Power-shaping of reaction systems: the CSTR case study

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Abstract

The non-isothermal continuous stirred tank reactor (CSTR) is a classical yet complex case study of nonlinear dynamical systems. Power-shaping control is a recent approach for the control of nonlinear systems based on the physics of the dynamical system. In this paper we apply the power-shaping control approach to the non-isothermal CSTR study case. A global Lyapunov function is derived for the open-loop exothermic CSTR. This Lyapunov function is then reshaped by the means of a controller in order to stabilize the process at a desired temperature. Some considerations on the local and global convergence to the desired state are presented.

Key words: non-isothermal CSTR, power-shaping control

1 Introduction

Thermodynamic systems, and among them chemical reaction systems, are usually nonlinear dynamical systems. They can therefore have a complex behaviour and be difficult to analyze and to control. Stability analysis of nonlinear systems requires the use of abstract mathematical tools such as the two Lyapunov methods or the passivity theory. Over the past years, several works have combined those abstract concepts with the underlying physical phenomena giving rise to the dynamical behaviour of the system. These works include for instance the study of port-Hamiltonian systems [4][7], energy-balancing...
passivity based control (PBC) [13][19] or the introduction of the contact formalism for expressing the dynamics of systems in which irreversible phenomena arise [5][6][11]. The non-isothermal continuous stirred tank reactor (CSTR) is a classical study case of nonlinear systems. Indeed the dynamical behaviour exhibits complex features, such as multiple equilibrium points. Up to now no precise physical interpretation of the complex behaviour of the non-isothermal reactor has been found [10].

Power-shaping control [18] has been developed in the past years as an extension of energy-balancing passivity-based control [13][19]. In energy-balancing passivity based control, the controller reshapes the energy function of the system so that it has a minimum at the desired equilibrium point. The controller provides the system a finite amount of energy so as to drive the system to the desired state. This concept has been applied to electro-mechanical systems [17][20] and also to thermodynamic systems where the storage function is the entropy instead of the energy [1][21]. Nevertheless energy-balancing passivity-based control cannot be applied to some systems (namely systems with pervasive dissipation). To overcome this difficulty the concept of power-shaping control has been introduced, firstly for the stabilization of nonlinear RLC circuits [18]. Contrary to energy-balancing passivity-based control, the storage function used for the control is related to the power and not to the energy. Power-shaping control has subsequently been applied to the control of mechanical and electromechanical systems [12].

This paper is dedicated to the analysis and control of the CSTR study case by the means of the power-shaping approach. First we shall briefly introduce the concepts of the power-shaping approach in Section 2. Then in Section 3 we shall explain the general methodology for writing the Brayton-Moser form on the particular case of a CSTR. This methodology shall then be applied to obtain the Brayton-Moser formulation of the dynamics of a CSTR with an irreversible reaction (Section 4). This formulation shall be used to design a control system for this CSTR using the power-shaping approach. Finally we shall give some considerations about the generalization of this work to more general cases. In Section 5 we shall extend the results obtained for the CSTR with an irreversible reaction to more complex reactions and reaction kinetics. Section 6.1 presents some primary results for the Brayton-Moser formulation for a general CSTR.

2 Power-shaping control

In this section we briefly explain the principles of the power-shaping approach. The statements are given without any proof. For more details, the reader can refer to [12][14][15][18].
2.1 The Brayton-Moser formulation

Let us consider a dynamic system of dimension $N$ with $m$ inputs. The state of the system is given by the vector $x \in \mathbb{R}^N$ and the input is given by vector $u_c \in \mathbb{R}^m$. The power-shaping control theory is based on the Brayton-Moser formulation of the system dynamics [3]. In this formulation the system dynamics are of following form:

$$ Q(x) \frac{dx}{dt} = \nabla \mathcal{P}(x) + G(x) u_c \quad (1) $$

where $Q(x) : \mathbb{R}^N \to \mathbb{R}^N \times \mathbb{R}^N$ is a non-singular square matrix, $\mathcal{P}(x) : \mathbb{R}^N \to \mathbb{R}$ is a scalar function of the state and $G(x) : \mathbb{R}^N \to \mathbb{R}^N \times \mathbb{R}^m$. Additionally the symmetric part of the matrix $Q(x)$ is negative semi-definite, i.e.:

$$ Q(x) + Q^t(x) \preceq 0 \quad (2) $$

The function $\mathcal{P}(x)$ is called the potential function. In electrical and mechanical systems, the potential function has the units of power and is related to the dissipated power in the system. In the first one it is related to the so-called content and co-content of the resistances [18][15]; in the latter it is related to the Rayleigh dissipation function [14].

Let us now assume the system dynamics is given by the following relation:

$$ \frac{dx}{dt} = f(x) + g(x) u_c \quad (3) $$

where $f(x) : \mathbb{R}^N \to \mathbb{R}^N$ and $g(x) : \mathbb{R}^N \to \mathbb{R}^N \times \mathbb{R}^m$. The system (3) can be written in the form (1) if there exists a non-singular matrix $Q(x)$ fulfilling (2) and that solves following partial differential equation:

$$ \nabla (Q(x) f(x)) = \nabla^t (Q(x) f(x)) \quad (4) $$

$\mathcal{P}(x)$ is the solution of the following partial differential equation system:

$$ \nabla \mathcal{P}(x) = Q(x) f(x) \quad (5) $$

and the function $G(x)$ is given by $G(x) = Q(x) g(x)$.

2.2 Power-shaping control

Let us assume that the system dynamics can be expressed by using the Brayton-Moser equations presented before. The desired equilibrium state is
denoted by $x^*$. The principle of power-shaping control is to choose the input $u_c(x)$ such that in closed loop the system dynamics are given by the following relation:

$$\frac{dx}{dt} = \nabla \mathcal{P}_d(x)$$

(6)

where $\mathcal{P}_d(x) : \mathbb{R}^N \rightarrow \mathbb{R}$ is the re-shaped potential function. The desired equilibrium point $x^*$ must be a local minimum of the potential function $\mathcal{P}_d(x)$ in order to be locally asymptotically stable. The function $\mathcal{P}_d(x)$ can be used as a Lyapunov function for the closed-loop system.

