISO links. If the signals transmitted over each of these links experience independent fading, then the diversity order of the system is given by $M_t \times M_r$. Thus the diversity order in a MIMO system scales linearly with the product of the number of receive and transmit antennas. Mathematically, diversity is defined to be equal to the slope of the symbol error rate (SER) versus SNR graph. This will be shown in greater detail in the following derivation.

The vector equation in (2) can be written as the following matrix equation

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{V} \tag{6}$$

where the channel input $\mathbf{X}$ is an $M_t \times T$ codeword spanning T sample times, the channel output is the $M_r \times T$ matrix $\mathbf{Y}$ observed on M_r receive antennas over T sample times and the receiver noise is the $M_r \times T$ matrix $\mathbf{V}$.

Consider two $M_t \times T$ codewords $\mathbf{X}^{(i)}$ and $\mathbf{X}^{(j)}$ that are transmitted over M_t transmit antennas across T sample times. If $\mathbf{X}^{(i)}$ was transmitted, the probability that $\mathbf{X}^{(j)} \neq \mathbf{X}^{(i)}$ is detected for a given realization of the channel $\mathbf{H}$ is equal to the following

$$\begin{aligned} \mathrm{PEP}_{|\mathbf{H}} &= \mathrm{Prob}(\mathbf{X}^{(i)} \rightarrow \mathbf{X}^{(j)}) \\ &= \mathrm{Prob}(\|\mathbf{Y} - \mathbf{H}\mathbf{X}^{(j)}\|_F^2 \leq \|\mathbf{Y} - \mathbf{H}\mathbf{X}^{(i)}\|_F^2) \ = \ \mathrm{Q}\left(\sqrt{D_{ij}\frac{\mathrm{SNR}}{2}}\right) \end{aligned} \tag{7}$$

where $\mathrm{Q}\left(x\right) = \int_x^\infty dt\, \frac{1}{\sqrt{2\pi}} \exp(-\frac{t^2}{2})$ is the complementary error function, $D_{ij} = \left\|\mathbf{H}\ (\mathbf{X}^{(i)} - \mathbf{X}^{(j)})\right\|_F^2$ is the pairwise Euclidean distance at the receiver and $\mathrm{SNR} = \frac{E_s}{N_0}$ is the ratio of the total transmitted signal power to the noise power per receive antenna.

This *conditional* pairwise error probability (PEP) is a function of the channel realization. Since the transmitter does not know the channel, the best it can do is optimize a criterion that takes channel statistics into account. One popular criterion is the *average PEP*, i.e. the average of the conditional PEP over channel statistics. It is difficult to compute the expectation of the expression in (7). A simpler alternative is to compute the average of a tight upper bound, in particular the Chernoff upper bound

$$\mathrm{PEP}_{|\mathbf{H}} \;=\; \mathrm{Q}\left(\sqrt{D_{ij}\frac{\mathrm{SNR}}{2}}\right) \;\leq\; e^{-D_{ij}\frac{\mathrm{SNR}}{4}} \tag{8}$$

For the i.i.d. Gaussian channel, the average Chernoff bound simplifies to the following as derived in [4], [5]

$$\mathrm{PEP} \;\leq\; \left(\frac{1}{\det(\mathbf{I}_{M_t}+\frac{\mathrm{SNR}}{4}(\mathbf{X}^{(i)}-\mathbf{X}^{(j)})(\mathbf{X}^{(i)}-\mathbf{X}^{(j)})^*)}\right)^{M_r} = \left(\frac{1}{\prod_{l=1}^{L}(1+\frac{\mathrm{SNR}}{4}\sigma_l^2)}\right)^{M_r} \tag{9}$$

where $\det$ is the determinant of a square matrix, $\{\sigma_l\}_{l=1}^{L}$ are the nonzero singular values of the difference matrix $\mathbf{\Delta}_{ij} = \mathbf{X}^{(i)} - \mathbf{X}^{(j)}$ and L is its rank. Taking the limit at high SNR,

$$\mathrm{PEP} \;\leq\; \left(\frac{\mathrm{SNR}}{4}\right)^{-M_rL}\left((\prod_{l=1}^{L}\sigma_l^2)^{\frac{1}{L}}\right)^{-M_rL} \tag{10}$$

and taking the logarithm of both sides, we have

$$\log\mathrm{PEP} \;\leq\; -M_rL\left(\log\left(\frac{\mathrm{SNR}}{4}\right)+\log\left((\prod_{l=1}^{L}\sigma_l^2)^{\frac{1}{L}}\right)\right) \tag{11}$$

Consider the logarithm of the PEP in (11). The right hand side is clearly linear in the logarithms of SNR and the product of squared singular values of the difference matrix. In addition, the slope of the r.h.s. is a product of the number of receive antennas and the rank of the difference matrix. The *diversity gain* of the space-time codebook is defined to be the minimum value of L over all pairs of codewords. For a given diversity gain, the *coding gain* is defined to be the minimum of the product $(\prod_{l=1}^{L}\sigma_l^2)^{\frac{1}{L}}$ over all pairs of codewords.

Performance of space-time codes is usually illustrated by plotting the SER versus SNR on a logarithmic scale. Since the PEP is closely related to SER, (11) is a good approximation to SER especially at high SNRs. Figure 5 illustrates the effect of each code metric on the SER curve. *Diversity gain* affects the asymptotic slope of the SER versus SNR graph - greater the diversity, the faster the SER drops with SNR. *Coding gain* affects the horizontal shift of the graph - greater the coding gain, the greater the shift to the left.

C. Multiplexing gain

The key differentiating advantage of MIMO systems is practical throughput enhancement which is not provided by SIMO or MISO systems. We refer to this leverage as multiplexing gain and it can be realized through a technique known as spatial multiplexing [7], [8]. Figure 6 shows the basic principle of spatial multiplexing for a system with two transmit and two receive antennas. The symbol stream to be transmitted is split into two half-rate sub-streams and modulated to form the signals x_1 and x_2 that are transmitted simultaneously from separate antennas. Under favorable channel conditions, the spatial signatures of these signals (denoted by $[y_{11}\ y_{12}]^T$ and $[y_{21}\ y_{22}]^T$) induced at the receive antennas are well separated (ideally orthogonal). The receiver can then extract the two sub-streams, x_1 and x_2, which it combines to give the original symbol stream, x.

This can be mathematically expressed as the theoretical channel capacity as derived in [9], [10]. Channel capacity of the memoryless MIMO channel in (2) is defined to be the "instantaneous" mutual information which is a function of the channel realization as follows

$$C_{|\mathbf{H}} \ = \ \log\det\left(\mathbf{I}_{M_r} + \mathrm{SNR}\ \mathbf{H}\mathbf{K}_{\mathbf{X}}\mathbf{H}^*\right) \qquad (12)$$

When the channel is square and orthogonal ($\mathbf{H}\mathbf{H}^* = \mathbf{I}$), then with an i.i.d.input distribution ($\mathbf{K}_{\mathbf{X}} = \frac{1}{M_t}\mathbf{I}_{M_t}$), (12) reduces to

$$C_{\perp} \ = \ M_t \log\left(1 + \frac{1}{M_t}\mathrm{SNR}\right) \qquad (13)$$

Hence $M = M_t = M_r$ parallel channels are created within the same frequency bandwidth for no additional transmit power. Capacity scales linearly with number of antennas for increasing SNR, i.e. capacity increases by M b/s/Hz for every 3 dB increase in SNR. In general, it can be shown that an orthogonal channel of the form described above maximizes the Shannon capacity of a MIMO system. For the i.i.d fading MIMO channel model described earlier, the channel realizations become approximately orthogonal when the number of antennas used is very large. When the number of transmit and receive antennas is not equal, $M_t \neq M_r$, the increase in capacity is limited by the minimum of M_t and M_r. This increase in channel capacity is called *multiplexing gain*.

