Improved method of multicriteria fuzzy decision-making based on vague sets

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Abstract

An improved method is presented, which provides improved score functions to measure the degree of suitability of each of a set of alternatives, with respect to a set of criteria presented with vague values. The improved algorithm for score functions is introduced by taking into account the effect of an unknown degree (hesitancy degree) of the vague values on the degree of suitability to which each alternative satisfies the decision-maker’s requirement. The meaning of the proposed function is more transparent than that of other existing functions, which are not reasonable in some cases. The proposed function illustrates that it has stronger discrimination in comparison with previous functions. The applicability of this improved multicriteria fuzzy decision-making approach is also demonstrated by means of examples. The improved method can be used to rank the decision alternatives according to the decision criteria. The functions proposed in this paper can provide a more useful technique than previous functions, in order to efficiently help the decision-maker.

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1. Introduction

On many occasions, an engineering decision needs to be made based on available data and information that are vague, imprecise, and uncertain, by nature. The decision-making process of engineering schemes in the conceptual design phase is one of these typical occasions, which frequently calls upon some method of treating uncertain data and information.

In a conceptual design phase, designers usually present many alternatives. However, the subjective characteristics of the alternatives are generally uncertain and need to be evaluated based on the decision-maker’s insufficient knowledge and judgments. The nature of this vagueness and uncertainty is fuzzy, rather than random, especially when subjective assessments are involved in the decision-making process. Fuzzy set theory offers a possibility for handing these sorts of data and information involving the subjective characteristics of human nature in the decision-making process.


Vague sets, which Cau and Buehrer [9] presented, are a generalized form of fuzzy sets. These sets were used by Chen and Tan [10]. They presented some new techniques for handling multicriteria fuzzy decision-making problems based on vague set theory, where the characteristics of the alternatives are represented by vague sets. The proposed techniques used a score function, S, to evaluate the degree of suitability to which an alternative satisfies the decision-maker’s requirement. Recently, Hong and Choi [11] proposed an accuracy function, H, to measure the degree of accuracy in the grades of membership of each alternative, with respect to a set of criteria represented by vague values. However, in some cases, these functions do not give sufficient information about alternatives.

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In this paper, an improved score function is proposed to efficiently evaluate the degree of suitability of each alternative, with respect to a set of criteria represented by vague values. The proposed function can provide another useful way to help the decision-maker. The rest of this paper is organized as follows. In Section 2, we briefly describe vague set theory and the main results of Chen and Tan [10] and Hong and Choi [11]. In Section 3, we propose an improved score function and some measures to handle multicriteria fuzzy decision-making problems. We conclude with some final remarks of the improved method in Section 4.

2. Vague set theory and its decision-making methods

2.1. Vague sets

The vague set, which is a generalization of the concept of a fuzzy set, has been introduced by Gau and Buehrer [9]. Vague set theory is introduced as follows.

A vague set, \( A(x) \), in \( X = \{x_1, x_2, \ldots, x_n\} \), is characterized by a truth-membership function, \( t_A \), and a false-membership function, \( f_A \), for the elements \( x_i \in X \) to \( A(x) \in X \), \( (i = 1, 2, \ldots, n) \),

\[
t_A : X \to [0, 1], \quad f_A : X \to [0, 1],
\]

where the functions \( t_A(x_i) \) and \( f_A(x_i) \) are constrained by the condition \( 0 \leq t_A(x_i) + f_A(x_i) \leq 1 \). The truth-membership function, \( t_A(x_i) \), is a lower bound on the grade of membership on the evidence for \( x_i \), and \( f_A(x_i) \) is a lower bound on the negation of \( x_i \) derived from the evidence against \( x_i \). The grade of membership of \( x_i \) in the vague set, \( A \), is bounded to a subinterval \([t_A(x_i), 1 - f_A(x_i)]\) of \([0, 1]\). The vague value \([t_A(x_i), 1 - f_A(x_i)]\) indicates that the exact grade of membership \( \mu_A(x_i) \) of \( x_i \) may be unknown, but it is bounded by \( t_A(x_i) \leq \mu_A(x_i) \leq 1 - f_A(x_i) \). Fig. 1 shows a vague set in the universe of discourse, \( X \).