The function $\mathcal{P}_d(x)$ cannot be chosen arbitrarily since the following relation must be fulfilled:

$$g^\perp(x) Q^{-1} \nabla \mathcal{P}_a(x) = 0$$

(7)

where $\mathcal{P}_a(x) = \mathcal{P}_d(x) - \mathcal{P}(x)$ and

$$g^\perp(x) : \mathbb{R}^N \rightarrow \mathbb{R}^{N-m} \times \mathbb{R}^N$$

is a full-rank left annihilator of $g(x)$ (i.e. $g^\perp(x) g(x) = 0$ with rank $(g^\perp(x)) = N - m$). Under these conditions, the control input $u(x)$ that re-shapes $\mathcal{P}(x)$ into $\mathcal{P}_d(x)$ is the following one:

$$u_c(x) = \left( g^l(x) Q^l(x) Q(x) g(x) \right)^{-1} g^l(x) Q^l(x) \nabla \mathcal{P}_a(x)$$

3 The Brayton-Moser PDE for a general CSTR

3.1 The dynamical model

3.1.1 The general CSTR

Let us consider a liquid-phase CSTR with constant volume $V$ containing $N_c$ species and in which $N_r$ reactions take place. The reactor is cooled/heated by a surrounding jacket. As it has been shown in [10] the dynamics of such a system are given as follows ($i = 1, \ldots, N_c$):

$$\frac{dn_i}{dt} = \frac{F}{V} (C_{i}^{\text{in}} V - n_i) + \sum_{l=1}^{N_r} \Gamma_{il} r_l (T, n)$$

(8a)

$$\frac{dU}{dt} = \frac{F}{V} (h^{\text{in}} V - H) + \dot{Q}$$

(8b)

with $n_i$ the quantity of species $i$, $n$ is a column vector with $n_i$ in the $i^{th}$ position, $U$ the internal energy of the mixture, $F$ the volume flow rate, $C_{i}^{\text{in}}$ is the concentration of $i$ in the inlet flow, $\Gamma_{il}$ is the stoichiometric coefficient of $i$
in reaction $l$, $r_l$ is the reaction rate of reaction $l$, $h^{in}$ is the volumetric enthalpy of the inlet flow, $T$ the temperature, $H$ the enthalpy of the mixture and $\dot{Q}$ the heat transferred into the reacting mixture. $H$ and $T$ are functions of $U$ and $n$ depending on the thermodynamic model of the liquid (e.g. an ideal liquid mixture):

$$H = \hat{H}(n, U), \quad T = \hat{T}(n, U)$$

Using the temperature function as a state transformation, (8b) can be replaced by the following relation:

$$\mathcal{C}_p V \frac{dT}{dt} = \dot{Q} + F h^{in} + \sum_{l=1}^{N_r} (-\Delta_r H)_l (T) \cdot r_l (T, n)$$

(9)

where $(-\Delta_r H)_l (T)$ is the molar reaction heat of reaction $l$ and $\mathcal{C}_p V = \sum_{i=1}^{N_o} C_{pi} n_i$ is the total heat capacity of the mixture (with $C_{pi}$ the volumetric heat capacity of species $i$).

The aim is to control the reactor temperature by acting on the heat transfer.

As explained in the previous section, the change from the usual form of the system dynamics (3) into its Brayton-Moser formulation (1) requires the solution of the partial differential equation system (4). Using (8) (4) is written as follows:

$$\frac{\partial}{\partial x_j} \left( \sum_{k=1}^{N_r} q_{jk}(x) \left[ \frac{F}{V} \left( C_{pi}^{in} V - n_i \right) + \sum_{l=1}^{N_r} \Gamma_{il} \right] \right) = \frac{\partial}{\partial x_i} \left( \sum_{k=1}^{N_r} q_{ik}(x) \left[ \frac{F}{V} \left( C_{pj}^{in} V - n_j \right) + \sum_{l=1}^{N_r} \Gamma_{jl} \right] \right)$$

(10)

The solution of these equations turns out to be quite intricate and we do not even know if a solution exists. In order to keep the resolution as simple as possible we shall first work with a simplified version of the CSTR model.

### 3.1.2 The simplified CSTR

In order to simplify the model, we consider the following additional assumptions:

- the specific heat $\mathcal{C}_p$ of the mixture are constant.
- the reaction enthalpy $(-\Delta_r H)_l$ of reaction $l$ is independent of the temperature.

Following usual thermodynamic notations $(-\Delta_r H)_l > 0$ for exothermic reactions and $(-\Delta_r H)_l < 0$ for endothermal reactions.
the heat exchange between the reactor and the jacket is proportional to the
temperature difference between them, with $\theta$ the the heat transfer coeffi-
cient.

The aim is to control the reactor temperature by acting on the heat transfer,
and more precisely by acting on the cooling fluid flow rate. Changing
the cooling fluid flow rate influences the system dynamics via the heat transfer
coefficient $\theta$ which is dependent on the cooling fluid flow rate. If the jacket
dynamics are fast enough compared to the reactor dynamics, this relation is
time independent. Therefore we shall consider in the sequel that the control
input $u_c$ is the quantity $\frac{\theta}{C_p V}$. However implementing this control law in practice
requires that the relation between the cooling fluid flow rate and the heat
transfer coefficient is know. The open-loop case corresponds to $\theta = 0$, i.e. an
adiabatic reactor.

This additional assumptions allows us to rewrite (8) as follows [2][23]:

$$
\frac{dn_i}{dt} = \frac{F}{V} \left( C_{i}^{in} V - n_i \right) + \sum_{l=1}^{N_r} \Gamma_{il} r_l(T, n) \\
\frac{dT}{dt} = \frac{F}{V} \left( T^{in} - T \right) + \sum_{l=1}^{N_r} \gamma_l r_l(T, n) + u_c (T_w - T)
$$

with $T_w$ the (constant) cooling fluid temperature and $T^{in}$ is the inlet temper-
ate.

