The Silver Lining Effect: Formal Analysis and Experiment

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The *silver lining effect* predicts that segregating a small gain from a larger loss results in greater psychological value than does integrating them into a smaller loss. Using a generic prospect theory value function, we formalize this effect and derive conditions under which it should occur. We show analytically that there exists a threshold such that segregation is optimal for gains smaller than this threshold. The threshold is increasing in the size of the loss and decreasing in the degree of loss aversion of the decision maker. Our formal analysis results in a set of hypotheses suggesting that the silver lining effect is more likely to occur when: (i) the gain is smaller (for a given loss), (ii) the loss is larger (for a given gain), (iii) the decision maker is less loss averse. We test and confirm these predictions in two studies of preferences, both in a non-monetary and a monetary setting. We analyze the empirical data in a hierarchical Bayesian framework.

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1. Introduction

Decision makers are often faced with mixed outcomes, famously captured by the saying "I have good news and I have bad news." In this paper we look at the case where the bad news is larger in magnitude than the good, and ask: Does the decision maker want these events combined or presented separately? Thaler, in his seminal paper (Thaler 1985), showed that a decision maker faced with such a mixed outcome consisting of a loss and a smaller gain should generally prefer to separate the loss and the gain. That is, evaluating the gain separately from the larger loss is seen more positively than reducing the loss by the same amount. The small gain becomes a “silver lining” to the dark cloud of the loss, and, pushing the analogy further, adding a silver lining to the cloud has a more beneficial impact than making the cloud slightly smaller. The separate evaluation is referred to as *segregation* and the joint evaluation as *integration*.

The silver lining effect has wide implications, as equivalent information may often be framed to decision makers as instances of either integration or segregation. Consider for example a retailer who decides to decrease the price of a product: he or she could simply lower the price, reducing the loss to the consumer, and announce the new discounted price. Another option would be to keep charging the same amount, but then give some of it back to the consumer in the form of a rebate (Thaler 1985). Similarly, a vacation resort could lower its average daily rates for stays of one week or more, or offer a free night for every six nights spent in the resort. The two methods can of course translate to the exact same dollar saving, and differ only in framing. Or consider an investor receiving a brokerage statement that contains a net loss: Would he or she want to see only the smaller total loss, or to see the winners separated from the losers, even though the balance would be the same?

Despite the relevance of the silver lining effect to both academics and practitioners, we are not aware of any formal study of the conditions under which it should occur, beyond Thaler (1985)’s intuitive argument that the silver lining effect is more likely when the gain is smaller relative to the loss. The primary contribution of the current paper is to fill this gap. We assume a generic prospect theory value function and formally show the existence of a threshold such that the silver lining effect should occur for all gains smaller than this threshold and only for gains smaller than this threshold. Next, we show how the value of this threshold is affected by both the magnitude of the loss, and by the loss aversion parameter of the value function. Our formal analysis provides a set of hypotheses suggesting that the silver lining effect is more likely to occur when: (i) the gain is smaller (for a given loss), (ii) the loss is larger (for a given gain), (iii) the decision maker is less loss averse. We test these hypotheses in two experiments.

Finally, we provide a methodological contribution to the literature on the measurement of loss aversion, by replacing the deterministic, individual-level approach traditionally used by behavioral economists with a hierarchical Bayes framework that accounts for measurement errors and similarities across decision makers.

This paper is structured as follows. In Section 2, we briefly review the silver lining effect and the prospect theory framework. In Section 3, we report our theoretical analysis of the silver lining effect. The context and results of our first empirical study, which is conducted in a non-monetary setting, is reported in Section 4. In Section 5 we present the second empirical study, which extends the findings from the first study to monetary decisions. Section 6 concludes and provides some directions for future research.

2. Prospect Theory and the Silver Lining Effect

2.1 The Silver Lining Effect

When faced with a decision that involves several pieces of information, decision makers don't always integrate them into a whole, but instead may use them in the form presented by the decision context; Slovic (1972) calls this the *concreteness principle*. Thus, an individual may treat two amounts of money, for instance, as separate entities instead of simply summing them. Thaler and Johnson (1990) provide evidence that these effects occur for monetary gambles, and Linville and Fischer (1991) demonstrate this phenomenon for life effects. Thaler's (1985) theory of mental accounting addresses what this phenomenon implies for subjective value and thus for decision making. The silver lining principle focuses on outcomes that consist of a loss and a smaller gain, known as *mixed losses*. When faced with such outcomes, Thaler's prescription—which he dubbed the "*silver lining principle*"—is to keep them separate in mind (i.e., segregate the gain from the loss), so that the small gain can provide a *silver lining* to the larger loss, rather than disappear if used to diminish the loss (i.e., if the gain were integrated with the loss). An important implication of concreteness is that decisions can be materially affected by the presentation of the outcomes, as either integrated or segregated, because decision-makers do not spontaneously combine them (Thaler and Johnson 1990).

Read, Loewenstein, and Rabin (1999) introduce the concept of *choice bracketing*; in broad bracketing, several decisions are considered jointly, whereas in narrow bracketing each individual choice is considered separately. While a related concept, bracketing is distinguished from segregation and integration. The latter are products of *outcome editing* (Kahneman and Tversky 1979; Thaler 1985) rather than bracketing, as they refer to the consideration of outcomes within a single choice, rather than to the consideration of whether to evaluate a group of choices jointly or separately.

2.2 Prospect Theory

Prospect theory (Kahneman and Tversky 1979, Tversky and Kahneman 1992) proposes a value function $v(\cdot)$ as an alternative to the utility functions assumed by expected utility theory. The value function is characterized by three key features: *reference dependence*, meaning that the arguments of the value function are positive and negative deviations from some reference level, defined as gains and losses respectively; *loss aversion*, reflecting the fact that losses loom larger than gains of the same magnitude; *diminishing sensitivity*, indicating that the marginal impact of both gains and losses decrease as they become larger. These three characteristics result in a value function shaped as in Figures 1, 2, and 3. The origin denotes the reference point relative to which outcomes are categorized as gains and losses. Loss aversion results in a "kink" in the function at the origin: for all $x > 0$, $-v(-x) > v(x)$. Moreover, diminishing sensitivity in both domains implies that the value function is concave for gains and convex for losses, that is $v''(x) < 0$ for $x > 0$, and $v''(x) > 0$ for $x < 0$. A large amount of evidence supporting this function, both from field studies and experimental work, can be found in Tversky and Kahneman (2000).

