Choice Disjunctive Queries in Logic Programming

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Abstract: One of the long-standing research problems on logic programming is to treat the cut predicate in a logical, high-level way. We argue that this problem can be solved by adopting linear logic and choice-disjunctive goal formulas of the form $G_0 \oplus G_1$ where $G_0, G_1$ are goals. These goals have the following intended semantics: choose the true disjunct $G_i$ and execute $G_i$ where $i (= 0 \text{ or } 1)$, while discarding the unchosen disjunct. Note that only one goal can remain alive during execution. These goals thus allow us to specify mutually exclusive tasks in a high-level way.

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1 Introduction

One of the long-standing research problems on logic programming is to treat the extra-logical primitive in a high-level way. The advances of logic programming have enriched Horn clauses with additional programming primitives in a high-level way (higher-order programming, modules, local constants, etc). Nevertheless some key constructs could not be dealt with in a high-level way, in particular when we are concerned with mutual exclusion (and the cut predicate).

Consequently, much attention [10, 11, 7] has been given to finding a semantics that captures the cut predicate. These proposals are quite interesting but somewhat complicated.

In this paper, inspired by the work in [2], we propose a purely logical solution to this problem. It involves the direct employment of linear logic [5] to allow for choice-disjunctive goals. A choice-disjunctive goal is of the form $G_0 \oplus G_1$ where $G_0, G_1$ are goals. (A more intuitive name would be $\text{choose}(G_0, G_1)$.) Executing this goal with respect to a program $\mathcal{P}$ – $\text{ex}(\mathcal{P}, G_0 \oplus G_1)$ – has the following intended semantics:

choose a true one between $\text{ex}(\mathcal{P}, G_0), \text{ex}(\mathcal{P}, G_1)$. 

For example, given a program \{male(kim), female(lee)\}, execution will succeed on the goal male(kim) ⊕ female(kim) by choosing male(kim). Similarly, execution will succeed on the goal male(lee) ⊕ female(lee) by choosing female(lee). On the other hand, consider a goal male(kim) ⊕ female(lee). In this case, both disjuncts can lead to a success, and, in our semantics, it does not matter which disjunct to use. For simplicity, we assume from now on that execution always chooses the first successful disjunct if there are many. Thus, back to the above, execution will succeed on the goal male(kim) ⊕ female(lee) by choosing male(kim). Note that the class of choice disjunctive goals is a superset of the class of mutually exclusive goals.

Another illustration of this construct is provided by the following definition of the relation \( son(X, Y) \) which holds if \( Y \) is a son of \( X \):

\[
son(X, Y) : = \neg (male(X) \land father(Y, X)) \oplus (female(X) \land mother(Y, X)).
\]

The body of the definition above contains a mutually exclusive goal, denoted by \( \oplus \). As a particular example, solving the query \( son(tom, Y) \) would result in selecting and executing the first goal \( male(tom) \land father(tom, Y) \), while discarding the second one. The given goal will succeed, producing solutions for \( Y \). Of course, we can specify mutually exclusive goals using cut in Prolog, but it is well-known that cuts complicates the declarative meaning of the program. Our language makes it possible to formulate mutually exclusive goals in a high-level way. The class of choice disjunctive goals is, in a sense, a high-level abstraction for the cut predicate.

As seen from the example above, choice-disjunctive goals can be used to perform mutually exclusive tasks. There are several well-designed linear logic languages [6, 12] in which goals of the form \( G_0 \oplus G_1 \) are present. A common aspect of these works is their treatment of the \( \oplus \)-goals: these goals are treated as inclusive-OR (or classical disjunctive) goals rather than exclusive-OR ones:

\[
ex(P, G_0 \oplus G_1) \text{ if } ex(P, G_0) \lor ex(P, G_1)
\]

where \( \lor \) represents classical disjunction. Hence, it is rather unfortunate that the declarative reading of \( \oplus \) – known as the machine’s choice – is missing in these languages.

A satisfactory solution can be obtained by adopting game semantics of [2], i.e., by adding an extra layer of the choice action, as discussed above, to their execution model of \( \oplus \). In this way, the execution respects the declarative reading of \( \oplus \), while maintaining provability. Hence, the main
difference is that, once a goal is chosen, the unchosen goal will be discarded in our language, while it will remain alive (typically through a creation of a choicepoint) in those languages.

This paper proposes Prolog⊕, an extension of Prolog with choice-disjunctive operators in goal formulas. The remainder of this paper is structured as follows. We describe Prolog⊕ in the next section and Section 3. In Section 4, we present some examples of Prolog⊕. Section 5 concludes the paper.