The time evolution equations of $n$ and $T$ have the same form and the dynamics
of the system can be written as follows:

$$
\frac{dx}{dt} = \frac{F}{V} \left( x^{in} - x \right) + \Gamma r(x) + u_c \left[ 0, \ldots, 0, T_w - T \right]^t
$$

where $x = [n_A, \ldots, n_{N_c}, T]^t$. $\Gamma \in \mathbb{R}^{N_c+1 \times N_r}$ is a matrix that contains the
stoichiometric coefficient of species $i$ in the reaction $l$ in position $(i, l)$ if $1 \leq
i \leq N_c$ and that contains $\gamma_l$ in the $l^{th}$ column if $i = N_c + 1$. Applying the
notations of Section 2 to (12), we have:

$$
f (x) = \frac{F}{V} \left( x^{in} - x \right) + \Gamma r(x) \quad \text{and} \quad g (x) = \left[ 0, \ldots, 0, T_w - T \right]^t
$$

3.2 Integrability condition: the Brayton-Moser partial differential equation

The first step to find the Brayton-Moser form of the dynamics is the solution
of a partial differential equation, i.e. find a matrix $Q (x)$ that fulfills (2) and
(4). By using (12) we can write the following relation (for \(i = 1, \ldots, N_c + 1\)):

\[
(Q(x)f(x))_i = \sum_{k=1}^{N_c+1} q_{ik} \left( \frac{F}{V} (x_k^n - x_k) + \Gamma_{kl} r_l(x) \right)
\]

By differentiating the above expression the condition (4) can be rewritten as follows for all \(i, j = 1, \ldots, N_c + 1\):

\[
- \frac{F}{V} q_{ij} + \sum_{k=1}^{N_c+1} \left( q_{ik} \sum_{l=1}^{N_r} \Gamma_{kl} \frac{\partial r_l}{\partial x_j} \right) + \sum_{k=1}^{N_c+1} \frac{\partial q_{ik}}{\partial x_j} \left( \frac{F}{V} (x_k^n - x_k) + \Gamma_{kl} r_l(x) \right) = - \frac{F}{V} q_{ji} + \sum_{k=1}^{N_c+1} \left( q_{jk} \sum_{l=1}^{N_r} \Gamma_{kl} \frac{\partial r_l}{\partial x_i} \right) + \sum_{k=1}^{N_c+1} \frac{\partial q_{jk}}{\partial x_i} \left( \frac{F}{V} (x_k^n - x_k) + \Gamma_{kl} r_l(x) \right)
\]

This partial differential equation is difficult to solve in general. So we shall now restrict ourselves to a particular set of possible matrices \(Q(x)\) to find a solution to (4). This set is such that the partial differential equation system is transformed into an algebraic one. (13) is transformed into an algebraic equation by setting:

\[
\forall i, j, k = 1, \ldots, N_c + 1, \ i \neq j : \quad \frac{\partial q_{ik}}{\partial x_j} = 0
\]

(13) becomes then as follows:

\[
- \frac{F}{V} q_{ij} + \sum_{k=1}^{N_c+1} \left( q_{ik} \sum_{l=1}^{N_r} \Gamma_{kl} \frac{\partial r_l}{\partial x_j} \right) = - \frac{F}{V} q_{ji} + \sum_{k=1}^{N_c+1} \left( q_{jk} \sum_{l=1}^{N_r} \Gamma_{kl} \frac{\partial r_l}{\partial x_i} \right)
\]

The above set of algebraic equations is still complex to solve due to the general form of the kinetics. Therefore, we shall also assume that the kinetics of reaction \(l\) can be written as a product as follows:

\[
r_l(x) = k_l(T) \prod_{m=1}^{N_r} \phi_m^{(l)}(n_m)
\]

The functions \(\phi_m^{(l)}(n_m)\) are smooth functions of their argument and \(\phi_m^{(l)}(0) = 0\) if the species \(n_m\) intervenes in the kinetics of reaction \(l\); otherwise \(\phi_m^{(l)}(n_m) = 1\). \(k_l(T)\) is the kinetic coefficient of the reaction \(l\). \(k_l(T)\) is such
that the reaction rate $r_l(x)$ fulfills the following conditions\textsuperscript{4}:

$$
k_l(T) > 0, \quad \lim_{T \to 0} k_l(T) = 0, \quad \lim_{T \to \infty} k_l(T) = k_l\infty > 0
$$

$$\frac{dk_l}{dT} > 0, \quad \lim_{T \to 0} \frac{dk_l}{dT} = 0, \quad \lim_{T \to \infty} \frac{dk_l}{dT} = 0 \tag{16}\text{Remark 1}$$

This assumption restricts the set of reaction kinetics that are considered since it excludes for example the Michaelis-Menten kinetics for enzymatic reactions. However the considered ones still cover a large number of practical cases, and among them the reactions with mass action law kinetics.

(15) is written as follows (for $i, j = 1, \ldots, N_c + 1$ with $i \neq j$):

\begin{itemize}
  \item for $i, j \neq N_c + 1$:
    $$
    - \frac{F}{V} q_{ij} (n_i) + \sum_{l=1}^{N_r} \left[ k_l(T) \phi_i^{(l)} (n_i) \frac{\partial \phi_i^{(l)}}{\partial n_j} \right. \\
    \times \left( \sum_{k=1}^{N_c+1} q_{ik} (n_i) \Gamma_{kl} \right) \left. \prod_{m \neq i, j}^{N_c} \phi_m^{(l)} (n_m) \right] = - \frac{F}{V} q_{ji} (n_j) + \sum_{l=1}^{N_r} \left[ k_l(T) \phi_j^{(l)} (n_j) \frac{\partial \phi_j^{(l)}}{\partial n_i} \right. \\
    \times \left( \sum_{k=1}^{N_c+1} q_{jk} (n_j) \Gamma_{kl} \right) \left. \prod_{m \neq i, j}^{N_c} \phi_m^{(l)} (n_m) \right] \tag{17a}\text{Remark 1}
    $$

  \item for $j = N_c + 1$:
    $$
    - \frac{F}{V} q_{ij} (n_i) + \sum_{l=1}^{N_r} \left[ \phi_i^{(l)} (n_i) \frac{dk_l}{dT} \right. \\
    \times \left( \sum_{k=1}^{N_c+1} q_{ik} (n_i) \Gamma_{kl} \right) \left. \prod_{m=1, m \neq i}^{N_c} \phi_m^{(l)} (n_m) \right] = - \frac{F}{V} q_{ji} (T) + \sum_{l=1}^{N_r} \left[ k_l(T) \frac{\partial \phi_i^{(l)}}{\partial n_i} \right. \\
    \times \left( \sum_{k=1}^{N_c+1} q_{jk} (T) \Gamma_{kl} \right) \left. \prod_{m=1, m \neq i}^{N_c} \phi_m^{(l)} (n_m) \right] \tag{17b}\text{Remark 1}
    $$
\end{itemize}

A general solution of (17a) and (17b) has not been found, yet. Despite of this, some characteristics of the solution (if it exists) can be derived. Indeed (17a) has to be satisfied for any $(n, T)$ and in particular for $T$ tending to 0. By taking the limit of (17a) for $T$ tending to 0, we find that the following relation

\textsuperscript{4} These assumptions on $k(T)$ are fulfilled by the commonly used Arrhenius law.
has to be fulfilled for $i, j \neq N_c + 1$ and $i \neq j$:

$$-\frac{F_v}{V} q_{ij} (n_i) = -\frac{F_v}{V} q_{ji} (n_j)$$

This means that $q_{ij} = q_{ji}$ and $q_{ij}$ and $q_{ji}$ have to be constants. Thus the only elements that can depend on the composition in the matrices $Q(x)$ that solve (17a) and (17b) are the diagonal terms. All non-diagonal elements (except for the last line and the last column) are constant.