In this paper, we assume the following generic specification of the value function:

$$v(x) = \begin{cases} g(x) & x \geq 0 \\ l(x) = -\lambda g(-x) & x < 0 \end{cases} \quad (1)$$

where $\lambda > 1$ and g is a function from $[0,\infty)$ to $[0,\infty)$ which is strictly concave, strictly increasing, twice differentiable, and such that $g(0)=0$. Note that we assume here a "reflective" value function (as in Kahneman and Tversky 1979), i.e., the value function for losses is the mirror image of the value function for gains. This assumption has received mixed empirical support (see for example Abdellaoui, Bleichrodt, and Paraschiv 2007). We leave the extension of our results to non-reflective value functions to future research. Such extension would be trivial under a power function specification.

3. Theoretical Analysis

In this section we derive some theoretical predictions regarding the silver lining effect. In particular, we characterize regions under which segregation of gains and losses is preferable to integration, and identify how the trade-off between integration and segregation is influenced by the magnitude of the gain, the magnitude of the loss, and the degree of loss aversion of the decision maker.

The proofs to all the results are provided in the appendix. Our analysis starts with the following proposition:

PROPOSITION 1**:** *For any fixed loss L, there exists a gain* $G^* \in [0, L[$ *such that the value derived from segregation is greater than that derived from integration for any gain G < G*, and the reverse is true for any gain G > G*.*

- *If* $\lim_{x \to 0^+} g'(x) = \infty$ *or if* $\lim_{x \to 0^+} g'(x) \neq \infty$ *and* $\lambda < \lambda^* = \dfrac{g'(0)}{g'(L)}$, *then* $G^* > 0$ *i.e., there exists a region in which the value derived from segregation is greater than that derived from integration.*
- *If* $\lim_{x \to 0^+} g'(x) \neq \infty$ *and* $\lambda > \lambda^* = \dfrac{g'(0)}{g'(L)}$, *then* $G^* = 0$ *, i.e., the value derived from integration is greater than that derived from segregation for any gain* $G \leq L$.

Proposition 1 shows the existence of a threshold *G** such that segregation is optimal for all gains smaller than this threshold, and integration is optimal for all gains larger than this threshold. The intuition behind the existence of a gain threshold is best explained graphically. Figure 1 gives an example of a mixed loss (L, G^*) for which the decision maker is indifferent between integration and segregation, i.e., $g(G^*)=l(-L+G^*)-l(-L)$. Let us consider a smaller gain G'. Both the corresponding gain $g(G')$ and the corresponding loss reduction $l(-L+G')-l(-L)$ are smaller under G' than under G^*. Whether integration or segregation is optimal hence depends upon which of the two quantities is decreased the least. Because of the concavity of the gain function g, the difference between the initial gain $g(G^*)$ and the smaller gain $g(G')$ corresponds to the flattest part of the gain function between 0 and G^*. In other words, the decrease in gain between G^* and G' is relatively *small* compared to the initial gain $g(G^*)$. On the other hand, the difference between the initial loss reduction and the smaller loss reduction corresponds to the steepest part of the loss function between $(-L+G^*)$ and $-L$. Therefore the decrease in loss reduction $l(-L+G')-l(-L)$ is relatively *large* compared to the initial loss reduction $l(-L+G^*)-l(-L)$, and hence compared to the initial gain $g(G^*)$ (recall that $g(G^*)=l(-L+G^*)-l(-L)$). Therefore, the new gain from segregation is larger than the new gain from integration, and segregation becomes optimal for G'. The same argument holds for G'' larger than G^*. In this case both the gain and loss reduction are *increased*. Because of the concavity of the gain function, the increase in gain is smaller than the increase in loss reduction, and integration becomes optimal for G''.

Proposition 1 also states that for some specifications of the value function, there may exist situations in which a decision maker who is very loss averse will always prefer integration over segregation, i.e., $G^* = 0$. Note however that such situations do not arise in the common specification in which g is a power function (of the form $g(x)=x^{\theta}$). Indeed, in that case $\lim_{x\to 0^+} g'(x) = \infty$ and there always exists a range of gains for which it is optimal to segregate, no matter how loss averse the decision maker is.

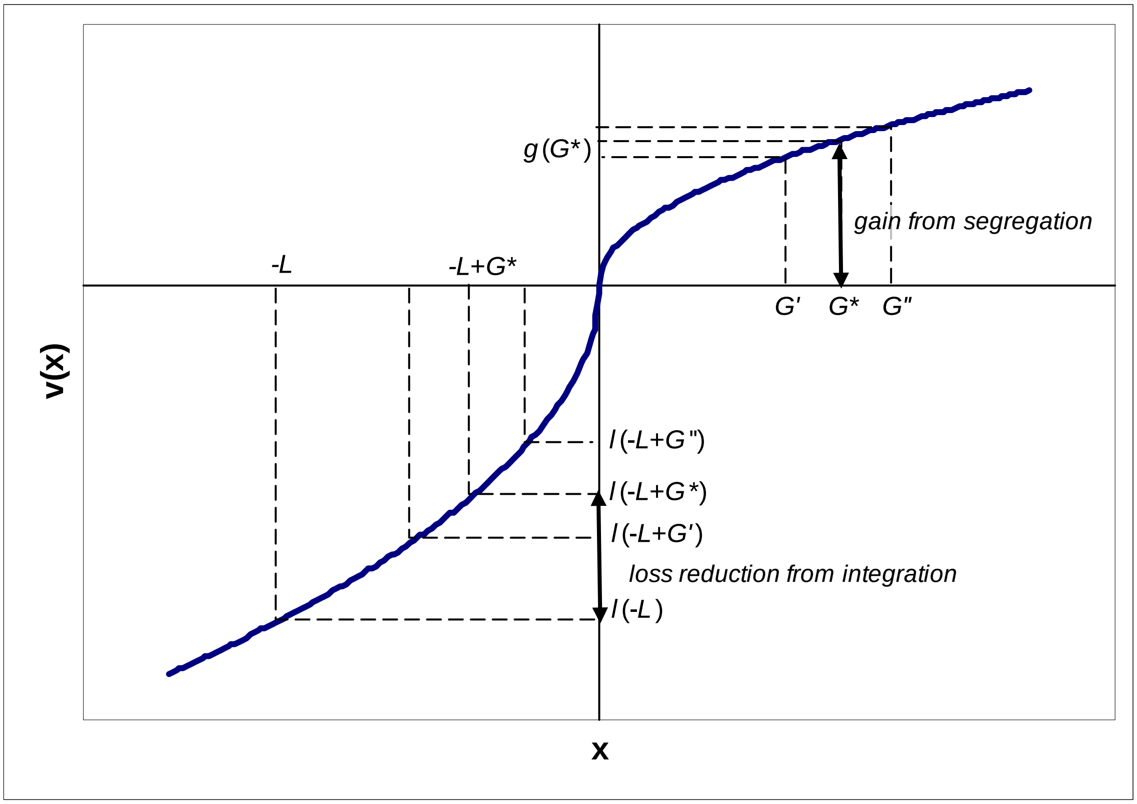

Figure 1: Illustration of Proposition 1

Proposition 1 has the following testable implication:

HYPOTHESIS 1 ("GAIN SIZE HYPOTHESIS"). *For a given mixed loss (L, G) with L > G, the smaller the gain G, the greater the value derived from segregation relative to integration, and conversely, the greater the gain, the greater the value derived from integration relative to segregation.*

Proposition 1 introduced a threshold gain, G^*, such that segregation is optimal for gains smaller than G^*, and integration is optimal for gains larger than G^*. G^* is a function of L and λ, as well as of any parameter of the value function. In the region in which $G^*>0$, G^* is defined by: $g(G^*)=l(-L+G^*)-l(-L)=-\lambda g(L-G^*)+\lambda g(L)$. Setting $F(G,L,\lambda)=g(G)+\lambda g(L-G)-\lambda g(L)$, G^* is defined by $F(G^*, L, \lambda) = 0$.

The following proposition uses the envelope theorem to evaluate the influence of L and λ on G^*.[1]

PROPOSITION 2: *In the region in which $G^*>0$, the following comparative statics hold:*

a) G^* is monotonically increasing in the amount of the loss L ($\frac{dG^*}{dL} > 0$)

b) G^* is monotonically decreasing in the loss aversion parameter λ ($\frac{dG^*}{d\lambda} < 0$)

Like Proposition 1, these results may be illustrated graphically. Let us first consider Proposition 2a, illustrated by Figure 2. The gain G_0^* is such that for a loss L_0 a decision maker is indifferent between integration and segregation, i.e., $g(G_0^*) = l(-L_0 + G_0^*) - l(-L_0)$. Let us consider a change from L_0 to $L_1 < L_0$. The gain function being unaffected by L, the gain from segregation $g(G_0^*)$ is unaffected as well. However, because of the concavity of the loss function, reducing the loss by a fixed amount G_0^* leads to a greater increase in value when the loss being reduced is smaller, i.e., $l(-L_1+G_0^*)-l(-L_1) > l(-L_0+G_0^*)-l(-L_0)$. As a result, while the decision maker is indifferent between segregation and integration for $G=G_0^*$ under L_0, he or she would prefer integration for $G= G_0^*$ under L_1, i.e., $g(G_0^*) < l(-L_1 + G_0^*) - l_1(-L_1)$, and therefore $G^*(L_1, \lambda) < G^*(L_0, \lambda)$.

Second, let us consider Proposition 2b. Figure 3 represents two value functions that differ only on the value of the loss aversion parameter λ. The gain function is unaffected by the parameter λ, and the two loss functions l_0 and l_1 on Figure 3 correspond respectively to $\lambda = \lambda_0$ and $\lambda = \lambda_1 > \lambda_0$. The gain G_0^* is such that a decision maker with a loss aversion parameter λ_0 is indifferent between integration and segregation,

[1] The implications of Proposition 2, captured by Hypotheses 2 and 3, extend to the domain in which $G^*=0$. The more loss averse the decision maker, the more likely $\lambda > \lambda^* = \frac{g'(0)}{g'(L)}$ is to hold, and therefore the more likely is G^* to be 0 and integration to be always preferred over segregation. Because of the concavity of the function g, $\lambda^* = \frac{g'(0)}{g'(L)}$ is increasing in L, and for a fixed λ, the larger the loss L, the less likely $\lambda > \lambda^*$ is to hold and therefore the less likely is integration to be always preferred to segregation.

i.e., $g(G_0^*) = l_o(-L + G_0^*) - l_0(-L)$. Let us consider a change from λ_0 to λ_1. The gain function being unaffected by λ, the gain from segregation $g(G_0^*)$ is unaffected as well. However, the loss reduction from integration $l(-L + G_0^*) - l(-L)$ being proportional to λ, it is greater when λ is greater. As a result, while a decision maker with loss aversion λ_0 would be indifferent between segregation and integration for $G = G_0*$, a decision maker with loss aversion λ_1 would prefer integration, i.e., $g(G_0^*) < l_1(-L + G_0^*) - l_1(-L)$, and therefore $G*(L, \lambda_1) < G*(L, \lambda_0)$.