The capacity-maximizing input $\mathbf{X}$ is a zero-mean, complex Gaussian vector with covariance $\mathbf{K}_{\mathbf{X}} = \mathrm{E}\mathbf{X}\mathbf{X}^*$. For example, when the channel $\mathbf{H}$ is fully known at the transmitter, $C_{|\mathbf{H}}$ is maximized by $\mathbf{K_X}$ that waterpours power over the dominant singular values of $\mathbf{H}$ [9]. The conditional capacity can be achieved by coding over longer and longer block lengths, assuming that the channel is time-invariant. In practice the fading channel does vary with time, and a commonly used measure of rate in this case is the *average channel capacity*. Average capacity of the memoryless MIMO channel is defined to be the average mutual information described as follows

$$C \;=\; \mathrm{E}_{\mathbf{H}} C_{|\mathbf{H}} \;=\; \mathrm{E}_{\mathbf{H}} \log\det\left(\mathbf{I}_{M_r} + \mathrm{SNR}\,\mathbf{H}\mathbf{K_X}\mathbf{H}^*\right) \tag{14}$$

where the average is computed over the channel distribution function. It has been shown in [9] that the iid input distribution is optimal for the iid Gaussian channel. Figure 7 shows the average capacity as a function of the SNR for the i.i.d fading channel model for different MIMO configurations. It is clear that the average capacity increases with the number of antennas in the system. At very low SNR, the MIMO multiplexing gain is low, but it increases with increasing SNR becoming asymptotically constant.

D. Interference reduction

In contrast to copper or optical fiber, the wireless medium is an unguided communication link as a result of which co-channel interference is a frequent problem due to the reuse of frequency spectrum in wireless networks. CCI adds to the overall noise in the system and deteriorates performance. Figure 8 illustrates the general principle of interference reduction for a receiver with two antennas. Typically, the desired signal (s) and the interference (i) arrive at the receiver with well separated spatial signatures - $[s_1\ s_2]^T$ and $[i_1\ i_2]^T$ respectively. The receiver can exploit the difference in spatial signatures to reduce the interference, thereby enhancing the signal to interference ratio (SIR). Interference reduction usually requires knowledge of the desired signal's spatial channel and benefits from knowledge of the interferers' spatial channels. Interference reduction can also be implemented at the transmitter via SDMA, where the goal is to enhance the signal power at the intended receiver and minimize the interference energy sent towards the co-channel users. Interference reduction allows the use of aggressive reuse factors and improves network capacity.

Having discussed the key advantages of smart antenna technology we note that it may not be possible to exploit all the leverages simultaneously in a smart antenna system. This is because the spatial degrees of freedom are limited and engineering tradeoffs must be made between each of the desired benefits. The optimal spatio-temporal signaling strategy is a function of the wireless channel properties and network requirements.

IV. Background on space-time codes

As described in the previous section, MIMO systems promise much higher spectral efficiency than SISO systems. MIMO systems can also be leveraged to improve the quality of transmission (reduce error rate). This section will focus on MIMO signaling schemes that assume perfect channel knowledge at the receiver and no channel knowledge at the transmitter.

A. Space-time trellis codes

For a given number of transmit antennas, the code design objective from (11) is to construct the largest possible codebook with full diversity gain ($L = M_t$) and the maximum possible coding gain. A number of hand-crafted STTCs (space-time trellis codes) with full diversity gain were first provided in [5]. Full diversity codes with greater coding gain were then reported in [13], where codes were found through exhaustive computer searches over a feedforward convolutional coding (FFC) generator. New codes were then presented in [15] by searching for the codes with the best distance spectrum properties. The distance spectrum of a codebook counts how many pairs of codewords are located at a given product distance (defined as $(\prod_{l=1}^{L} \sigma_l^2)^{\frac{1}{L}}$ for the ij codeword pair in (11)).

To provide some insight into all these codes, Figures 9 and 10 show the frame error rate for two transmit antennas over an iid Gaussian channel. As a reference, a linear space-time block code (STBC) from [12] is also shown, with and without a concatenated AWGN trellis code (STBC + TCM). The STBC does not provide coding gain but does provide full diversity, which is of order $2M_r$ for both the STBC and the STTCs considered here.

In Figure 9 (a) note that STBC by itself performs slightly better than all the STTCs, even though it provides no coding gain. This is explained by the multidimensional structure of STBC, i.e., each input symbol is spread over two time samples which improves performance against AWGN. With one receive antenna, concatenated STBC performs significantly better than all the 4-state STTCs. The performance gap reduces with increasing receive antennas in Figure 10. With three receive antennas and 8 trellis states, in fact, STTC-Yan outperforms STBC-TCM.

This performance loss is explained by the well-known capacity loss incurred by STBC [18], and will be addressed in greater detail in the next section.

The performance gap between STBC-TCM and STTC is also less noticeable with 8-state codes for all receive antennas. We conjecture that the distance spectrum of the concatenated scheme degrades in comparison to the distance spectrum of STTC with increasing number of TCM states. While we will not directly address the distance spectra of space-time codes here, we will introduce a tighter error criterion than the worst-case PEP which reflects the effects of the distance spectrum on error probability.

Let us summarize the three main observations made so far. Space-time block codes which are linear in the input information symbols are simple to encode and decode. Since STBCs outperform STTCs for small number of antennas, they are of great interest in such MIMO architectures. In general, the distance spectrum of the codebook is a better measure of error performance than the maximum pairwise error probability, and should be incorporated into code design metrics. Finally, linear block codes with the best error performance do not always demonstrate the best capacity performance.

B. Linear space-time block codes

Since linear codes are easier to encode and decode, we will focus on the design of linear codes here. A *linear code* is defined as a set of codewords that are linear in the scalar input symbols. Complex valued $M_t \times T$ *modulation matrices* $\{\mathbf{A}_k\}_{k=1}^{K}$ are used to spread the input information symbols over M_tT spatio-temporal dimensions. The real and imaginary part of each input symbol s_k is modulated separately with the matrices $\mathbf{A}_k$ and $\mathbf{A}_{k+\frac{K}{2}}$. Define $x_k = \Re s_k$ and $x_{k+\frac{K}{2}} = \Im s_k$ where $1 \leq k \leq K$. The modulated matrices are summed to obtain the $M_t \times T$ codeword $\mathbf{X}$ as follows

$$\mathbf{X} \;=\; \sum_{k=1}^{K/2}(\mathbf{A}_k\Re s_k + \mathbf{A}_{k+K/2}\Im s_k) \;=\; \sum_{k=1}^{K}\mathbf{A}_k x_k \tag{15}$$

The number of modulation matrices K is usually upper bounded by the total number of spatio-temporal degrees of freedom $2M_tT$. When $K < 2M_tT$, the modulation matrices can be designed to be orthogonal (as defined in (20), (21)) in order to optimize error performance as shown in the next section. For optimal capacity performance, however, in general $K = 2M_tT$ and the modulation matrices are not orthogonal.

In order to normalize transmit power, the modulation matrices are scaled as follows

$$\begin{aligned} x_k \in \{\text{QAM}\ ,\ \text{PSK}\} \ &\Rightarrow\ \|\mathbf{A}_k\|_F^2 \ \le\ c\ =\ \frac{2T}{K}\ \text{ for all } k \\ x_k \in \{\text{PAM}\} \ &\Rightarrow\ \|\mathbf{A}_k\|_F^2 \ \le\ c\ =\ \frac{T}{K}\ \text{ for all } k \end{aligned} \tag{16}$$

for different input constellations such as quadrature amplitude modulation (QAM), phase shift keying (PSK) and pulse amplitude modulation (PAM). Most of the current spatial modulation techniques can be interpreted as linear codes, the key exceptions being space-time trellis codes.

Example 1—Spatial multiplexing: Spatial multiplexing [7], also called BLAST [10], is the simplest example of linear codes. Each incoming symbol is transmitted only once on one antenna and at one symbol time. The modulation matrices are Kronecker delta matrices, i.e., each $M_t \times T$ matrix is equal to 1 in the i^{th} row and the j^{th} column and zero elsewhere. For example, for the case $M_t = 2, T = 1$, the modulation matrices are as follows

$$\mathbf{X} \ =\ \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \ =\ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Re s_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Re s_2 + \begin{bmatrix} j \\ 0 \end{bmatrix} \Im s_1 + \begin{bmatrix} 0 \\ j \end{bmatrix} \Im s_2$$

Example 2—Orthogonal space-time block codes: Orthogonal space-time block codes are a special example of linear codes, where the codeword $\mathbf{X}$ is designed to be a unitary matrix. Such modulation matrices exist for limited values of K, M_t and T [6]. For example, the orthogonal block code for $M_t = 2$, $T = 2$ is the Alamouti code [12] consisting of four matrices shown below

$$\begin{aligned} \mathbf{X} \ =\ \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} \ &=\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Re s_1 + \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix} \Im s_1 \\ &+ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \Re s_2 + \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \Im s_2 \end{aligned} \tag{17}$$

Example 3—Delay diversity: Delay diversity [3], [16] is really a trellis code that can also be classified as a linear code. The $M_t \times T$ modulation matrices are proportional to $[0 I_{M_t} 0]$, where the order M_t identity matrix is shifted right by $k-1$ columns in the k^{th} modulation matrix. For any $T \ge M_t$, the total number of complex symbols thus encoded is $M_t + T - 1$. The case $M_t = 2, T = 2$ is shown below

$$\mathbf{X} \ =\ \begin{bmatrix} s_2 & s_3 \\ s_1 & s_2 \end{bmatrix} \ =\ \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \Re s_1 + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Re s_2 + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \Re s_3$$

$$+\begin{bmatrix} 0 & 0 \\ j & 0 \end{bmatrix}\Im s_1+\begin{bmatrix} j & 0 \\ 0 & j \end{bmatrix}\Im s_2+\begin{bmatrix} 0 & j \\ 0 & 0 \end{bmatrix}\Im s_3$$

This framework can be extended to non-linear codes by modulating the modulation matrices with nonlinear functions of the input symbols. For example, for $K=3$, the following vector of input symbols leads to a nonlinear space-time code

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \Re s_1 \\ \Im s_1 \\ \Re s_1 \Im s_1 \end{bmatrix}$$

This is similar to the analytic representation of TCM [17], except that the coefficients of the analytic expansion are now modulation matrices instead of scalars.