4 The non-isothermal CSTR with first order irreversible reaction

As already mentioned, a general solution of (17a) and (17b) has not been found, yet. Furthermore to this stage we do not even know if a solution exists. Therefore we shall in a first instance work with the most simple case, namely the case of an irreversible reaction with one reactant.

4.1 The dynamical model

In order to simplify the system dynamics to reduce its dimension as much as possible, we shall the particular case of an irreversible reaction $A \rightarrow \ldots$ with reaction kinetics that follow the mass action law. As a consequence the reaction kinetics can be written as follows:

$$r (n_A, T) V = k(T) n_A$$

where $k(T)$ is the kinetic coefficient. $k(T)$ fulfills (16).

From (11) the dynamical behaviour of the CSTR can be represented by a two-dimensional system as follows [2][23]:

$$\begin{cases} \frac{dn_A}{dt} = \frac{F}{V} \left( C^{\text{in}}_AV - n_A \right) - k(T) n_A \\ \frac{dT}{dt} = \frac{F}{V} \left( T^{\text{in}} - T \right) + \gamma k(T) n_A + u_c (T_w - T) \end{cases}$$

(18)

with $\gamma = \frac{(-\Delta_r H)}{\rho c_p V}$. It can be shown that this system can have several equilibrium points depending on the numerical values of the parameters. In this work we do not consider the cases where limit cycles can occur. In the endothermic case there can be only one asymptotically stable equilibrium, whereas in the exothermic case an odd number of equilibria can occur. In this last case, there is an alternance of asymptotically stable and unstable equilibrium points.
The most common case which can be observed for instance with Arhenius type kinetics is the case of three equilibrium points where the temperature of the unstable equilibrium is intermediate to the two other (asymptotically stable) equilibrium temperatures.

**Proposition 2** The dynamics of the CSTR can be expressed in the form of the Brayton-Moser equations with:

\[
Q(n_A, T) = \left( \frac{\gamma}{k(T) + 4 \frac{F}{V} \alpha} \frac{1}{k(T)^2 + 4 \left( \frac{F}{V} \right)^2 \alpha + 4 \alpha F k(T)} \right) q_{12} \tag{19}
\]

where \(0 < \alpha \leq 1\) and \(q_{12}\) are two real coefficients such that \(\frac{q_{12} \gamma}{k^2} < 0\).

**PROOF.** It is sufficient to check that the matrix \(Q(x)\) given in (19) is nonsingular and fulfills (2) and \((17b)^5\). Indeed the determinant of \(Q(n_A, T)\) is non-zero:

\[
det Q(n_A, T) = \frac{4 \alpha}{k^2} q_{12} \left( \frac{F}{V} \right)^2 > 0
\]

Concerning (2) it is sufficient to check the sign of the trace and the determinant of the symmetric part of \(Q(n_A, T)\) (the dependence in \(T\) of \(k(T)\) has not been written for the sake of shortness of the notations):

\[
tr(Q + Q^t) = 2q_{12} \frac{k^2 (\gamma^2 + 1) + 4 \left( \frac{F}{V} \right)^2 \alpha + 4 \alpha \frac{F}{V} k}{\gamma k^2} \neq 0
\]

\[
det(Q + Q^t) = 16 \left( \frac{F}{V} \right)^2 q_{12}^2 k^2 \alpha (1 - \alpha) \geq 0
\]

As a consequence the condition \((17b)\) is fulfilled [8].

The potential function \(\mathcal{P}(n_A, T)\) is the solution of (5):

\[
\mathcal{P}(n_A, T) = q_{12} \left[ -\frac{F}{V} \frac{1}{2 \gamma} \left( \gamma \left( C_A^{in} V - n_A \right) + (T^{in} - T) \right)^2 
+ 4 \alpha \left( \frac{F}{V} \right)^2 \times \int_{T_0}^{T} \frac{T \gamma C_A^{in} V k(\tau) + \left( \frac{F}{V} + k(\tau) \right) (T^{in} - \tau) d\tau}{\gamma k(\tau)^2} \right]
\]

where \(T_0 > 0\) is an arbitrary reference temperature. The first term is related to the convection whereas the other terms are linked to the reaction kinetics.

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\( (17a) \) does not exist because \(N_c = 1\).
The equilibrium points of the system \((\bar{n}_A, \bar{T})\) are the solutions of \(f(\bar{n}_A, \bar{T}) = 0\). As a consequence:

\[
Q(\bar{n}_A, \bar{T}) f(\bar{n}_A, \bar{T}) = \nabla \mathcal{P}(\bar{n}_A, \bar{T}) = 0
\]

Hence the equilibrium points of the system are the critical points of \(\mathcal{P}(n_A, T)\).

**Proposition 3** The asymptotically stable equilibria are local minima of the function \(\mathcal{P}(n_A, T)\) and the unstable equilibria are saddle points.

**PROOF.** First let us determine the nature of the critical points of \(\mathcal{P}(n_A, T)\) by computing the eigenvalues of the Hessian matrix \(\mathcal{H}(\mathcal{P})\) at the equilibrium points:

\[
\text{tr} (\mathcal{H}(\mathcal{P})) = -q_1 \frac{F}{V} \left[ k\left(\bar{T}\right)^2 \left(1 + \gamma^2\right) + 4\alpha \frac{F}{V} \left(k\left(\bar{T}\right) + \frac{F}{V} - \gamma \bar{n}_A \frac{dk}{dT}\right)\right]
\]

\[
\det (\mathcal{H}(\mathcal{P})) = 4\alpha q_1^2 \left(\frac{F}{V}\right)^3 \left(-\gamma \bar{n}_A \frac{dk}{dT} + k\left(\bar{T}\right) + \frac{F}{V}\right)
\]

Besides the eigenvalues of the linearized system around the equilibria \(\lambda_1\) and \(\lambda_2\) are given as follows:

\[
\lambda_1 = -\frac{F}{V} \quad \text{and} \quad \lambda_2 = \gamma \bar{n}_A \frac{dk}{dT} - k\left(\bar{T}\right) - \frac{F}{V}
\]

At an unstable equilibrium \(\lambda_2 > 0\) whereas at the stable ones \(\lambda_2 < 0\) [8]. As a consequence, \(\mathcal{H}(\mathcal{P})\) is positive definite at the stable equilibria, whereas it indefinite at the unstable ones.