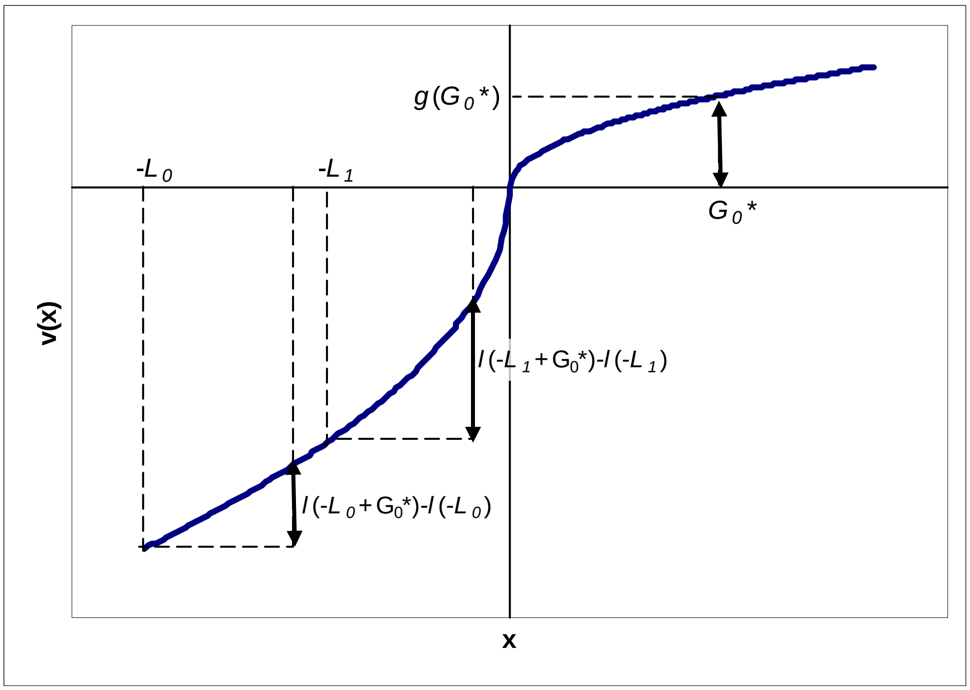

Figure 2: Varying the magnitude of the loss

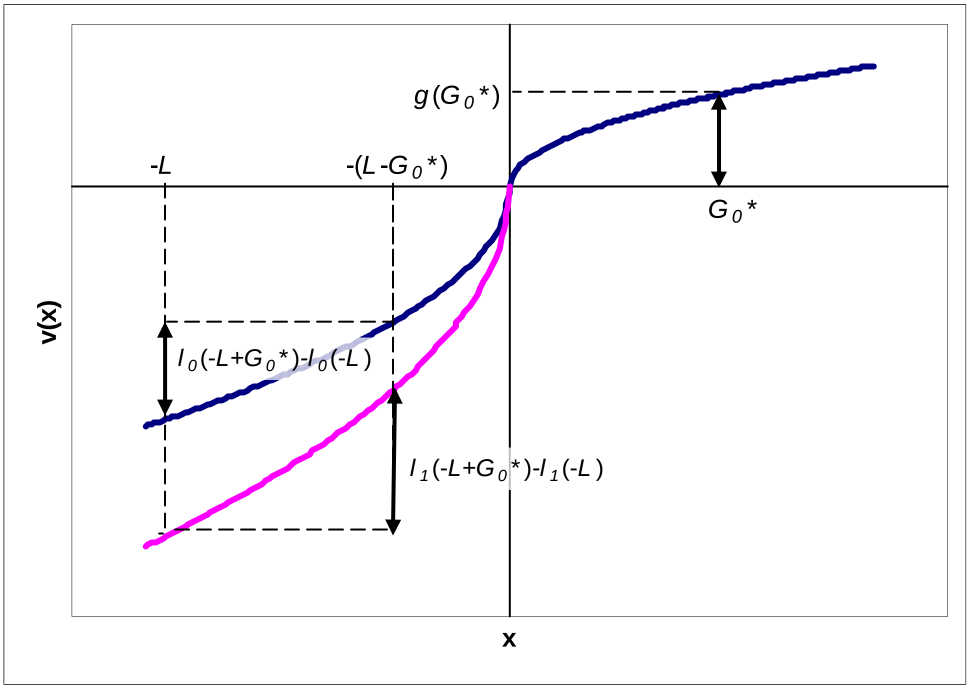

Figure 3: Varying the degree of loss aversion

Like Proposition 1, Proposition 2 has the following testable implications.

HYPOTHESIS 2 ("LOSS SIZE HYPOTHESIS"): *For a given mixed loss (L, G) with L > G, the larger the loss L, the greater the value derived from segregation relative to integration, and conversely, the smaller the loss L, the greater the value derived from integration relative to segregation.*

HYPOTHESIS 3 ("LOSS AVERSION HYPOTHESIS"): *For a given mixed loss (L, G) with L > G, the more loss averse a decision maker, the greater the value derived from integration relative to segregation, and conversely, the less loss averse a decision maker, the greater the value derived from segregation relative to integration.*

The remainder of the paper will focus on testing the above hypotheses experimentally.

4. Experiment 1

Our analysis led to a set of hypotheses which together suggest that the segregation of a small gain from a larger loss is more appealing when: (i) the gain is smaller (for a given loss), (ii) the loss is larger (for a given gain), (iii) the decision maker is less loss averse. Our first study tests these hypotheses in the context of choices between different amounts of vacation days.

4.1 Method

The experiment was conducted using a large online panel of pre-registered individuals, to which our survey task was advertised; participants performed the task over the internet on their own time, and were compensated $2 for their assistance. A total of 53 participants completed the survey. In order to ensure that only participants who paid proper attention were kept for analysis, three methods were employed. First, completion times were recorded and screened for participants who were outside two standard deviations away from the mean; this led to the exclusion of one subject. Second, one subject was excluded for answering "yes" to all 16 of our gamble items (presented below); this response does not provide usable data, and also suggests inattention. The third screening method was a "trick" item inserted in the middle of our survey. This item consisted of a block of text apparently containing the instructions for how to answer the following questions; however, in the middle of the block participants were told that this was an attention-check, and given instructions to ignore the surrounding text and instead answer the questions in a particular way to show that they were indeed paying attention. The "real" instructions were positioned so that briefly skimming or ignoring the text altogether would lead to an identifiable response to the question; this method eliminated as inattentive a further 15 participants. The analyzed sample thus consisted of 36 individuals, with a median age of 37 and a median income of $42 500. Our sample goes well beyond the student populations often used in experimental studies: only 14% of our subjects described themselves as students, 11% as unemployed, and 64% as employed outside of the household.

Our three hypotheses were tested using a rating task in which each participant read four scenarios. Each scenario involved a change of jobs necessitated by the current employer going out of business, and an associated change in the allocation of vacation days. In each of the four scenarios, the decision maker faced two job offers, both of which involved a net loss of vacation days, but which differed in their distribution (see Table 1 for details of each scenario).

	Pair 1		Pair 2		Pair 3		Pair 4	
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Change in number of summer vacation days	−4	−1	−4	−3	−7	−4	−7	−6
Change in number of winter vacation days	3	0	1	0	3	0	1	0
Net change	−1	−1	−3	−3	−4	−4	−6	−6

Table 1: The four pairs of job offers and their outcomes.

For instance, in scenario 1, offer A would mean a loss of four summer vacation days, and a gain of three winter vacation days, while offer B would mean only a loss of one summer vacation day. Both offers thus resulted in a net loss of one vacation day; in one case, this was presented as a larger loss (−4 days) and a separate (segregated) smaller gain (3 days), and in the other only as a smaller loss (−4+3=−1 day). After reading the scenario, the participant rated her preference for one offer over the other on a five-point scale from −2 (strongly prefer the integrated offer) to 2 (strongly prefer the segregated offer).

The design of the experiment was a 2 (loss: small (4 days) vs. large (7 days)) × 2 (gain: small (1 day) vs. large (3 days)) factorial design, with all manipulations within-subject, and the order of presentation of the pairs of job offers varying between-subjects in a Latin square pattern.

In addition to the main preference measure, two other individual-level measures were taken: relative preference for summer vacations over winter vacations, and loss aversion. Since each choice was between a loss in summer vacation days and a gain in winter vacation days (segregated option) vs. a smaller loss in summer vacation days (integrated option), it is important to control for a general preference for summer vacation days. To do this, we asked participants to allocate 20 vacation days between summer, winter, and the rest of the year. Our measure of participant *i*'s relative preference for summer vacation days, $summer_i$, was then defined as the ratio of days allocated to summer, divided by the total days allocated to summer and winter (days allocated to the rest of the year were ignored as they do not appear in our scenarios).