In this paper we present the syntax and semantics of this language called Prolog⊕. The remainder of this paper is structured as follows. We describe a subset of LINC logic in the next section. Section 3 describes the new semantics. Section 4 concludes the paper.

2 Prolog⊕ with the Old Semantics

The language is a version of Horn clauses with choice-disjunctive goals. Note that we disallow linear clauses here, thus allowing only reusable clauses. It is described by G- and D-formulas given by the syntax rules below:

\[
G ::= \top | A | t = s | G \land G | \exists x \ G | G \oplus G \\
D ::= A | G \supset A | \forall x \ D
\]

In the rules above, \(t, s\) represent terms, and \(A\) represents an atomic formula. A D-formula is called a Horn clause with choice-disjunctive goals. A set of D-formulas is called a program.

The logic programming paradigm such as Prolog was originally founded on the resolution method. But this approach was difficult to extend to richer logics. The use of sequent calculus allows us to overcome this limit. We adopt this approach below.

We will present a machine’s strategy for this language as a set of rules. These rules in fact depend on the top-level constructor in the expression, a property known as uniform provability[9, 8]. Note that execution alternates between two phases: the goal-reduction phase and the backchaining phase. In the goal-reduction phase, the machine tries to decompose a goal \(G\). If \(G\) becomes an atom, the machine switches to the backchaining mode. This is encoded in the rule (2).

**Definition 1.** Let \(\sigma\) be an answer substitution and let \(G\) be a goal and let \(\mathcal{P}\) be a set of D-formulas. Then the task of proving \(G\) with respect to \(\sigma, \mathcal{P}\)
– $pv(\sigma, \mathcal{P} \vdash G)$ – is defined as follows:

1. $pv(\sigma, \mathcal{P} \vdash \top)$. % success
2. $pv(\sigma, \mathcal{P} \vdash A)$ if $A' : - B \in \mathcal{P}$ and $A'\theta = A\sigma$ and $pv(\sigma\theta, \mathcal{P} \vdash B)$. % DefR (backchaining)
3. $pv(\sigma, \mathcal{P} \vdash G_0 \land G_1)$ if $pv(\sigma, \mathcal{P} \vdash G_0)$ and $pv(\sigma, \mathcal{P} \vdash G_1)$.
4. $pv(\sigma, \mathcal{P} \vdash G_0 \oplus G_1)$ if $pv(\sigma, \mathcal{P} \vdash G_i)$ where $i$ is 0 or 1.
5. $pv(\sigma, \mathcal{P} \vdash \exists x G)$ if $pv(\sigma\sigma_1, \mathcal{P} \vdash [w/x]G)$ where $w$ is a new free variable, $\sigma_1 = \{\langle w, t \rangle\}$ and $t$ is a term.

Initially, $\sigma$ is an empty substitution. Most rules are straightforward to read.

As an illustration of this approach, let us consider the following program $\mathcal{P}$.

{ \( \text{emp}(\text{tom}) : - \top \), \( \text{emp}(\text{pete}) : - \top \), \( \text{harvard}(\text{tom}) : - \top \), \( \text{mit}(\text{pete}) : - \top \) }

Now, consider a goal task $\exists x((\text{yale}(x) \oplus \text{harvard}(x)) \land \text{emp}(x))$.

The following is a proof tree of this example. Below the proof tree is represented as a list. Now, given $\sigma, \mathcal{P}$ and $G$, a proof tree of a proof formula $(\sigma, \mathcal{P} \vdash G)$ is a list of tuples of the form $(E, Ch)$ where $E$ is a proof formula and $Ch$ is a list of the form $i_1 :: \ldots :: i_n :: \text{nil}$ where each $i_k$ is the address of its $k$th child (actually the distance to $E$’s $k$th child in the proof tree).

\[
\{(w_0, \text{tom})\}, \mathcal{P} \vdash \top, \text{nil} \% \text{success} \\
\{(w_0, \text{tom})\}, \mathcal{P} \vdash \text{harvard}(w_0), 1::\text{nil} \% \text{defR} \\
\{(w_0, \text{tom})\}, \mathcal{P} \vdash \text{yale}(w_0) \oplus \text{harvard}(w_0), 1::\text{nil} \% \oplus-R \\
\{(w_0, \text{tom})\}, \mathcal{P} \vdash \top, \text{nil} \% \text{success} \\
\{(w_0, \text{tom})\}, \mathcal{P} \vdash \text{emp}(w_0), 1::\text{nil} \% \text{defR} \\
\{(w_0, \text{tom})\}, \mathcal{P} \vdash ((\text{yale}(w_0) \oplus \text{harvard}(w_0)) \land \text{emp}(w_0)), 3::1::\text{nil} \\
\emptyset, \mathcal{P} \vdash \exists x((\text{yale}(x) \oplus \text{harvard}(x)) \land \text{emp}(x)), 1::\text{nil} \% \exists-R
\]

3 The Execution Phase

Adding game semantics requires another execution phase beside the proof phase. To be precise, our new execution model – adapted from [2] – actually
solves the goal relative to the program using the proof tree built in the proof phase.