As shown in [8] there exist a particular choice of the reference temperature \(T_0\) such that \(\mathcal{P}(n_A, T) > 0\) for all \((n_A \geq 0, T > 0)\). More precisely this value is equal to one of the stable equilibrium temperatures. With this particular choice of \(T_0\), \(\mathcal{P}(n_A, T) > 0\) is a Lyapunov function of the open-loop system.

### 4.2 Controller design

The aim is to design a control law \(u_c = u_c(n_A, T)\) such that the closed loop system has an asymptotically stable equilibrium at the desired state \((\bar{n}_A^*, T^*)\). Let us apply the methodology of Section 2.2.
Fig. 1. Level curves of $\mathcal{P}(n_A, T)$ for the case with three equilibrium points. The low temperature equilibrium lies outside the frame.

**Lemma 4** The desired set-point $(n_A^*, T^*)$ can be an equilibrium point of the closed-loop system if and only if:

$$n_A^* = \frac{F C_A^{\text{in}} V}{V + k(T^*)}$$  \hspace{1cm} (21)

**PROOF.** $(n_A^*, T^*)$ is an equilibrium of the closed loop system, if and only if:

$$\frac{dn_A}{dt} = \frac{F}{V} \left(C_A^{\text{in}} V - n_A^* \right) - k(T^*) n_A^* = 0$$

(21) can be deduced directly from this equation.

**Lemma 5** With $Q(n_A, T)$ given in Proposition 2 there exists a control input $u_c = u_c(n_A, T)$ such that the closed-loop dynamics can be written in the form (6) with the re-shaped potential function $\mathcal{P}_a(n_A, T) = \mathcal{P}_a(n_A, T) + \mathcal{P}(n_A, T)$ if and only if $\mathcal{P}_a(n_A, T)$ is of the following form:

$$\mathcal{P}_a(n_A, T) = w \left( n_A + \int_{T_0}^T v(\tau) d\tau \right)$$  \hspace{1cm} (22)

where $v(T) = \frac{k(T)^2 + 4 (\frac{F}{V})^2 \alpha + 4 \alpha \frac{F}{V} k(T)}{\gamma k(T)^2}$. $w(z) : \mathbb{R} \to \mathbb{R}$ is some continuously differentiable function.
PROOF. The closed-loop dynamics can be written in the form (6) if and only if there exists a solution $u_c(n_A, T)$ to the equation

$$\mathcal{P}_a(n_A, T) = Q(n_A, T) g(n_A, T) u_c(n_A, T)$$

This is the case if and only if $\mathcal{P}_a(n_A, T)$ is the solution of the differential equation (7). In our case $g(x) = \begin{bmatrix} \varphi & 0 \end{bmatrix}$ with $\varphi \in \mathbb{R}^*$ so that (7) becomes as follows:

$$\frac{q_{12} \varphi}{\det(Q(x))} \left( v(T) \frac{\partial \mathcal{P}_a}{\partial n_A} - \frac{\partial \mathcal{P}_a}{\partial T} \right) = 0$$

The solution of this partial differential equation is of the form given in (22).

**Proposition 6** Let us denote by

$$u^*_c(T_w - T^*) = -\frac{F}{V} \left( T^{\text{in}} - T^* \right) - \gamma k(T^*) n^*_A$$

the equilibrium value of the control input. If the conditions of Lemmas 4 and 5 are fulfilled and if:

$$\left. \frac{dw}{dz} \right|_{z^*} = -|q_{12}| u^*_c(T_w - T^*)$$

with

$$\frac{\omega}{\gamma} = -\frac{1}{\gamma q_{12}} \left. \frac{d^2w}{dz^2} \right|_{z^*} > \frac{k^*^2 \frac{F}{V} \Psi^*}{4 \alpha \frac{F}{V} \left( k^* + \frac{F}{V} \right)^2 - k^*^2 \Psi^*}$$

where

$$\Psi^* = -\left( k^* + \frac{F}{V} \right) + \left. \frac{dk}{dT} \right|_{T^*} \gamma n^*_A$$

then $(n^*_A, T^*)$ is an asymptotically stable equilibrium of the closed-loop dynamics.

If

$$4 \alpha \frac{F}{V} \left( k^* + \frac{F}{V} \right)^2 - k^*^2 \Psi^* < 0$$

then the closed-loop system has no asymptotically stable equilibrium.

PROOF. The closed-loop system (6) has an asymptotically stable equilibrium at the desired set-point $(n^*_A, T^*)$ if and only if $(n^*_A, T^*)$ is a local minimum of $\mathcal{P}_d(n_A, T)$. Lemmas 4 and 5 combined with (23) ensure that $(n^*_A, T^*)$ is such that $\nabla \mathcal{P}_d(n^*_A, T^*) = 0$. To have a minimum the Hessian matrix $\mathcal{H} (\mathcal{P}_d)$ of the function $\mathcal{P}_d(n_A, T)$ must be positive definite at $x^* = (n^*_A, T^*)$.

$$\mathcal{H} (\mathcal{P}_d)(x^*) = \begin{pmatrix} -\gamma \frac{F}{V} - \omega & -\frac{F}{V} - \omega v(T^*) \\ -\frac{F}{V} - \omega v(T^*) & H_{22} \end{pmatrix} q_{12}$$
with \( H_{22} = -\frac{F}{k_{12}} \left( k^2 - 4\alpha F \Psi \right) - \omega v (T^*)^2 \). If (25) is fulfilled, then the Hessian matrix \( \mathcal{H}(\mathcal{P}_d) \) cannot be positive definite [8]. In the other cases \( \omega \) must fulfill the condition (24) so that the Hessian matrix \( \mathcal{H}(\mathcal{P}_d) \) is definite positive [8].