The other additional measure was loss aversion for vacation days (the currency used in this experiment). To estimate this parameter, we used two sequences of gambles, each of which participants were asked to accept or reject (see Goette, Huffman and Fehr 2004 and Tom et al. 2007 for similar measures of loss aversion). The gambles were introduced by a scenario in which the decision maker had the option of joining a new project at work, which if successful would result in extra vacation days (five days in one sequence, 12 in the other), while if it failed would result in a loss of vacation days. The project's probability of success was estimated at .5, and because it depended on competitors and clients, the decision maker herself would not be able to influence it. The gambles varied the amount of lost vacation days that the project's failure would mean; see Table 2 for the complete sequences of gambles used.

	Sequence 1		Sequence 2	
Gamble	*Gain*	*Loss*	*Gain*	*Loss*
1	5	0.5	12	1
2	5	1	12	2
3	5	1.5	12	2.5
4	5	2.5	12	3
5	5	3	12	3.5
6			12	4.5
7			12	5
8			12	6
9			12	8
10			12	10

Table 2: The gambles used to measure loss aversion, and their outcomes; all probabilities are 1/2 and all amounts are in numbers of vacation days.

For each subject, each sequence of gambles provided an amount of vacation days lost equivalent to five and 12 days gained, respectively. For instance, if a subject accepted Gamble 3 but rejected Gamble 4 in the first sequence, and accepted Gamble 7 but rejected Gamble 8 in the second sequence, we coded these responses as if a gain of five days was equivalent to a loss of 2 (the average of 1.5 and 2.5) days, setting x_{5days}=2, and as if a gain of 12 days was equivalent to a loss of 9 (the average of 8 and 10) days setting x_{12days}=9. This provided us with two measures of the parameter λ: $\hat{\lambda}_5 = \frac{5}{x_{5days}}$ and $\hat{\lambda}_{12} = \frac{12}{x_{12days}}$. These measures assume that the value function is approximately linear for small amounts, and that the probability weighing function is similar for gains and losses.[2] In the analysis below, we use both of these

[2] The effect of diminishing sensitivity would be slight, producing somewhat smaller estimates of λ. For example, given the commonly cited value of .88 in v(x)=$x^{.88}$, a value of 2 would become $2^{.88}$≈1.84. Different probability

measures and take into account the existence of measurement error. The two measures correlated at Kendall's τ=.45 across subjects. Responses were screened for monotonicity; that is, the analysis only included subjects with at most one switch from acceptance to rejection as the size of the loss increased. This eliminated one participant. Six participants indicated that they would accept all gambles in the first sequence; in those cases, only the 12-day measure was used in the analysis.

4.3 Results and Discussion

There was an overall preference for integration in our sample ($M=-.49$ on our scale from −2 to 2; $t=-4.72$, $p<.0001$), as well as an overall preference for summer over winter vacations ($M=.62$ on our scale from 0 to 1, with 0.5 being the indifference point; $t=-7.95$, $p<.0001$).

In order to test all three hypotheses simultaneously, and in particular to estimate the effect of loss aversion (Hypothesis 3) we analyze the data using a hierarchical linear model estimated in a Bayesian framework (Gelman et al., 1995; Rossi and Allenby 2003). This model allows us to capture measurement errors in the loss aversion parameter λ, as well as similarities in λ across subjects (by allowing λ to be shrunk towards a population mean). The details of the model and its estimation can be found in Appendix 2. Similar results were obtained with a simpler non-Bayesian linear model (details are available from the authors). We report the Bayesian model here as its assumptions better capture the structure of our data.

At the subject-level, we model the preference of subject i on gamble pair j, $pref_{ij}$, as a function of the size of the gain, the size of the loss (both coded orthogonally, with small=−1 and large=1), their interaction, a subject-specific intercept a_i, and a normally distributed error term, ε_i; we also include indicator variables for the position (within the Latin square design) at which pair j was presented:

$$pref_{ij} = a_i+\beta_1 gain_{ij}+\beta_2 loss_{ij}+\beta_3 gain_{ij}\times loss_{ij}+\beta_4\times 1(\text{position}_{ij}=1)+\beta_5\times 1(\text{position}_{ij}=2)+\beta_6\times 1(\text{position}_{ij}=3)+\varepsilon_{ij}$$

with $\varepsilon_{ij} \sim N(0,\sigma^2)$

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weighing functions for gains and losses would require multiplying our estimate of λ by $\frac{w^+(0.5)}{w^-(0.5)}$. To the extent that these assumptions are inaccurate, we would see increased noise and worse fit of our model to the data; it would not create a false appearance of support for our hypotheses.

Our two measures of individual *i*'s loss aversion, $\hat{\lambda}_{5,i}$ and $\hat{\lambda}_{12,i}$ are modeled as functions of a true underlying value, λ_i, plus normally distributed errors δ_i and ζ_i:

$$\hat{\lambda}_{5,i} = \lambda_i + \delta_i$$

$$\hat{\lambda}_{12,i} = \lambda_i + \varsigma_i$$

with $\delta_i \sim N(0, \nu_5^2), \zeta_i \sim N(0, \nu_{12}^2)$

We captured the effect of loss aversion and relative preference for summer vacations on the preference for segregation versus integration by allowing λ_i and $summer_i$ to impact the subject-specific intercept a_i. In particular, we used the following Bayesian prior for a_i:

$$a_i \sim N(a_0 + \lambda_i a_1 + summer_i a_2, \eta^2)$$

Finally, we specified a prior on λ_i that captures similarities across subjects and allows shrinking towards a population means:

$$\lambda_i \sim N(\lambda_0, \tau^2)$$

We estimated this model using Markov Chain Monte Carlo, with 100 000 iterations, the first 50 000 being used as burn-in. Convergence was assessed informally from the time-series plots of the parameters. The results are shown in Table 3. The estimates shown are the means of the posterior distributions of each parameter; the reported *p*-values are posterior *p*-values, based on the draws from the posterior distribution (one-tailed where our hypotheses make directional predictions).

The results generally support our hypotheses; greater gains predict increased preference for integration (Hypothesis 1), greater losses (marginally) predict increased preference for segregation (Hypothesis 2), and greater loss aversion predicts increased preference for integration (Hypothesis 3). As expected, we also see a significant impact of a general preference for summer over winter vacations.

Parameter	**Estimate**	***p*-value**
a_1 (loss aversion)	−.402	.022
a_2 (summer preference)	−4.82	<.0002
β_1 (gain: large=1, small=−1)	−.113	.032
β_2 (loss: large=1, small=−1)	.088	.071
β_3 (gain x loss interaction)	.034	.558
β_4 (position=1)	.339	.052
β_5 (position=2)	.027	.876
β_6 (position=3)	.074	.664

Table 3: Estimates from the hierarchical Bayes model.