V. New design criteria

In this section we will first discuss design criteria for optimizing error performance and capacity performance separately. Then we will bring the two together to design capacity-efficient codes that also provide good error performance.

A. Error performance

Evaluating the distance spectrum of space-time codes is difficult. Optimizing their distance spectrum is even harder, and possibly not very insightful. We propose a simple new criterion that efficiently incorporates the effects of the distance spectrum, i.e., the *union bound on error probability*. The union bound is an upper bound on average error probability, where the average is taken over the entire matrix constellation. Moreover, the union bound is a tighter upper bound than the worst case PEP. This follows because the union bound is an average whereas the worst case PEP is the maximum, and can be seen from the following inequalities

$$P_e \;\leq\; P_U \;=\; \sum_{i=1}^{R} p_i \sum_{j\neq i}^{R} \mathrm{PEP}_{ij} \;\leq\; (R-1)\max_{ij} \mathrm{PEP}_{ij} \tag{18}$$

for a constellation consisting of R equally likely codewords, where p_i is the probability that the i^{th} codeword was transmitted. Therefore it is advisable to minimize the scaled union bound, i.e.

$\frac{1}{R-1}P_U$, rather than the maximum PEP, i.e. $\max_{ij}\mathrm{PEP}$. To provide a contrast we illustrate the two bounds in Figure 11 for the Alamouti code and spatial multiplexing.

The union bound on error probability for linear codes can be averaged over the iid Gaussian channel and written as follows [19]

$$P_A \leq \sum_{i=1}^{R} p_i \sum_{j\neq i}^{R} \left(\det\left(\mathbf{I}_{M_t} + \frac{SNR}{4}\sum_{k=1}^{K}\sum_{l=1}^{K}\mathbf{A}_k\mathbf{A}_l^*\epsilon_k^{(ij)}\epsilon_l^{(ij)}\right)\right)^{-M_r} \tag{19}$$

where R is the size of the codebook ($R = M^{\frac{K}{2}}$ for an input M-QAM constellation), and $\epsilon_k^{(ij)} = x_k^{(i)} - x_k^{(j)}$ is the difference of the input sequences $x^{(i)}$ and $x^{(j)}$ at the k^{th} location. See [19] for a detailed proof.

Let the modulation matrices be unitary [4], i.e. $\mathbf{A}_k\mathbf{A}_k^* = \mathbf{I}_{M_t}$ for $M_t \leq T$ and $\mathbf{A}_k^*\mathbf{A}_k = \mathbf{I}_T$ for $M_t \geq T$. Then we have the following necessary and sufficient condition for code design.

Theorem 1: A linear code consisting of *wide* unitary modulation matrices minimizes P_A iff it satisfies (20)

$$\mathbf{A}_k\mathbf{A}_l^* + \mathbf{A}_l\mathbf{A}_k^* = \mathbf{0} \qquad \text{for } 1 \leq k \neq l \leq K \tag{20}$$

A linear code consisting of *tall* unitary modulation matrices minimizes P_A iff it satisfies (21)

$$\mathbf{A}_k^*\mathbf{A}_l + \mathbf{A}_l^*\mathbf{A}_k = \mathbf{0} \qquad \text{for } 1 \leq k \neq l \leq K \tag{21}$$

See [19] for a detailed proof. This gives us the conditions required for optimal error performance. Note that these conditions precisely describe orthogonal space-time block codes [6].

B. Capacity performance

We can rewrite the complex channel in (6) as the following real channel

$$\mathbf{y} = \mathcal{H}\mathcal{A}\mathbf{x} + \mathbf{v} \tag{22}$$

where $\mathbf{y}$ ($2M_rT \times 1$) is the channel output vector, $\mathcal{H}$ ($2M_rT \times 2M_tT$) is the block diagonal channel, $\mathcal{A}$ ($2M_tT \times 2K$) is the linear code matrix to be designed, $\mathbf{x}$ ($2K \times 1$) is a block of uncoded input symbols with each entry of power $E_s/2$, and $\mathbf{v}$ ($2M_rT \times 1$) is the noise vector

[4]Unitary modulation matrices achieve the matched filter bound, i.e. they minimize error probability when $K = 1$.

with each entry of power $N_0/2$. These quantities are defined as follows [5]

$$
\begin{aligned}
\mathbf{y} &= \mathrm{vec}(\begin{bmatrix} \Re\mathbf{Y} \\ \Im\mathbf{Y} \end{bmatrix}) \\
\mathcal{H} &= \mathbf{I}_T \otimes \overline{\mathbf{H}}\,,\ \mathrm{where}\ \overline{\mathbf{H}} \ = \begin{bmatrix} \Re\mathbf{H} & -\Im\mathbf{H} \\ \Im\mathbf{H} & \Re\mathbf{H} \end{bmatrix} \\
\mathcal{A} &= \begin{bmatrix} \mathrm{vec}\left(\begin{bmatrix} \Re\mathbf{A}_1 \\ \Im\mathbf{A}_1 \end{bmatrix}\right) & \ldots & \mathrm{vec}\left(\begin{bmatrix} \Re\mathbf{A}_{2K} \\ \Im\mathbf{A}_{2K} \end{bmatrix}\right) \end{bmatrix} \\
\mathbf{x} &= [\Re x_1, \ldots, \Re x_K, \Im x_1, \ldots, \Im x_K]^T \\
\mathbf{v} &= \mathrm{vec}(\begin{bmatrix} \Re\mathbf{V} \\ \Im\mathbf{V} \end{bmatrix})
\end{aligned}
\tag{23}
$$

and are all real. This reformulation cleanly separates the channel from the linear code, thereby enabling us to analytically maximize channel capacity.

Average capacity of this input-output equation can be written as

$$
C \ = \ \mathrm{E}_{\mathbf{H}} \frac{1}{2T} \log\det(\mathbf{I}_{2M_rT} + \mathrm{SNR}\ \mathcal{H}\mathcal{A}\mathcal{A}^T\mathcal{H}^T) \tag{24}
$$

Theorem 2: In order to maximize the average channel capacity over all normalized linear codes $\mathcal{A}$ for *any number of receive antennas,* it is sufficient that the optimal linear code $\mathcal{A}$ satisfy the following factorization

$$
\mathcal{A} \ = \ (\mathbf{I}_T \otimes \mathcal{B})\mathcal{Q} \tag{25}
$$

where $\mathcal{B}$ is a ($2M_t \times r$) factor of the input covariance $\mathcal{K}_{\mathcal{A}} = \mathcal{A}\mathcal{A}^T = \mathcal{B}\mathcal{B}^T$, and $\mathcal{Q}$ is any ($rT \times 2K$) unitary matrix such that $\mathcal{Q}\mathcal{Q}^T = \mathbf{I}$ (where $rT \leq 2K$). See [19] for a detailed proof.