The simplest function \( w(z) \) that fulfills the above proposition is the following one:

\[
\mathcal{P}_a (n_A, T) = w(z) = |q_{12}| \left[ \omega (z - z^*)^2 - u_c^* (T_w - T^*) (z - z^*) \right]
\]

The final expression of the control input is finally deduced from (8):

\[
u_c (n_A, T) (T_w - T) = \omega \left( n_A^* - n_A - \int_{T^*}^T v (\tau) d\tau \right) + u^* (T_w - T^*) \tag{26}
\]

The control law has two parameters to be chosen: \( \alpha \) and \( \omega \). Note that the possible values are constrained by (24) and (25). Finally let us mention that the control action \( u(n_A, T) \) acts in (18) via the term

\[
g(n_A, T) u_c(n_A, T) = \begin{pmatrix} 0 \\ \omega \left( n_A^* - n_A - \int_{T^*}^T v (\tau) d\tau \right) + u^* (T_w - T^*) \end{pmatrix}
\]

which does not depend on the difference \( (T_w - T) \) anymore. This means that the actual control input is the transferred heat.

4.3 Considerations on local and global convergence

The controller designed in the previous section only guarantees a local convergence to the desired equilibrium point, i.e. the system will converge to desired point if and only if the initial conditions are close enough to it. Global convergence is ensured if and only if the shaped potential function \( \mathcal{P}_d (n_A, T) \) does not have other local minima, i.e. if and only if \( x \) is a global minimum of the function \( \mathcal{P}_d (n_A, T) \).

By analogy with (21) let us define the function \( n_A^{eq} (T) \) by the following relation:

\[
n_A^{eq} (T) = \frac{E C_A^u V}{F + k(T)}
\]
Let us also consider the following function:

\[
\Delta(T) = \frac{C^i_{AV}}{V} \left( C^i_{AV} V - n^{eq}_A(T) \right) + \frac{F}{V} \left( T_{in} - T \right) \\
+ u^*_c (T_w - T^*) - \omega \left( n_A^{eq}(T) - n_A^* + \int_{T^*}^{T} v(\tau) d\tau \right)(27)
\]

**Lemma 7** A state \((n_A^#, T^#)\) is an equilibrium state of the closed loop system (18) if and only if it is contained in the following set:

\[
\Omega = \left\{ \Delta(T^#) = 0, n_A^# = n_A^{eq}(T^#) \right\}
\]

**PROOF.** To prove the "only-if" part, let \((n_A^#, T^#)\) be an equilibrium state of the closed loop system. Hence we have:

\[
\begin{align*}
\frac{dn_A}{dt} &= 0 = \frac{F}{V} \left( C^i_{AV} V - n_A^# \right) - k(T^#) n_A^# \\
\frac{dT}{dt} &= 0 = \frac{F}{V} \left( T_{in} - T^# \right) + \gamma k(T^#) n_A^# \\
&+ \omega \left( n^*_A - n_A^#(T^#) - \int_{T^*}^{T^#} v(\tau) d\tau \right) \\
&+ u^*_c (T_w - T^*)
\end{align*}
\]

The first equation can be rewritten as \(n_A^* = n_A^{eq}(T^*)\). When replacing this expression into the second equation of (28), the right-hand term is equal to \(\Delta(T^#)\).

To prove the "if" part, let us consider a state \((n_A^#, T^#)\) that is contained in \(\Omega\). By replacing \(n_A\) by the expression of \(n_A^{eq}(T^#)\) in the first equation of (28), it is obvious that \(\frac{dn_A}{dt} = 0\) if \((n_A^#, T^#) \in \Omega\). Let us now consider the temperature dynamics:

\[
\begin{align*}
\frac{dT}{dt} &= \frac{F}{V} \left( T_{in} - T^# \right) + \gamma k(T^#) n_A^# \\
&+ u^*_c \left( n_A^#(T^#) \right) (T_w - T^#) \\
&= \frac{F}{V} \left( T_{in} - T^# \right) + \gamma k(T^#) n_A^{eq}(T^#) \\
&+ \omega \left( n^*_A - n_A^{eq}(T^#) - \int_{T^*}^{T^#} v(\tau) d\tau \right) \\
&+ u^*_c (T_w - T^*)
\end{align*}
\]
The right-hand term is equal to $\Delta \left( T^* \right)$ and consequently, we have $\frac{dT}{dt} = 0$.

The function $\Delta (T)$ is linear in the parameter $\omega$. Therefore it can be written as follows:

$$\Delta (T) = \Delta_0 (T) + \omega \Delta_\omega (T)$$

As it can be deduced from the work in [10] the existence of the three open-loop equilibria implies that $\Delta_0 (T)$ is decreasing for low and high temperatures, but increasing on one determined interval (see also [8]). As a consequence, $\Delta (T)$ can have several zeros, depending on the term $\omega \Delta_\omega (T)$.

**Proposition 8** If the parameter $\alpha$ fulfills the following condition:

$$\forall T \in \mathbb{R}^+ : 1 \geq \alpha > \frac{\Psi (T) k^2 (T)}{4 \frac{E}{\gamma} (k(T) + \frac{E}{\gamma})^2} \quad (29)$$

where $\Psi (T) = - \left( k(T) + \frac{F}{\gamma} \right) + \frac{dV}{dt} \gamma n_{eq}^* (T)$, then there exists a lower bound on $\frac{\omega}{\gamma}$ such that $\Delta (T)$ has only one zero.

**PROOF.** The derivative of $\Delta (T)$ with respect to $\frac{\omega}{\gamma}$ is given as follows:

$$\frac{d\Delta}{d \left( \frac{\omega}{\gamma} \right)} = - \int_{T^*}^{T} \frac{-k^2 (\tau) \Psi (\tau) + 4 \alpha \frac{E}{\gamma} \left( k(\tau) + \frac{E}{\gamma} \right)^2}{k^2 (\tau) \left( \frac{E}{\gamma} + k(\tau) \right)} d\tau$$

If the numerator is positive, i.e. if (29) is fulfilled, then $\Delta_\omega (T)$ is of the sign of $-(T - T^*)$. As a consequence for $\frac{\omega}{\gamma} > 0$, we have $\Delta (T) > \Delta_0 (T)$ (resp. $\Delta (T) < \Delta_0 (T)$) for $T > T^*$ (resp. $T < T^*$). As it can be seen in Figure 2 if the value of $\omega$ is sufficiently high, then $\Delta (T)$ has only one zero: the desired closed-loop equilibrium.