Experiment 1 provides initial support for our three hypotheses in a non-monetary setting, and also introduces our modeling framework. We conducted an additional experiment to look for stronger support for our Hypothesis 2 (loss size) in a larger sample size and a more canonical context. The second experiment also extends the initial findings to a monetary setting.

5. Experiment 2

The three hypotheses were tested here in the context of mixed monetary gambles.

5.1. Method

The experiment was conducted in the virtual lab of a large East Coast university, with participants accessing the experiment over the internet. The present stimuli and measures were embedded in a longer series of surveys. Invitations to participate were sent out to a group of pre-registered individuals, who had not previously participated in surveys from the lab, and were offered $8 for completing the series of surveys.

The sample consisted of 163 individuals, with a median age of 35.5; the reported median income range was between $50,000 and $100,000. Again the sample is composed of individuals of varied occupations:

13% of our subjects were students, 13% were unemployed, and 57% were employed outside of the household.

We tested our hypotheses using four pairs of gambles, all of which had three possible outcomes, each with probability 1/3. Each of the four pairs corresponded to one cell in a 2 (loss: small ($30) vs. large ($60)) × 2 (gain: small ($5) vs. large ($20)) factorial design. See Table 4 for a complete description of the gambles. Within each pair, one gamble presented the gain and the loss separately (as outcomes 1 and 2) while the other combined them in one single outcome (outcome 2). In each pair, a third outcome was added to both gambles in order to equate the expected value of all gambles to $20; this ensured that differences across pairs were not due to differences in the expected values of the gambles.

	Pair 1		Pair 2		Pair 3		Pair 4	
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Outcome 1 (gain)	5	0	20	0	5	0	20	0
Outcome 2 (loss)	−30	−25	−30	−10	−60	−55	−60	−40
Outcome 3 (equalizer)	85	85	70	70	115	115	100	100
Expected value	20	20	20	20	20	20	20	20

Table 4: The four pairs of gambles and their outcomes; all probabilities are 1/3 and all amounts are in dollars.

For each pair of gambles, subjects were asked to rate their preference for one of the gambles versus the other on a five point scale from −2 (strongly prefer the integrated gamble) to 2 (strongly prefer the segregated gamble), with zero indicating indifference. The order of presentation of the gambles was counterbalanced between subjects using a Latin square design.

The loss aversion parameter was measured using gambles in a similar way to Experiment 1, the main difference being the use of gambles for money rather than for vacation days. Subjects were shown two sequences of ten gambles each and for each gamble were asked to indicate whether they would accept to play it. Again, all gambles were binary, with one loss and one gain, and probabilities were held constant at .5. Each of the two sequences held the gain amount constant, at \$6 and \$20, respectively, while the loss amounts increased as the sequence progressed. See Table 5 for the complete set of gambles.

	Sequence 1		Sequence 2	
Gamble	*Gain*	*Loss*	*Gain*	*Loss*
1	6	0.5	20	2
2	6	1	20	4
3	6	2	20	6
4	6	2.5	20	8
5	6	3	20	10
6	6	3.5	20	12
7	6	4	20	14
8	6	5	20	16
9	6	6	20	18
10	6	7	20	20

Table 5: The two sequences of gambles used to measure loss aversion; all amounts in dollars.

For each subject, each sequence of gambles then provided an amount of loss equivalent to a gain of \$6 and \$20, respectively. This provided us with two (noisy) measures of the parameter λ: $\hat{\lambda}_6 = \frac{6}{x_6}$ and $\hat{\lambda}_{20} = \frac{20}{x_{20}}$. The two measures correlated at Kendall's τ=.60 across subjects. Responses were screened for monotonicity; that is, the analysis only includes subjects with at most one switch from acceptance to rejection of the gambles as the size of the loss increased. This led to the elimination of seven subjects.

5.2. Results and Discussion

As in Experiment 1, we used a hierarchical linear model to analyze the data. The only difference was in the specification of the prior distribution for the subject-specific intercepts a_i, where we eliminated the term that captured a preference for summer over winter vacations, leaving only the intercept and the effect of loss aversion:

$$a_i \sim N(a_0 + \lambda_i\, a_1, \eta^2)$$

We again estimated this model using Markov Chain Monte Carlo, with 100 000 iterations, the first 50000 being used as burn-in. Convergence was assessed informally from the time-series plots of the parameters. The results are shown in Table 6.[3]

Parameter	**Estimate**	***p*-value**
a_1 (loss aversion)	−0.400	.0050
β_1 (gain: large=1, small=−1)	−0.115	.0024
β_2 (loss: large=1, small=−1)	0.099	.0042
β_3 (gain x loss interaction)	0.105	.0074
β_4 (position=1)	0.257	.017
β_5 (position=2)	−.006	.96
β_6 (position=3)	0.155	.16

Table 6: Estimates from the full hierarchical Bayes model.

The estimates support all three hypotheses, including the loss size hypothesis. As before, a large gain predicts greater preference for integration compared to a small gain (Hypothesis 1), and greater loss aversion predicts greater preferences for integration (Hypothesis 3). Our loss size hypothesis now also receives significant support, as an increase in loss size predicts greater preference for integration (Hypothesis 2).[4]

In conclusion, our second experiment strongly supports all three of our hypotheses. We take this as confirmation of the accuracy of our analytical results, and ultimately of the underlying assumptions of prospect theory.

[3] It is important to note that according to Cumulative Prospect Theory (Tversky and Kahneman 1992), the difference between the value of the "integrated" and "segregated" gamble is only approximately proportional to $l(-L+G) -l(-L) -g(G)$. We also note that Wu and Markle (2008) have recently documented that the assumption of gain-loss separability in mixed gambles is questionable. These caveats do not apply to Experiment 1.

[4] The interaction between gain size and loss size is significant in this experiment. This interaction term was included in the model for completeness and was not motivated by our theory. Moreover it was not significant in the first experiment. Therefore we do not elaborate on it.

6. General Discussion and Conclusion

In this paper we have formalized the silver lining effect identified by Thaler in 1985, using the basic assumptions of prospect theory. We have identified analytically and tested experimentally a set of conditions under which decision makers are more likely to segregate gains from losses. We have shown that segregating a gain from a larger loss is more appealing when (i) the gain is smaller (for a given loss), (ii) the loss is larger (for a given gain), (iii) the decision maker is less loss averse.