If the channel is i.i.d.Gaussian, then average capacity is maximized by choosing $\mathcal{K} = \frac{1}{M_t}\mathbf{I}_{2M_tT}$ [9]. In this case, $\mathcal{B}$ is a unitary matrix and the optimal linear code is unitary. A code that maximize average channel capacity is called *capacity-efficient*. For the i.i.d.Gaussian channel, any code that satisfies $\mathcal{A}\mathcal{A}^T = 1/M_t\mathbf{I}$ is capacity-efficient. Note that Theorem 2 is a sufficient condition to design capacity-efficient codes for *any number of receive antennas*. This condition is not necessary for limited cases, e.g. the Alamouti code does not satisfy Theorem 2 but is

[5] $\mathrm{vec}(\mathbf{A})$ is the column-by-column vectorization of matrix $\mathbf{A}$, $\otimes$ is the matrix kronecker product, and $\overline{\mathbf{H}}$ is the entrywise complex conjugate of $\mathbf{H}$.

capacity-efficient for one receive antenna (it is not capacity-efficient for two or more receive antennas).

C. Unified design

In this section we will put together the criteria derived in the previous two sections to design new linear space-time block codes. First, define the following metrics for code design :

- *Deviation from unitarity* is defined as follows

$$d_1 = \frac{1}{K}\sum_{k=1}^{K}\kappa(\mathbf{A}_k) - 1 \tag{26}$$

  where κ is the condition number with respect to the spectral norm. When each of the modulation matrices is unitary, $d_1 = 0$.
- *Deviation from pairwise skew-Hermitianity* is defined as follows

$$d_2 = \frac{1}{K}\sum_{k<l}\|\mathbf{A}_k\mathbf{A}_l^* + \mathbf{A}_l\mathbf{A}_k^*\|_F^2 \tag{27}$$

  where $\|.\|_F^2$ is the squared Frobenius norm. When all the modulation matrices are pairwise skew-Hermitian, $d_2 = 0$, and when they also satisfy $d_1 = 0$, the code is error-optimal.
- *Deviation from capacity-efficiency* is defined as follows

$$d_3 = \|\mathcal{A}\mathcal{A}^T - \frac{1}{M_t}\mathbf{I}\|_F^2 \tag{28}$$

  When $d_3 = 0$, the code is capacity-efficient for any number of receive antennas. For codes that are capacity-efficient for limited values of M_r, such as the Alamouti code with one receive antenna, it is possible that $d_3 > 0$.

Using these metrics, we will design capacity-efficient codes that also perform well with respect to error probability. Our method of code generation is via random search, which is explained in detail in the following.

We will focus on the case of two transmit antennas and block length of two. The minimum number of modulation matrices required for capacity-efficient codes is $K = 2M_tT = 8$. A large number of real, 8×8 random matrices with zero-mean, unit-variance Gaussian entries were generated, their singular value decompositions were computed, and their left singular matrix was extracted as a candidate for the code matrix $\mathcal{A}$. Since the left singular matrix is orthogonal by definition, it is automatically capacity-efficient and $d_3 = 0$. For each such code, its non-zero deviations d_1 and d_2 were evaluated. After evaluating all candidate codes, we chose a good code that had the lowest values of both d_1 and d_2 over the ensemble generated.

The performance of this chosen code is demonstrated over the i.i.d. Gaussian channel by means of simulated BER in Figure 12 for two receive antennas and a data rate of 4 bits per sample time. The new code is labeled "random CE" and is evaluated against other capacity-efficient codes such as spatial multiplexing ("SM") and Linear Dispersion ("LD"). For reference we have also plotted the performance of the Alamouti code ("STBC"). With two receive antennas and a rate of eight bits per codeword, CE outperforms both LD and SM at high SNRs. At a BER of 10^{-5}, CE is at an advantage of about 5 dB. It therefore has a much higher diversity than LD and SM, although not as much as STBC which outperforms all these codes at high enough SNRs. In Figure 13, with three receive antennas CE outperforms STBC by about 1 dB at a BER of 10^{-3}.

What this demonstrates is that all capacity-efficient codes don't perform equally well with respect to BER, and that the modulation matrices must also be pairwise skew-Hermitian and unitary in order to minimize BER. Note that in general, the relative performance of these codes is a function of SNR, number of receive antennas and data rate, and a code that is the best for a given set of conditions may not be the best under different conditions.

For reference we have reproduced the LD code [14]

$$
\begin{aligned}
\mathbf{A}_1 &= \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \mathbf{A}_2 = \frac{i}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
\mathbf{A}_3 &= \frac{1}{2}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \ \mathbf{A}_4 = \frac{i}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\
\mathbf{A}_5 &= \frac{1}{2}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \ \mathbf{A}_6 = \frac{i}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\
\mathbf{A}_7 &= \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \ \mathbf{A}_8 = \frac{i}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}
\end{aligned} \tag{29}
$$

and our new CE code below

$$
\mathbf{A}_1 = \begin{bmatrix} 0.2654 + 0.0803i & -0.2580 + 0.3128i \\ 0.2302 - 0.1754i & -0.3130 + 0.2775i \end{bmatrix}
$$

$$\mathbf{A}_2 = \begin{bmatrix} -0.2864 + 0.0865i & -0.2270 - 0.0092i \\ 0.3268 - 0.2990i & 0.0075 - 0.4033i \end{bmatrix}$$
$$\mathbf{A}_3 = \begin{bmatrix} 0.0212 - 0.0311i & 0.2838 - 0.0042i \\ 0.2274 + 0.3188i & -0.4540 - 0.2420i \end{bmatrix}$$
$$\mathbf{A}_4 = \begin{bmatrix} -0.2408 + 0.4767i & 0.1235 - 0.2776i \\ -0.0470 - 0.1162i & -0.2049 + 0.2546i \end{bmatrix}$$
$$\mathbf{A}_5 = \begin{bmatrix} 0.0909 - 0.0949i & -0.2835 - 0.1838i \\ -0.4402 - 0.1035i & -0.3475 - 0.2076i \end{bmatrix}$$
$$\mathbf{A}_6 = \begin{bmatrix} -0.3673 - 0.4376i & 0.1715 + 0.0808i \\ -0.0303 - 0.2390i & -0.1797 + 0.2176i \end{bmatrix}$$
$$\mathbf{A}_7 = \begin{bmatrix} 0.2721 + 0.0950i & 0.4197 + 0.1225i \\ -0.1112 - 0.4209i & 0.0174 - 0.1896i \end{bmatrix}$$
$$\mathbf{A}_8 = \begin{bmatrix} -0.2683 + 0.2198i & -0.0064 + 0.5193i \\ -0.2811 + 0.1404i & -0.0238 - 0.1030i \end{bmatrix} \quad (30)$$

What is interesting about this code is that the singular values of all modulation matrices are very similar and are listed in Table 6.1. This suggests that non-unitary modulation matrices may

Matrix	σ_1	σ_2	$\kappa = \sigma_1/\sigma_2$
$\mathbf{A}_1$	0.6843	0.1784	3.84
$\mathbf{A}_2$	0.6847	0.1769	3.87
$\mathbf{A}_3$	0.6867	0.1685	4.08
$\mathbf{A}_4$	0.6642	0.2428	2.74
$\mathbf{A}_5$	0.6472	0.2846	2.27
$\mathbf{A}_6$	0.6691	0.2288	2.93
$\mathbf{A}_7$	0.6754	0.2094	3.23
$\mathbf{A}_8$	0.6358	0.3094	2.06

TABLE II

SINGULAR VALUES FOR “RANDOM CE”

outperform unitary modulation matrices in some cases. The relative importance of different design metrics may be determined by formal minimization of the average union bound on error probability and will not be addressed here.

VI. Receiver design

For a general MIMO channel, the receiver receives a superposition of the transmitted signals and must separate the constituent signals based on channel knowledge. The method of spatial deconvolution determines the computational complexity of the receiver. This problem is similar in nature to the multi-user detection problem in CDMA and parallels can be drawn between the receiver architectures in these two areas.

The signal design in this chapter assumed maximum likelihood (ML) decoding, which amounts to exhaustive comparisons of the received signal to all possible transmitted signals. This is computationally prohibitive for higher order constellations such as 64-QAM, which for example would require $64^2 = 4096$ complex multiplications, Euclidean distance computations and comparisons for two transmit antennas. While ML detection is optimal, receiver complexity grows exponentially with the number of transmit antennas making this scheme impractical. Lower complexity sub-optimal receivers include the zero-forcing receiver (ZF) or the minimum mean-square error (MMSE) receiver, the design principles of which are similar to equalization principles for SISO links with inter-symbol interference (ISI). An attractive alternative to ZF and MMSE receivers is the vertical BLAST (V-BLAST) algorithm described in [11], which is essentially a successive cancellation technique. An exciting new algorithm that yields ML-like performance with cubic instead of exponential complexity is the sphere decoding algorithm [20].