**Proposition 9** If (29) is fulfilled and

$$\forall T \in \mathbb{R}^+ : \frac{\omega}{\gamma} > \frac{\frac{E}{\gamma} \Psi (T) k^2}{-k^2 \Psi (T) + 4 \alpha \frac{E}{\gamma} \left( k + \frac{E}{\gamma} \right)} \quad (30)$$

then the controller (26) achieves global stability of the desired equilibrium point.

**PROOF.** (29) and (30) imply (25) and (24), respectively. Using Proposition 6 the desired equilibrium is locally stable.
Moreover, the derivative of $\Delta (T)$ is given as follows:

$$\frac{d\Delta}{dT} = \frac{\frac{E}{\Psi (T)} k (T)^2}{k (T)^2 (k (T) + \frac{E}{\Psi})} - \frac{\frac{\omega}{\gamma} \left(-k (T)^2 \Psi (T) + 4\alpha \frac{E}{\Psi} (k + \frac{E}{\Psi})\right)}{k (T)^2 (k (T) + \frac{E}{\Psi})}$$

Conditions (29) and (30) imply that $\frac{d\Delta}{dT} < 0$ for all $T > 0$. $\Delta (T)$ is monotonically increasing and hence has a unique zero, namely the desired closed-loop equilibrium.

**Remark 10** The conditions for the global convergence (29) and (30) are very similar to the ones for the local convergence (25) and (24), respectively, but instead of being satisfied only at the desired set point temperature $T^*$, they have to be satisfied at any $T$.

The shape of the function $\mathcal{P}_d (n_A, T)$ is shown in Figures 3 and 4. In the first case, the parameters $\alpha$ and $\omega$ were chosen such that local convergence is achieved (i.e. (24) is fulfilled but not (30)). The function $\mathcal{P}_d (n_A, T)$ has two local minima and consequently the convergence to the desired state is only local. In the second case the parameters $\alpha$ and $\omega$ were chosen such that global convergence is achieved (i.e. (30) is fulfilled). The function $\mathcal{P}_d (n_A, T)$ has a single minimum and the convergence to the desired state is global.

**Remark 11** Even if the conditions of Proposition 9 are satisfied, global convergence can be lost if the input is bounded. Local convergence will however be kept. It would be interesting to discuss the size of the attraction region with the proposed controller in case of input constraints compared to other controller proposed in the literature (e.g. in [24]).
5 Extension to more complex systems

The power-shaping approach has given interesting results on the case study of a CSTR with an irreversible reaction. It is therefore of major interest to see if this approach can be extended to more complex systems, and more particularly to systems with more than one reacting chemical species and/or with multiple reactions.

We shall now look at some particular cases of more complex reactions to get
a first intuition of the existence of non-singular and negative semi-definite solutions of (17a) and (17b). The solution of (17a) and (17b) in the case of parallel reactions, consecutive reactions and one reaction with two reactants is given in [8][9]. In the three cases it has been assumed that the kinetic functions $\phi_m^{(l)}(n_m)$ can be expressed as powers of $n_m$, i.e.:

$$
\phi_m^{(l)}(n_m) = (n_m)^{c_m^{(l)}}
$$

It has been shown that in some cases (17a) and (17b) have no solution. This does not mean that the dynamics cannot be put into the Brayton-Moser form. But if the Brayton-Moser form exists, then the relation (14) is not fulfilled and the partial differential equation (13) has to be solved. In the other cases a general solution has been found, but the obtained matrix may have a non-negative symmetric part. Therefore the corresponding potential functions cannot be used in general as a Lyapunov functions for the open-loop system because they may be non-decreasing along the system trajectories. As shown in [12], given a matrix $Q(x)$ that fulfills (4) other solutions to (4) can be built departing from the first one. Applying this methodology on the matrices that have been found could lead to another matrix $Q(x)$ that would be negative-definite and hence give a Lyapunov function for the open-loop system.

6 Some considerations on the general CSTR dynamics

6.1 General solution

As it has been explained in Section 3.1.1 the solution to (10) is quite intricate. One of the difficulties is due to the term in $E$. This difficulty can be overcome by also considering the dilution coefficient $\frac{1}{V}$ as an input of the system. This corresponds to the system studied in [22]. The Brayton-Moser formulation of the CSTR has been found with the same assumptions as in Section 3.1.2. However, as the authors mention, they did not manage to extend their result to more general cases.

Contrary to [22] who used the simplified form of the dynamics (12) with mass action law dynamics we shall use the more general form (8). Let us consider
a reversible reaction \( A \rightleftharpoons \nu B \). Using the notations of (3) we have:

\[
\begin{pmatrix}
-f(n_A, n_B, U) \\
g(n_A, n_B, U)
\end{pmatrix} = 
\begin{pmatrix}
-r(T, n_A, n_B) V \\
\nu r(T, n_A, n_B) V \\
0 \\
C^n_A V - n_A \\
C^n_B V - n_B \\
V h^{in} - \hat{H}(n_A, n_B, U) 1
\end{pmatrix}
\]

with \( u_c = \left[ \begin{array}{c} \xi \\ \eta \end{array} \right] \). The partial differential system (4) becomes (for \( i, j = 1, 2, 3 \)):

\[
\frac{\partial}{\partial x_j}[a_i(x) \hat{r}(x) V] = \frac{\partial}{\partial x_i}[a_j(x) \hat{r}(x) V]
\]