Our first empirical study tested the analytic predictions in a non-monetary setting, in the context of vacation days, and found initial support for our hypotheses. The second study extended and generalized the results to monetary decisions. Together, the two studies suggest that the basic phenomenon of the silver lining effect, and the moderators we have documented here, are likely to be quite general.

Our predictions are highly relevant to researchers, practitioners and policy-makers in a variety of domains where different frames of presentation of the same underlying information may provide different subjective values. Our analysis could for instance provide guidance to marketers wishing to design promotion schemes (where a rebate could provide a silver lining to a base price) or inform economic policy ('stimulus checks' or tax refunds may be silver linings to overall tax payments). We hope that future research will test our predictions in such decision environments.

A final contribution of our current work is the use of a small set of choices to estimate, at the individual level, the degree to which a decision-maker is loss-averse. Our results show that loss aversion is an important individual difference in predicting the reactions of decision-makers to integration and segregation as predicted by our model. Recent work shows that individual differences in loss aversion are related to demographic variables (Gaechter, Johnson and Herrmann 2008), reactions to changes in wages (Goette, Huffman, and Fehr 2004) and underlying neural signals in the striatum (Tom et al. 2007). Our estimation method provides a framework for estimating these differences and modeling this source of heterogeneity.

7. Appendix 1: Proof of the Propositions.

7.1. Proof of Proposition 1

PROPOSITION 1**:** *For any fixed loss L, there exists a gain* $G^* \in [0, L[$ *such that the value derived from segregation is greater than that derived from integration for any gain G < G*, and the reverse is true for any gain G > G*.*

- *If* $\lim_{x \to 0^+} g'(x) = \infty$ *or if* $\lim_{x \to 0^+} g'(x) \neq \infty$ *and* $\lambda < \lambda^* = \dfrac{g'(0)}{g'(L)}$*, then* $G^* > 0$ *i.e., there exists a region in which the value derived from segregation is greater than that derived from integration.*
- *If* $\lim_{x \to 0^+} g'(x) \neq \infty$ *and* $\lambda > \lambda^* = \dfrac{g'(0)}{g'(L)}$*, then* $G^* = 0$ *, i.e., the value derived from integration is greater than that derived from segregation for any gain* $G \leq L$*.*

7.1.1 Preparatory Material

LEMMA 1: *For any fixed loss L,*

- *If segregation is optimal for a gain G, it is optimal for any smaller gain G'<G.*
- *If integration is optimal for a gain G, it is optimal for any larger gain G''>G.*

PROOF OF LEMMA 1: Let us define y_0 and y_1 respectively as the loss reduction achieved by integration under G_0 and G_1 such that $G_1 < G_0$:

$y_0 = l(-L+G_0) - l(-L)$

$y_1 = l(-L+G_1) - l(-L)$

and x_0 and x_1 as the analogous gains from segregation:

$x_0 = g(G_0)$

$x_1 = g(G_1)$

Monotonicity and concavity of gains and monotonicity and convexity of losses imply that

$$\frac{y_1}{G_1} < \frac{y_0}{G_0} \quad \Leftrightarrow \quad y_1 < y_0 \frac{G_1}{G_0},$$

and
$$\frac{x_1}{G_1} > \frac{x_0}{G_0} \quad \Leftrightarrow \quad x_1 > x_0 \frac{G_1}{G_0}.$$

Thus, $x_0 > y_0 \Rightarrow x_1 > x_0 \frac{G_1}{G_0} > y_0 \frac{G_1}{G_0} > y_1 \Rightarrow x_1 > y_1$.

Recalling that $x_0 > y_0$ means that it is optimal to segregate, (i.e., the gain from segregation is greater than the loss reduction from integration), we see that if it is optimal to segregate a gain G_0 from a loss L, it is also optimal to segregate any smaller gain $G_1 < G_0$.

Conversely, $x_1 < y_1 \Rightarrow x_0 < x_1 \frac{G_0}{G_1} < y_1 \frac{G_0}{G_1} < y_0 \Rightarrow x_0 < y_0$, hence if it is optimal to integrate for G_1, it is optimal to integrate for any $G_0 > G_1$.

7.1.2 Proof of the Main Result

Proof of Proposition 1. Lemma 1 insures us that there are at most 2 regions: one in which it is optimal to segregate (smaller gains) and one in which it is optimal to integrate (larger gains). Clearly, the latter region is never empty as it is always optimal to integrate if $G=L$ (as long as $\lambda>1$). The former region is non-empty if and only if it is optimal to segregate as G goes to 0. The condition under which segregation is preferred is $g(G) > l(-L+G) - l(-L)$.

Let us first consider the case in which $g'(0)$ is a finite number. In that case we can write the following Taylor series expansions:

$$g(G) = G.g'(0) + o(G)$$
$$l(-L+G) - l(-L) = G.L'(-L) + o(G)$$

As G goes to 0, segregation is preferred if $g'(0) > l'(-L)$ and integration is preferred if $g'(0) < l'(-L)$. Because $l'(-L) = \lambda.g'(L)$, segregation is preferred as G goes to 0 if $\lambda < \frac{g'(0)}{g'(L)}$ and integration is preferred if $\lambda > \frac{g'(0)}{g'(L)}$.

Let us next consider the case in which $\lim_{G\to 0} g'(G) = +\infty$.

In that case, there exists M>0 and ε'>0 such that for all $\varepsilon<\varepsilon'$ and for all $G<L$:

$$g'(\varepsilon) > 3.M; \quad |L(-L+\varepsilon) - L(-L)| < M.(G-\varepsilon); \quad L'(-L+\varepsilon) < M$$

For any $\varepsilon<\varepsilon'$, we can write the following Taylor series expansions:

$$g(G) = g(\varepsilon) + (G-\varepsilon).g'(\varepsilon) + o(G-\varepsilon) > (G-\varepsilon).3.M + o(G-\varepsilon)$$

$$l(-L+G) - l(-L) = L(-L+\varepsilon) - L(-L) + (G-\varepsilon).L'(-L+\varepsilon) + o(G-\varepsilon) < (G-\varepsilon).2.M + o(G-\varepsilon)$$

Segregation is preferred as G goes to ε, i.e., the region in which segregation is preferred is non-empty.