A. Modulation and coding for MIMO

The signal design in this chapter did not include effects of concatenated coding. MIMO technology is compatible with a wide variety of coding and modulation schemes. In general, the best performance in achieved by generalizing standard (scalar) modulation and coding techniques to matrix channels. MIMO has been proposed for single-carrier (SC) modulation, direct-sequence code division multiple access (DS-CDMA) and orthogonal frequency division multiplexing (OFDM) modulation techniques. MIMO has also been considered in conjunction with concatenated coding schemes. Application of turbo codes and low density parity codes to

MIMO has recently generated a great deal of interest, as have simpler coding and interleaving techniques such as bit-interleaved coded modulation (BICM) along with iterative decoding. Inclusion of concatenated codes along with soft Viterbi decoding is in fact essential for realizing the full diversity gain of practical MIMO systems.

VII. Concluding Remarks

Smart antenna wireless communication systems provide significant gains in terms of spectral efficiency and link reliability. These benefits translate to wireless networks in the form of improved coverage and capacity. MIMO communication theory is an emerging area and full of challenging problems. Some promising research areas in the field of MIMO technology include channel estimation, new coding and modulation schemes, low complexity receivers, MIMO channel modeling and multiuser SDMA network design.

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Acronym	Expansion
AWGN	additive white Gaussian noise
BER	bit error rate
BICM	bit interleaved coded modulation
BLAST	Bell Labs layered space-time transceiver
CCI	co-channel interference
CDMA	code division multiple access
CE	capacity efficient
CW	continuous wave
DS	direct sequence
i.i.d.	independent, identically distributed
LAN	local area network
LD	linear dispersion
MAC	media access layer
MIMO	multiple input, multiple output
MISO	multiple input, single output
ML	maximum likelihood
MMSE	minimum mean squared error
NIC	network interface card
OFDM	orthogonal frequency division multiplexing
PAM	pulse amplitude modulation
PEP	pairwise error probability
PHY	physical layer
PSK	phase shift keying
QAM	quadrature amplitude modulation
QoS	quality of service
Rx	receive
SC	single carrier
SDMA	spatial division multiple access
SER	symbol error rate
SIMO	single input, multiple output
SISO	single input, single output
SM	spatial multiplexing
SNR	signal to noise ratio
STBC	space time block code
STTC	space time trellis code
s.v.d.	singular value decomposition
TCM	trellis coded modulation
Tx	transmit
ZF	zero forcing