(31)

where the following notations have been used:

\[
a_i(U, n_A, n_B) = \nu q_{i2}(U, n_A, n_B) - q_{i1}(U, n_A, n_B)
\]

\[
\hat{r}(U, n_A, n_B) = r(T(U, n_A, n_B), n_A, n_B)
\]

and \( x = (n_A, n_B, U) \). The general solution of (31) is given as follows:

\[
a_1(x) \hat{r}(x) V = \int \phi(U, n_A, n_B) dn_B + \psi(U, n_A)
\]

\[
a_2(x) \hat{r}(x) V = \int \phi(U, n_A, n_B) dn_A + \varphi(U, n_B)
\]

\[
a_3(x) \hat{r}(x) V = \int \int \frac{d\phi}{dU} dn_A dn_B + \int \frac{d\psi}{dU} dn_A
\]

\[
+ \int \frac{d\varphi}{dU} dn_B + \rho(U)
\]

where \( \phi(U, n_A, n_B) \), \( \psi(U, n_A) \), \( \varphi(U, n_B) \) and \( \rho(U) \) are real scalar functions. The corresponding matrix \( Q(n_A, n_B, U) \) and the potential function \( \varphi(x) \) are given respectively by:

\[
Q(x) = \begin{pmatrix}
\nu q_{i2}(x) - a_1(x) q_{i2}(x) q_{i3}(x) \\
\nu q_{22}(x) - a_2(x) q_{22}(x) q_{23}(x) \\
\nu q_{32}(x) - a_3(x) q_{32}(x) q_{33}(x)
\end{pmatrix}
\]

\[
\varphi(n_A, n_B, U) = \int \phi(n_A, n_B, U) dn_A dn_B + \int \psi(n_A, U) dn_A
\]

\[
+ \int \varphi(n_B, U) dn_B + \int \rho(U) dU + \omega
\]
where $\omega$ is some real constant. The functions $q_{ij}(x)$ can be chosen arbitrarily as far as $Q(x)$ is non-singular and (2) is fulfilled. Finally let us mention that the solution to the partial differential equation system has been obtained without restricting to a particular subset of solutions, contrary to what has been done in the previous sections.

6.2 Particular solution

Let us choose the following particular solution:

$$
\phi(n_A, n_B, U) = -\frac{\partial^2 \hat{S}}{\partial n_A \partial n_B} \omega = 0
$$

$$
\psi(U, n_A) = 0 \quad \varphi(U, n_B) = 0 \quad \rho(U) = 0
$$

$$
q_{32} = 0 \quad q_{13} = 0 \quad q_{23} = 0
$$

where $\hat{S}(n_A, n_B, U)$ is the entropy function. The potential $\Phi(x)$ is the opposite of the entropy function $-\hat{S}(n_A, n_B, U)$ with the following matrix:

$$
Q(x) = \begin{pmatrix}
\frac{\Lambda}{T V} & \frac{\mu_B}{T V} & 0 \\
\frac{4\alpha}{\nu} \left( \frac{\Lambda}{T V} \right) - \frac{4\alpha}{\nu} \left( \frac{\Lambda}{T V} \right) & 0 & 0 \\
\frac{\mu_i}{T V} & 0 & \frac{\beta}{4(1-\alpha)T V \Lambda}
\end{pmatrix}
$$

(32)

where $0 \leq \alpha < 1$ and $\beta \geq 1$ are two real constants and $\Lambda = \nu \mu_B - \mu_A$. $\mu_i$ is the chemical potential of the species $i$ with $\frac{\mu_i}{T} = \frac{\partial \hat{S}}{\partial n_i}$. Following thermodynamics the reaction kinetics must fulfill the condition$^6$:

$$
\text{sign} \left[ \hat{r}(x) V \right] = \text{sign} \left[ \mu_A - \nu \mu_B \right]
$$

Using this property it can be seen that $Q(x)$ in (32) fulfills (2). The dynamics can be written as follows:

$$
\frac{dx}{dt} = -Q^{-1}(x) \nabla \hat{S}
$$

(33)

Using (2) it is possible to see that $-\hat{S}(n_A, n_B, U)$ is always decreasing along the systems trajectories. This is in accordance with the second principle of thermodynamics which claims that the entropy production is always positive. Indeed the open-loop system $u_c = 0$ is an isolated system and the entropy variation is equal to the entropy production.

$^6$ i.e. the reaction evolves in the direction of decreasing affinity $\mu_A - \nu \mu_B$. 

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The open-loop equilibria $\bar{x} = (\bar{n}_A, \bar{n}_B, \bar{U})$ are the set of points such that $\mu_A(\bar{x}) = \nu B(\bar{x})$. There are an infinity of points $\bar{x}$ contained in this set. The open-loop equilibria are locally stable but not asymptotically stable. Indeed let us consider an initial point $(n^0_A, n^0_B, U^0)$. From (8) it can be seen easily that along the trajectories we have:

$$\nu n_A + n_B = \nu n^0_A + n^0_B \quad \text{and} \quad U = U^0 \quad (34)$$

Using (34) as the invariant set, $-\dot{S}(x)$ as a Lyapunov function candidate and (33), the stability is deduced from La Salle’s theorem [16].

However the power-shaping methodology cannot be applied using the above proposed matrix $Q(x)$ and potential $-\dot{S}(x)$. Indeed the matrix $Q(x)$ is not defined at the open-loop equilibria and $Q^{-1}(x)$ is singular at these points. Contrary to what has been shown in Section 4 the gradient of the potential function is not zero at the open-loop equilibria.

**Remark 12** Note that the availability function proposed by Alonso, Ydstie and coworkers (e.g. [1]) can also be used as a potential function for the Brayton-Moser form of the dynamics:

$$\mathcal{P}(x) = S^{ref} - S(x) - \left(\frac{\mu_A^{ref}}{T^{ref}} - \frac{\mu_B^{ref}}{T^{ref}} \frac{1}{T^{ref}}\right) \begin{pmatrix} n_A^{ref} - n_A \\ n_B^{ref} - n_B \\ U^{ref} - U \end{pmatrix}$$

where $x^{ref} = (n_A^{ref}, n_B^{ref}, U^{ref})$ is a reference state and $S^{ref} = S(x^{ref})$. The corresponding matrix $Q(x)$ is very similar to (32) and is also not defined at the open-loop equilibria.

7 Conclusion

In this paper we apply the power-shaping approach to the control of a CSTR. This approach requires to write the system dynamics in the Brayton-Moser form. This particular form of the dynamics also allows to find a Lyapunov function for the open-loop system. However the dynamics can be written in the Brayton-Moser form only if a partial differential equation system is solved. We have given the form of this partial differential system in the case of a CSTR with general kinetics and we have proposed a methodology for finding some of its solutions. This methodology has been successfully applied to a CSTR with irreversible kinetics with one reactant. The Lyapunov function for the open-loop CSTR that has been found using the Brayton-Moser formulation of the dynamics has been related to the physical phenomena. We have also
shown that this methodology needs to be extended in the case of more complex reaction kinetics. Finally first results for applying the power-shaping approach in the case of a general CSTR with two control inputs have been presented. These results show how the power-shaping approach can be related to the entropy of the system. The work presented in [12] offers promising tools to extend these results, by instance for finding alternative potential functions departing from the proposed one.

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References


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