Let \( X \) be the universe of discourse, \( X = \{x_1, x_2, \ldots, x_n\} \), \( x_i \in X \). A vague set, \( A \), of \( X \) can be represented by

\[
A = \sum_{i=1}^{n} [t_A(x_i), 1 - f_A(x_i)]/x_i, \quad x_i \in X.
\]

For example, let \( X = \{6, 7, 8, 9, 10\} \). A vague set “LARGE” of \( X \) may be defined by

\[
\text{LARGE} = [0.1, 0.2]/6 + [0.3, 0.5]/7 + [0.6, 0.8]/8 + [0.9, 1]/9 + [1, 1]/10.
\]

Note that we will omit the argument \( x_i \) of \( t_A(x_i) \) and \( f_A(x_i) \) throughout this paper, unless they are needed for clarity.

Definition 1. The intersection of two vague sets, \( A(x) \) and \( B(x) \), is a vague set, \( C(x) \), written as \( C(x) = A(x) \land B(x) \). The truth-membership and false-membership functions are \( t_C \) and \( f_C \), respectively, where \( t_C = \min(t_A, t_B) \), and \( 1 - f_C = \min(1 - f_A, 1 - f_B) \). That is,

\[
[t_C, 1 - f_C] = [t_A, 1 - f_A] \land [t_B, 1 - f_B] = [\min(t_A, t_B), \min(1 - f_A, 1 - f_B)].
\]

Definition 2. The union of two vague sets, \( A(x) \) and \( B(x) \), is a vague set, \( C(x) \), written as \( C(x) = A(x) \lor B(x) \). The truth-membership function and false-membership functions are \( t_C \) and \( f_C \), respectively, where \( t_C = \max(t_A, t_B) \), and \( 1 - f_C = \max(1 - f_A, 1 - f_B) \). That is,

\[
[t_C, 1 - f_C] = [t_A, 1 - f_A] \lor [t_B, 1 - f_B] = [\max(t_A, t_B), \max(1 - f_A, 1 - f_B)]
\]

2.2. Existing decision-making methods

This section reviews the proposed functions and some measures of Chen and Tan [10], and Hong and Choi [11], to handle multi-criteria fuzzy decision-making problems.

Let \( A \) be a set of alternatives and let \( C \) be a set of criteria, where

\[
A = \{A_1, A_2, \ldots, A_m\}, \quad C = \{C_1, C_2, \ldots, C_n\}.
\]

Assume that the characteristics of the alternative \( A_i \) are presented by the vague set:

\[
A_i = (\{C_1, [t_{i1}, 1 - f_{i1}]\}, \{C_2, [t_{i2}, 1 - f_{i2}]\}, \ldots, \{C_n, [t_{in}, 1 - f_{in}]\}),
\]

where \( t_{ij} \) indicates the degree to which the alternative \( A_i \) satisfies criteria \( C_j \), \( f_{ij} \) indicates the degree to which the alternative \( A_i \) does not satisfy criteria \( C_j \), \( t_{ij} \in [0, 1] \), \( f_{ij} \in [0, 1] \), \( t_{ij} + f_{ij} \leq 1 \), \( 1 \leq j \leq n \), and \( 1 \leq i \leq m \). Let \( t_{ij} = 1 - f_{ij} \), where \( 1 \leq j \leq n \) and \( 1 \leq i \leq m \). In this case, \( A_i \) can be rewritten as

\[
A_i = (\{C_1, [t_{i1}, t_{i1}^*]\}, \{C_2, [t_{i2}, t_{i2}^*]\}, \ldots, \{C_n, [t_{in}, t_{in}^*]\}),
\]

where \( 1 \leq i \leq m \). In this case, the characteristics of these alternatives can be represented as shown in Table 1.