TABLE III

GLOSSARY

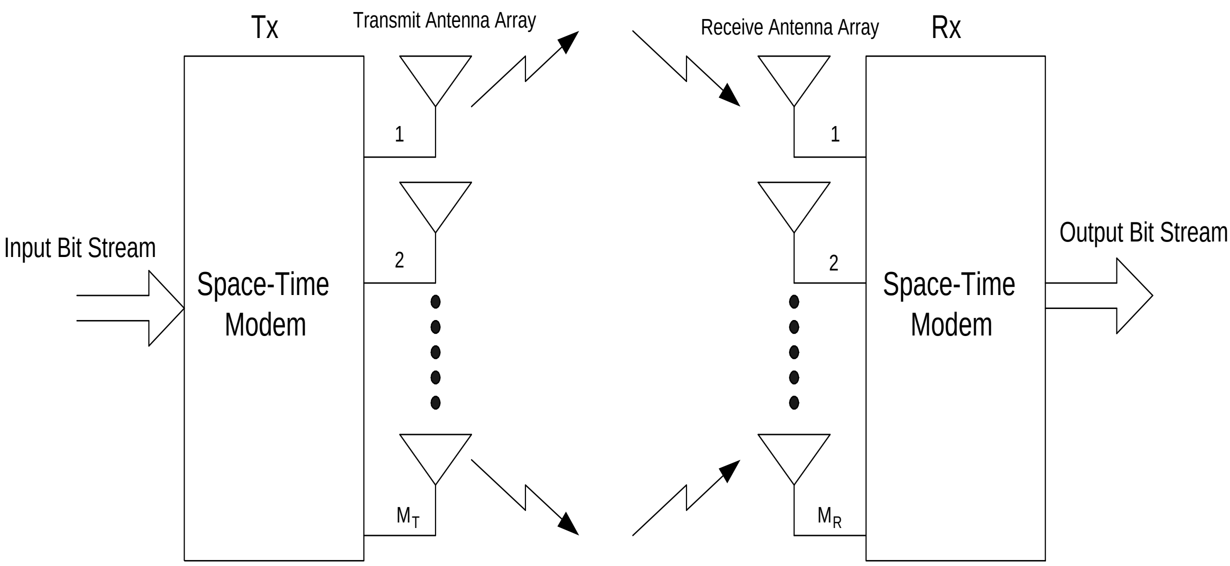

Fig. 1. *Schematic of a MIMO Communication System*

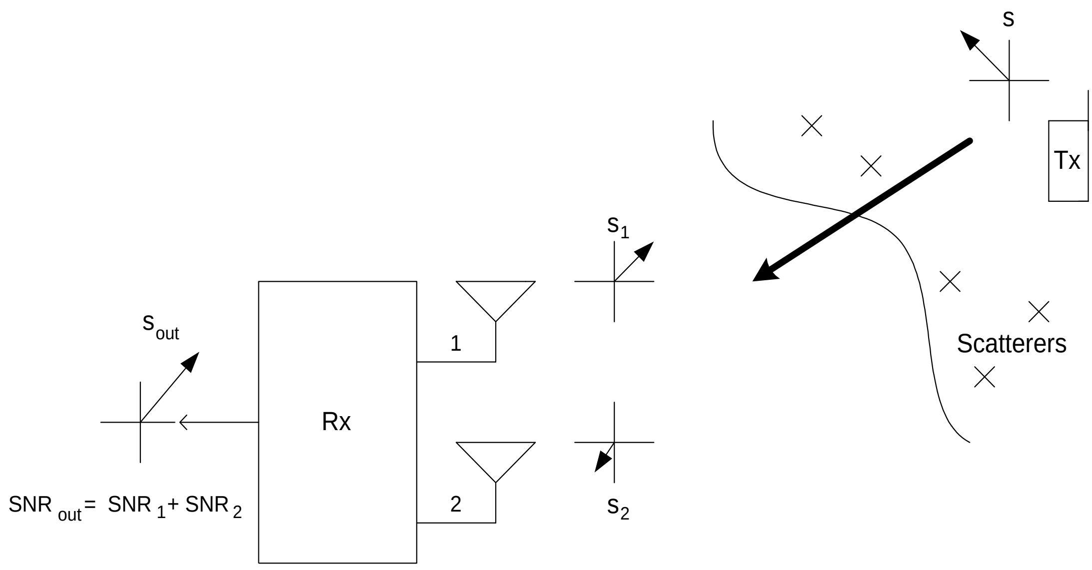

Fig. 2. *Array Gain*

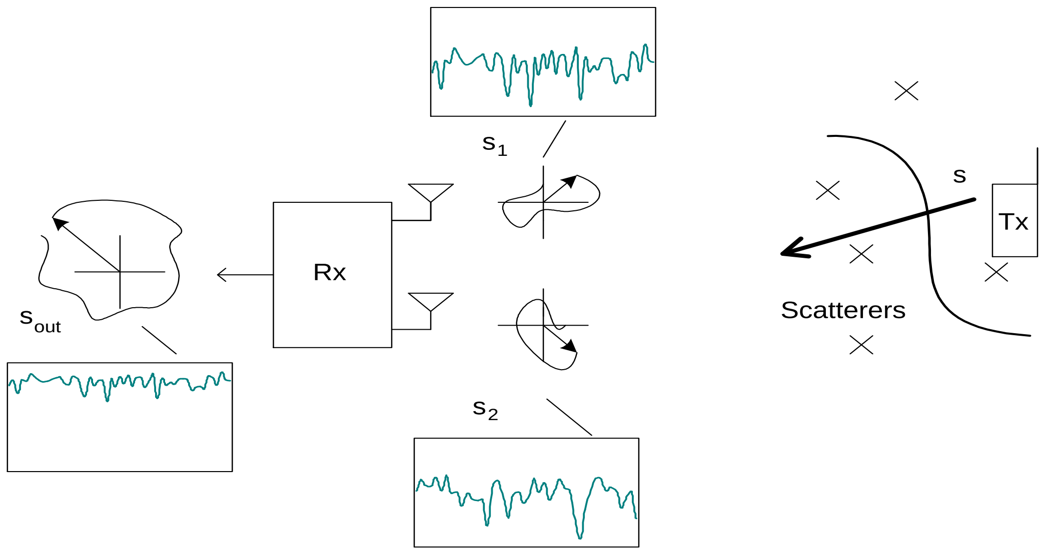

Fig. 3. *Receive Diversity*

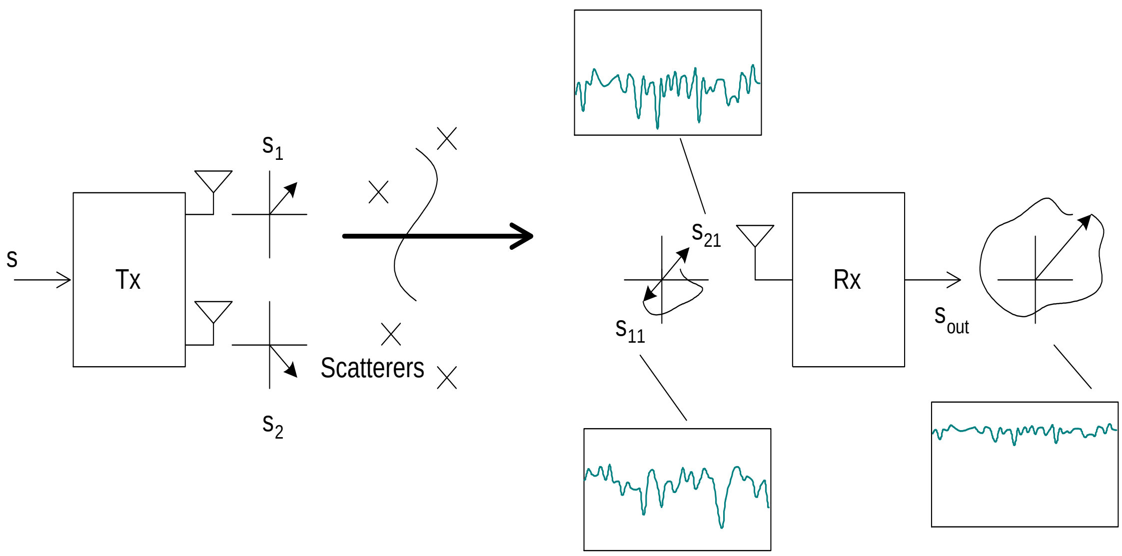

Fig. 4. *Transmit Diversity*