7.2. Proof of Proposition 2

PROPOSITION 2: *In the region in which G*>0, the following comparative statics hold:*

a) G is monotonically increasing in the amount of the loss L (*$\frac{dG^*}{dL} > 0$*)*

b) G is monotonically decreasing in the loss aversion parameter λ (*$\frac{dG^*}{d\lambda} < 0$*)*

Proof of Proposition 2. As mentioned in the text, G^* is defined by: $F(G^*, L, \lambda) = 0$, where:

$$F(G, L, \lambda) = g(G) - \lambda g(L) + \lambda g(L-G).$$

Using the envelope theorem, $\frac{dG^*}{dL} = \frac{-\partial F/\partial L}{\partial F/\partial G}$ and $\frac{dG^*}{d\lambda} = \frac{-\partial F/\partial \lambda}{\partial F/\partial G}$.

We have

- $\frac{\partial F}{\partial G}(G^*) = g'(G^*) - \lambda g'(L-G^*) < 0$ because

  $g'(G^*)G^* < g(G^*) = \lambda(g(L) - g(L-G^*)) < \lambda g'(L-G^*)G^*$ where the inequalities hold because g is monotonically increasing and concave and $g(0)=0$, and the equality holds by definition of G^*.

- $\frac{\partial F}{\partial L} = -\lambda g'(L) + \lambda g'(L-G) > 0$, because g' is monotonically decreasing

- $\frac{\partial F}{\partial \lambda} = -g(L) + g(L-G) < 0$ because g is monotonically increasing

Thus, we have $\frac{dG^*}{dL} > 0$, and $\frac{dG^*}{d\lambda} < 0$.

8. Appendix 2: Details of the Hierarchical Bayes Model

8.1 Model

8.1.1

Likelihood

$$pref_{ij} = a_i + \beta_1 gain_{ij} + \beta_2 loss_{ij} + \beta_3 gain_{ij} \times loss_{ij} + \beta_4 \times 1(position_{ij} = 1) + \beta_5 \times 1(position_{ij} = 2) + \beta_6 \times 1(position_{ij} = 3) + \varepsilon_{ij}$$

$$\hat{\lambda}_{6,i} = \lambda_i + \delta_i$$

$$\hat{\lambda}_{20,i} = \lambda_i + \zeta_i$$

$$\varepsilon_{ij} \sim N(0,\sigma^2), \delta_i \sim N(0,\nu_5^2), \zeta_i \sim N(0,\nu_{12}^2)$$

8.2.2. First-stage Prior

$$a_i \sim N(a_0 + \lambda_i.a_1 + summer_i.a_2, \eta^2)$$

8.2.3. Second-stage Prior

$$\lambda_i \sim N(\lambda_0, \tau^2)$$

8.2.4. Third-stage Prior

Diffuse on a_0, a_1,a_2,β_1, β_2, β_3 , β_4, β_5, β_6, λ_0

$$\sigma^2 \sim IG(\frac{r_0}{2},\frac{s_0}{2}), \nu_5^2 \sim IG(\frac{r_0}{2},\frac{s_0}{2}), \nu_{12}^2 \sim IG(\frac{r_0}{2},\frac{s_0}{2}), \eta^2 \sim IG(\frac{r_0}{2},\frac{s_0}{2}), \tau^2 \sim IG(\frac{r_0}{2},\frac{s_0}{2})$$

with r_0=s_0=1.

8.3 Markov Chain Monte Carlo Estimation

- L(a_i | rest) ~ N(m_i,V_i) where

  $V_i = (\frac{J}{\sigma^2} + \frac{1}{\eta^2})^{-1}$ and

  $$m_i = V_i.(\frac{\sum_{j=1}^{J} pref_{ij} - (\beta_1 gain_{ij} + \beta_2 loss_{ij} + \beta_3 gain_{ij} loss_{ij} + \beta_4 1(position_{ij} = 1) + \beta_5 1(position_{ij} = 2) + \beta_6 1(position_{ij} = 3))}{\sigma^2}$$

  $$+ \frac{a_0 + a_1.\lambda_i + a_2.summer_i}{\eta^2})$$

- L(η^2 | rest) ~ $IG(\frac{r_0 + \sum_{i=1}^{I}(a_i - a_0 - \lambda_i.a_1 - summer_i.a_2)^2}{2}, \frac{s_0 + I}{2})$

- L(σ^2 | rest) ~ $IG(\frac{r_0 + \sum_{i=1}^{I}\sum_{j=1}^{J}(pref_{ij} - \hat{p}_{ij}))^2}{2}, \frac{s_0 + I.J}{2})$

where

$$\hat{p}_{ij} = a_i + \beta_1 gain_{ij} + \beta_2 loss_{ij} + \beta_3 gain_{ij} loss_{ij} + \beta_4 1(position_{ij} = 1) + \beta_5 1(position_{ij} = 2) + \beta_6 1(position_{ij} = 3)$$

- L([β_1; β_2; β_3; β_4; β_5; β_6] | rest) ~$N(X'X)^{-1}.X'.Y;\sigma^2.(X'X)^{-1}$]

  where X_{ij}=[gain$_{ij}$, loss$_{ij}$, gain$_{ij}$.loss$_{ij}$, 1(position$_{ij}$=1), 1(position$_{ij}$=2), 1(position$_{ij}$=3)],

  $Y_{ij}=pref_{ij}-a_i$

- L([a_0; a_1;a_2]| rest) ~ $N(X'X)^{-1}.X'.Y;\eta^2.(X'X)^{-1}$]

  where X_{ij}=[1, λ_i, $summer_i$], $Y_{ij}=a_i$

- L(λ_i | rest) ~ N(m_i,V_i)

  where $V_i = (\frac{1}{\nu_5^2} + \frac{1}{\nu_{12}^2} + \frac{a_1^2}{\eta^2} + \frac{1}{\tau^2})^{-1}$

  $$m_i = V_i.(\frac{\hat{\lambda}_{6,i}}{\nu_5^2} + \frac{\hat{\lambda}_{20,i}}{\nu_{12}^2} + \frac{(a_i - a_0 - summer_i.a_2).a_1}{\eta^2} + \frac{\lambda_0}{\tau^2})$$

- L(λ_0 | rest) ~ N(m,V) where $V = \frac{\tau^2}{I^2}$ and $m = \frac{\sum_{i=1}^{I}\lambda_i}{I}$

- L(τ^2 | rest) $\sim IG(\frac{r_0 + \sum_{i=1}^{I}(\lambda_i - \lambda_0)^2}{2}, \frac{s_0 + I}{2})$

- L(v_5^2 | rest) $\sim IG(\frac{r_0 + \sum_{i=1}^{I}(\hat{\lambda}_{6,i} - \lambda_i)^2}{2}, \frac{s_0 + I}{2})$

- L(v_{12}^2 | rest) $\sim IG(\frac{r_0 + \sum_{i=1}^{I}(\hat{\lambda}_{20,i} - \lambda_i)^2}{2}, \frac{s_0 + I}{2})$

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