In the execution phase, it just follows the path in the proof tree, printing the output values occasionally.

**Definition 2.** Let \( i \) be an index, and let \( L \) be a proof tree. Then executing \( L_i \) (the \( i \) element in \( L \)) – written as \( ex(i, L) \) – is defined as follows:

1. \( ex(i, L) \) if \( L_i = (E, nil) \). % no child, success.
2. \( ex(i, L, F) \) if \( L_i = (\sigma, P \vdash G_0 \land G_1, m :: 1 :: nil) \) and \( ex(i - m, L, F) \) and \( ex(i - 1, L, F) \). % two children
3. \( ex(i, L, F) \) if \( L_i = (\sigma, P \vdash \exists x \ G, 1 :: nil) \) and \( L_{i-1} = (\sigma, P \vdash [w/x]G, Ch) \) and \( print(x = w\sigma) \) and \( ex(i - 1, L) \)
4. \( ex(i, L) \) if \( L_i = (E, 1 :: nil) \) and \( ex(i - 1, L) \). % otherwise

Here, \( i \) is initialized to the length of \( L \). Dealing with existentially quantified goals requires the machine to choose a right term \( t \). This process typically requires unification. It can be easily seen that adding unification to our model poses no problem at all.

### 4 Some Examples

Let us first consider the relation \( f(X, Y) \) specified by two rules:

1. if \( X < 2 \), then \( Y = 0 \).
2. if \( X \geq 2 \), then \( Y = 3 \).

The two conditions are mutually exclusive which is expressed by using the cut in traditional logic programming as shown below:

\[
f(X, 0) : \neg X < 2, !.
\]

\[
f(X, 3) : \neg X \geq 2.
\]

Using cut, we can specify mutually exclusive goals, but cuts affect the declarative meaning of the program. Our language makes it possible to formulate mutually exclusive goals through the choice-disjunctive goals as shown below:
\[
f(X, Y) : \quad (X \geq 2 \land Y = 3) \oplus \\
            (X < 2 \land Y = 0)
\]

The new program, equipped with \(\oplus\)-goals, is more readable than the original version with cuts, while preserving the same efficiency. A similar example is provided by the following “max” program that finds the larger of two numbers.

\[
\text{max}(X, Y, \text{Max}) : \quad (X \geq Y \land \text{Max} = X) \oplus \\
            (X < Y \land \text{Max} = Y)
\]

These two goals in the body of the above clause are mutually exclusive. Hence, only one of these two goals can succeed. For example, consider a goal \(\text{max}(3, 9, \text{Max})\). Solving this goal has the effect of choosing and executing the second goal \((3 < 9) \land \text{Max} = 9\), producing the result \(\text{Max} = 9\).

As another example, we consider the relation \(\text{member}(X, L)\) for establishing whether \(X\) is in the list \(L\). A typical Prolog definition of \(\text{member}(X, L)\) is shown below:

\[
\text{member}(X, [Y|L]) : \quad (Y = X) \lor \text{member}(X, L)
\]

This definition is nondeterministic in the sense that it can find any occurrence of \(X\). Our language in Section 2 makes it possible to change \(\text{member}\) to be deterministic and more efficient: only one occurrence can be found. An example of this is provided by the following program.

\[
\text{member}(X, [Y|L]) : \quad (Y = X) \oplus \text{member}(X, L)
\]

As a final example, we consider the relation \(\text{rprime}\) for establishing whether the keyboard input data \(X\) is prime or not. An example of this is provided by the following program.

\[
\text{rprime} : \quad \text{read}(X) \land \\
        (\text{prime}(X) \land \text{write}(\text{prime})) \oplus \\
        (\text{composite}(X) \land \text{write}(\text{composite}))
\]
5 Conclusion

In this paper, we have considered an extension to Prolog with choice-disjunctive goals. This extension allows goals of the form $G_0 \oplus G_1$ where $G_0, G_1$ are goals. These goals are particularly useful for replacing the cut in Prolog, making Prolog more concise and more readable.

In the near future, we plan to investigate the connection between Prolog$\oplus$ and Japaridze’s Computability Logic(CL)$[2, 3]$. CL is a new semantic platform for reinterpreting logic as a theory of tasks. Formulas in CL stand for instructions that can carry out some tasks. We plan to investigate whether our operational semantics is sound and complete with respect to the semantics of CL.

References