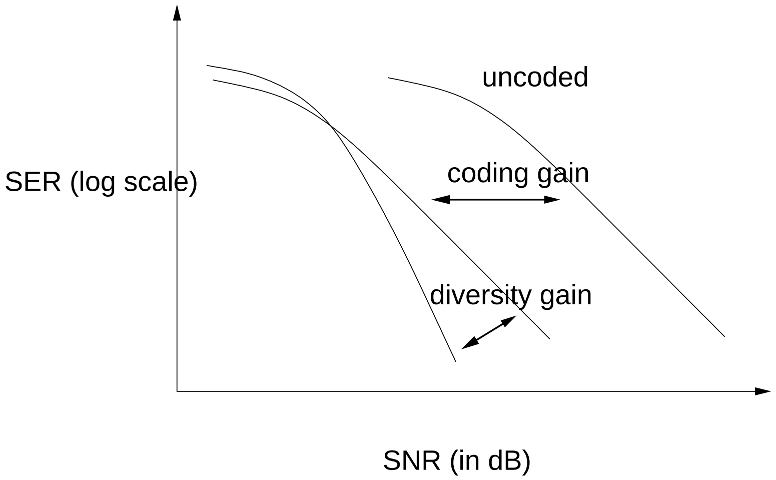

Fig. 5. Diversity gain and coding gain

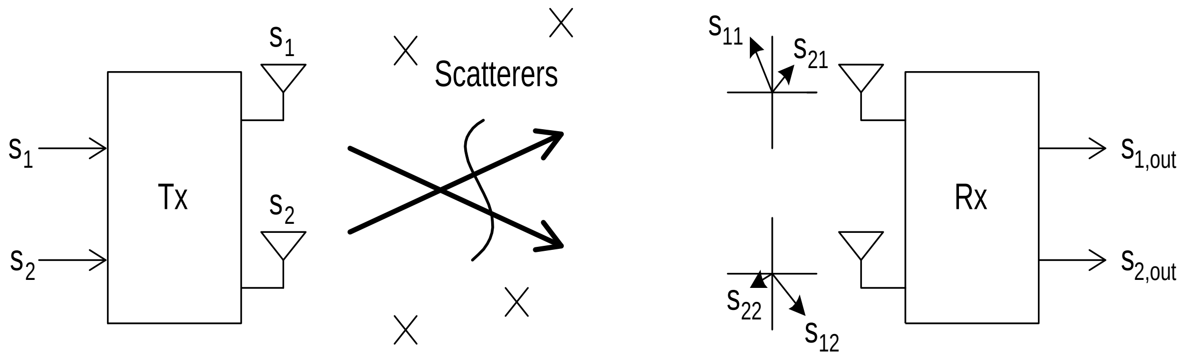

Fig. 6. *Spatial Multiplexing*

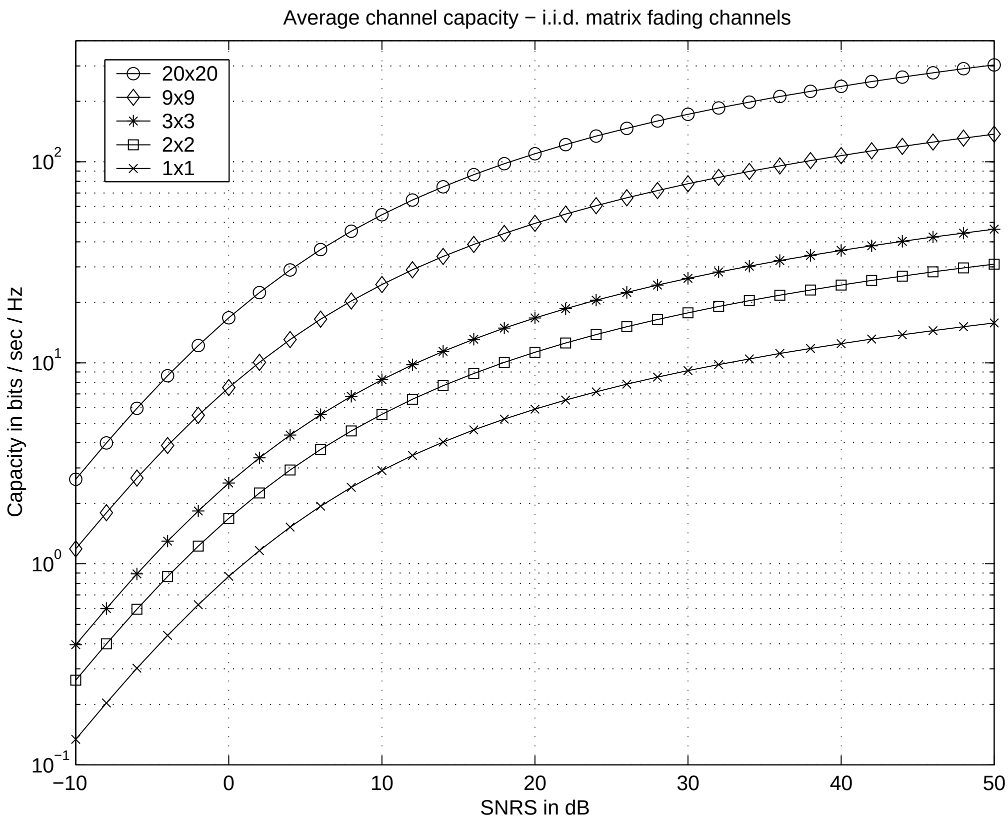

Fig. 7. Capacity

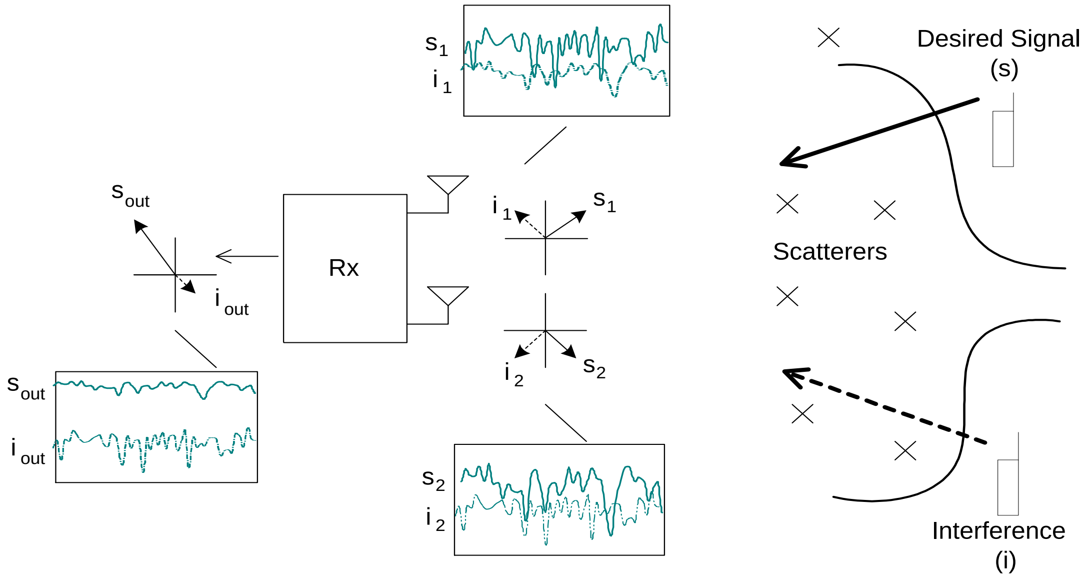

Fig. 8. *Interference Reduction*

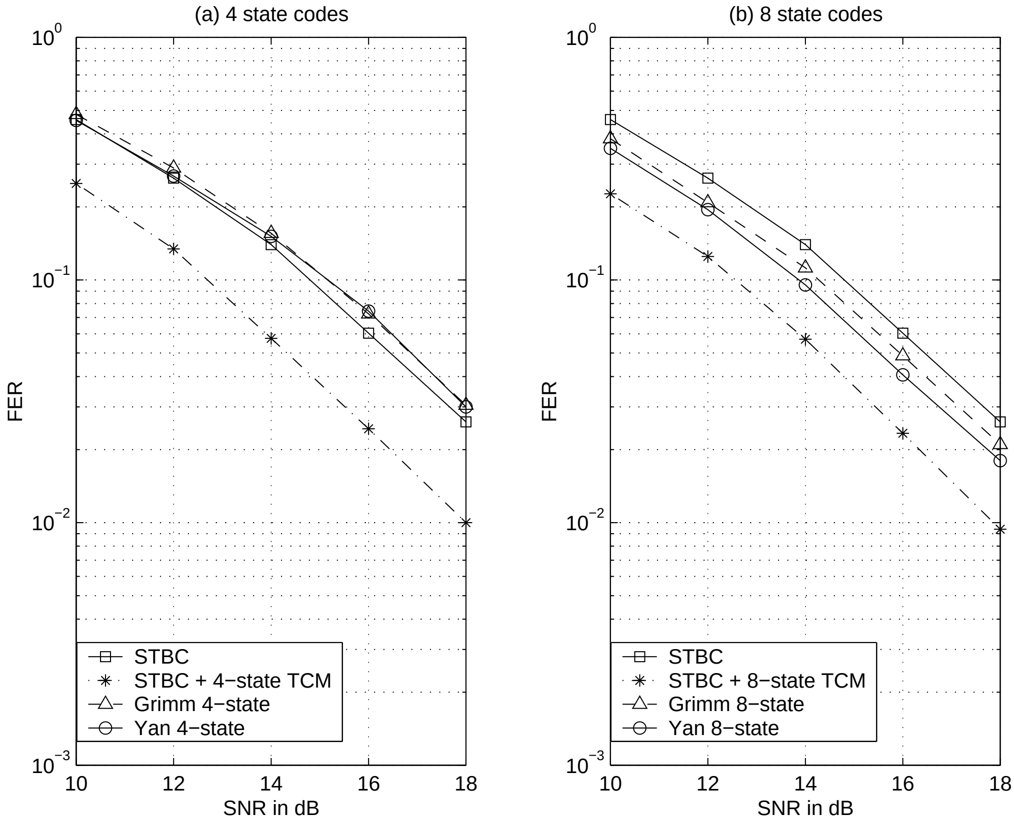