Assume that there is a decision-maker who wants to choose an alternative that satisfies the criteria \( C_j, C_k, \ldots, C_p \), or which satisfies the criteria \( C_s \). This decision-maker’s requirements are represented by the following expression:

\[
C_j \text{ AND } C_k \text{ AND } \cdots \text{ AND } C_p \text{ OR } C_s.
\]

In this case, the degrees to which the \( A_i \) satisfies or does not satisfy the decision-maker’s requirements can be measured by
the evaluation function $E$,

$$E(A_i) = ([t_{ij}, t_{ij}^*] \wedge [t_{ik}, t_{ik}^*] \wedge \cdots \wedge [t_{ip}, t_{ip}^*]) \lor [t_{is}, t_{is}^*]$$

$$= [\min(t_{ij}, t_{ik}, \ldots, t_{ip}), \min(t_{ij}^*, t_{ik}^*, \ldots, t_{ip}^*)]$$

$$\lor [t_{is}, t_{is}^*]$$

$$= [\max(\min(t_{ij}, t_{ik}, \ldots, t_{ip}), t_{is}),$$

$$\max(\min(t_{ij}^*, t_{ik}^*, \ldots, t_{ip}^*), t_{is}^*)]$$

$$= [A_{i1}, A_{i2}] = [A_{is}, 1 - f_{Ai}],$$  \hspace{1cm}\text{(1)}

where $\wedge$ and $\lor$ denote the minimum operator and the maximum operator of the vague values, respectively; $E(A_i)$ is a vague value, $1 \leq i \leq m$; $A_{i1}$ and $A_{i2}$ are as follows:

$$A_{i1} = \max(\min(t_{ij}, t_{ik}, \ldots, t_{ip}), t_{is})$$

$$A_{i2} = \max(\min(t_{ij}^*, t_{ik}^*, \ldots, t_{ip}^*), t_{is}^*).$$

Chen and Tan [10] presented a score function $S$ to measure the degree of suitability of each alternative, with respect to a set of criteria presented by vague values.

Let $x = [t_s, 1 - f_s]$ be a vague value, where $t_s \in [0, 1]$, $f_s \in [0, 1]$, $t_s + f_s \leq 1$. The score of $x$ can be evaluated by the score function, $S$, which is shown as follows:

$$S(x) = t_s - f_s,$$  \hspace{1cm}\text{(2)}

where $S(x) \in [-1, +1]$. Based on the score function, $S$, the degree of suitability to which the alternative $A_i$ satisfies the decision-maker’s requirements can be measured as follows:

$$S(E(A_i)) = A_{i1} + A_{i2}^* - 1,$$  \hspace{1cm}\text{(3)}

where $S(E(A_i)) \in [-1, +1]$. The larger the value of $S(E(A_i))$, the more the suitability to which the alternative $A_i (1 \leq i \leq m)$ satisfies the decision-maker’s requirement. Let

$$S(E(A_1)) = p_1,$$

$$S(E(A_2)) = p_2,$$

$$\vdots$$

$$S(E(A_m)) = p_m.$$  \hspace{1cm}\text{If $S(E(A_i)) = p_i$ and $p_i$ is the largest value among the values $p_1, p_2, \ldots, p_m$, then the alternative $A_i$ is the best choice. Assuming that the characteristics of the alternatives are shown in Table 1, and assuming that there is a decision-maker who wants to choose an alternative which satisfies the criteria $C_j$, $C_k$, \ldots, and $C_p$, or which satisfies the criteria $C_s$, the decision-maker’s requirements can be represented by the following expression:}

$$C_j \text{ AND } C_k \text{ AND } \cdots \text{ AND } C_p \text{ OR } C_s.$$  

Assume that the weights of the criteria $C_j$, $C_k$, \ldots, and $C_p$, presented by the decision-maker, are $w_j$, $w_k$, \ldots, and $w_p$, respectively, where $w_j \in [0, 1]$, $w_k \in [0, 1]$, \ldots, $w_p \in [0, 1]$, and $w_j + w_k + \cdots + w_p = 1$. Then, the degree of suitability to which the alternative $A_i$ satisfies the decision-maker’s requirements can be measured by the weighting function $W$,

$$W(A_i) = \max[S(t_{ij}, t_{ij}^*) \cdot w_j + S(t_{ik}, t_{ik}^*) \cdot w_k + \cdots + S(t_{ip}, t_{ip}^*) \cdot w_p, S(t_{is}, t_{is}^*)].$$  \hspace{1cm}\text{(4)}