Fig. 9. One receive antenna

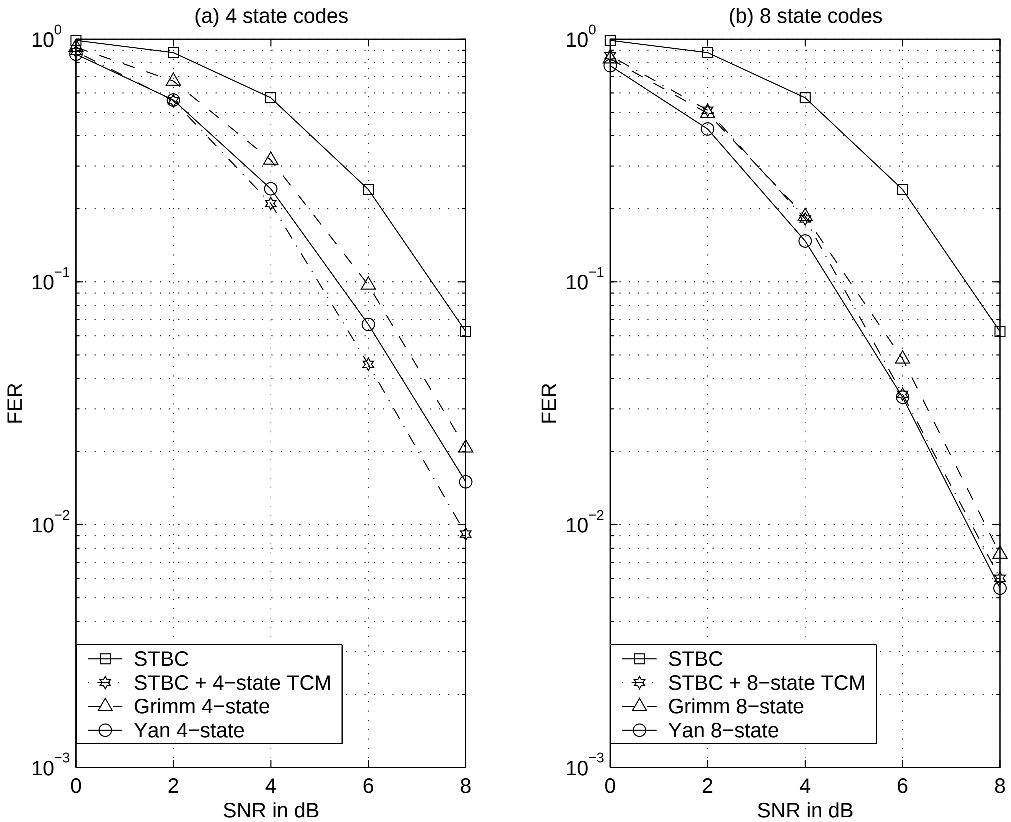

Fig. 10. Three receive antennas

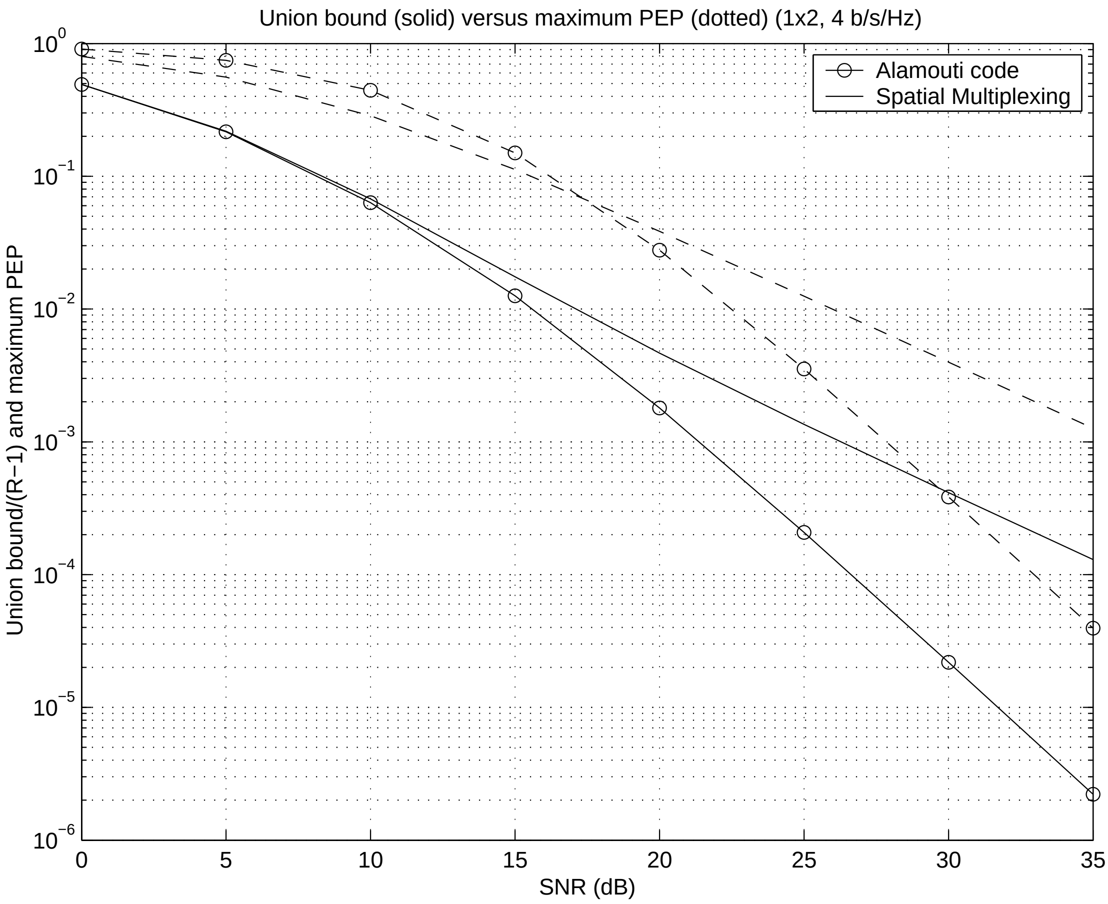

Fig. 11. Union bound compared to maximum PEP

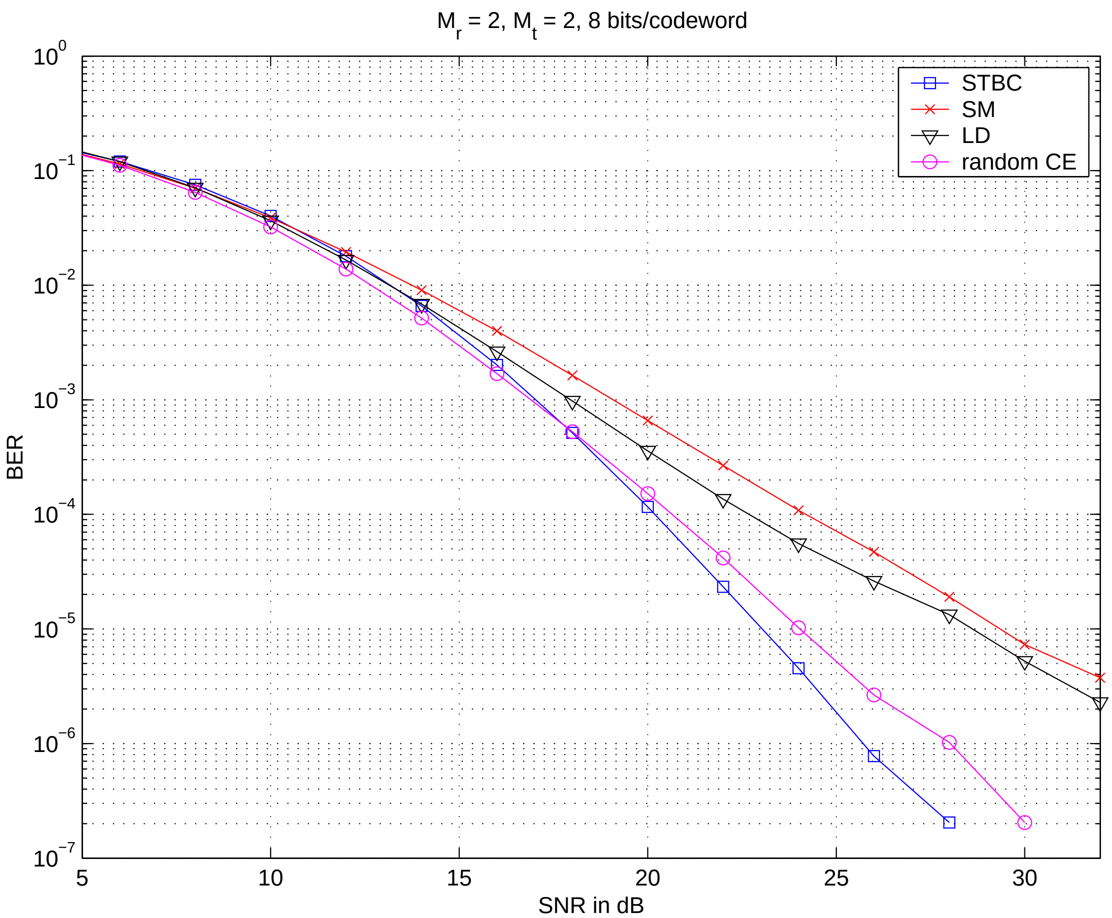

Fig. 12. BER for 2 receive antennas

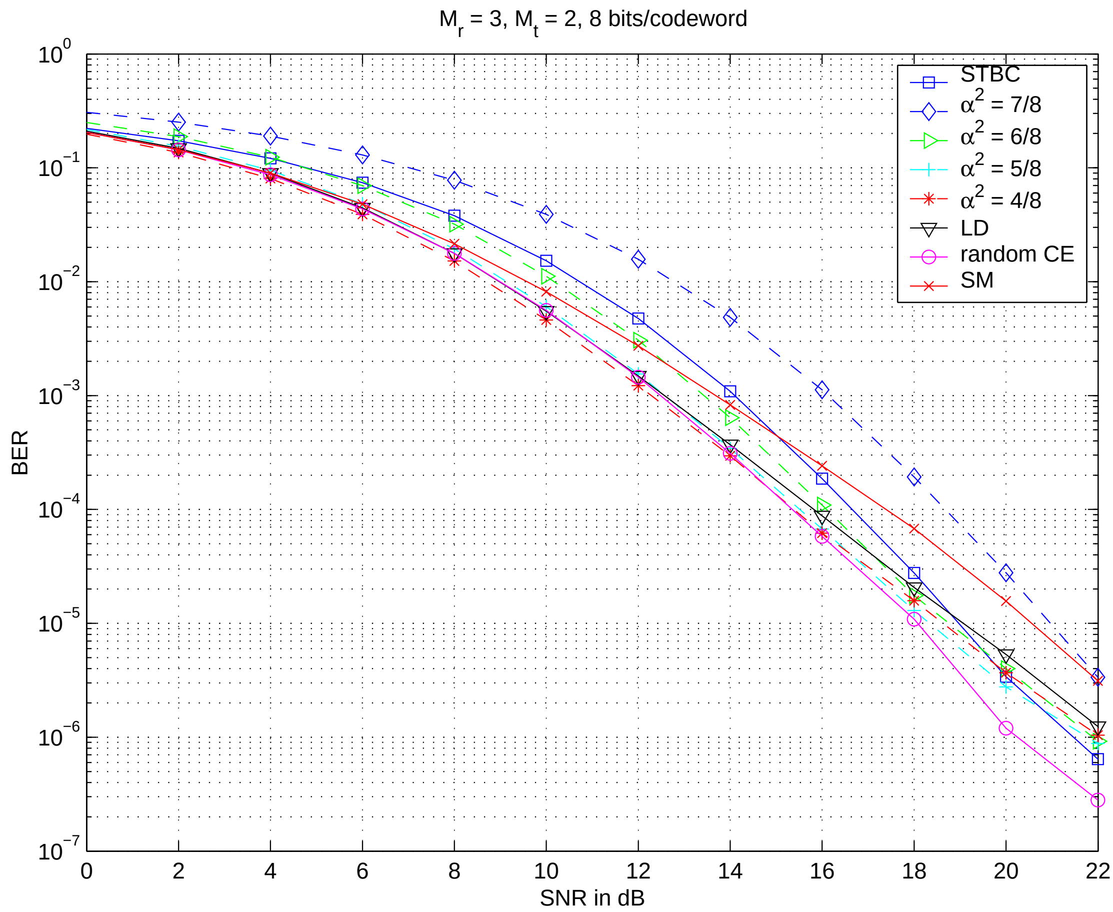

Fig. 13. BER for 3 receive antennas