By applying Eq. (3), Eq. (4) can be rewritten as

$$W(A_i) = \max[(t_{ij} + t_{ij}^* - 1) \cdot w_j + (t_{ik} + t_{ik}^* - 1) \cdot w_k + \cdots + (t_{ip} + t_{ip}^* - 1) \cdot w_p, (t_{is} + t_{is}^* - 1)],$$  \hspace{1cm}\text{(5)}

where $W(A_i) \in [-1, +1]$ and $1 \leq i \leq m$. Let

$$W(A_1) = p_1,$$

$$W(A_2) = p_2,$$

$$\vdots$$

$$W(A_m) = p_m.$$  \hspace{1cm}\text{If $W(A_i) = p_i$ and $p_i$ is the largest value among the values $p_1, p_2, \ldots, p_m$, then the alternative $A_i$ is the best choice.}

We now consider the following example.

**Example 2.1.** Let $A_1$ and $A_2$ be two alternatives, and let $C_1$, $C_2$, and $C_3$ be three criteria. Assuming that the characteristics of the alternatives are represented by the vague sets

$$A_1 = \{(C_1, [0, 1]), (C_2, [0, 1]), (C_3, [0, 1])\},$$

$$A_2 = \{(C_1, [0.5, 0.5]), (C_2, [0.5, 0.5]), (C_3, [0.5, 0.5])\},$$

and assuming that the decision-maker wants to choose an alternative that satisfies the criteria $C_1$ and $C_2$, or which satisfies the criteria $C_3$, then by applying Eq. (1), we can get

$$E(A_1) = ([0, 1] \lor [0, 1]) \lor [0, 1] = [0, 1],$$

$$E(A_2) = ([0.5, 0.5] \lor [0.5, 0.5]) \lor [0.5, 0.5] = [0.5, 0.5].$$

By applying Eq. (3), we can get

$$S(E(A_1)) = 0 + 1 - 1 = 0,$$

$$S(E(A_2)) = 0.5 + 0.5 - 1 = 0.$$  \hspace{1cm}\text{We do not know which is better.}
In Example 2.1, if the weights of the criteria $C_1$ and $C_2$ entered by the decision-maker are 0.7 and 0.3, respectively, then by applying Eq. (5), we can get

$$W(A_1) = \max((0 + 1 - 1) \times 0.7 + (0 + 1 - 1) \times 0.3, 0 + 1 - 1)$$
$$= \max(0, 0) = 0,$$

$$W(A_2) = \max((0 + 0.5 + 0.5 - 1) \times 0.7 + (0 + 0.5 + 0.5 - 1) \times 0.3, 0 + 0.5 + 0.5 - 1)$$
$$= \max(0, 0) = 0.$$

It is impossible to know which is better. What is the difference between alternative $A_1$ and alternative $A_2$? Here, some other measures are needed.

In the above case, Hong and Choi [11] presented an accuracy function, $H$, to evaluate the degree of accuracy in the grades of membership of each alternative, with respect to a set of criteria represented by vague values.

Let $x = [t_x, 1 - f_x]$ be a vague value, where $t_x \in [0, 1]$, $f_x \in [0, 1]$, $t_x + f_x \leq 1$. The degree of accuracy of $x$ can be evaluated by the accuracy function, $H$, as follows:

$$H(x) = t_x + f_x,$$  \hspace{1cm} (6)

where $H(x) \in [0, 1]$. The larger the value of $H(x)$, the greater the degree of accuracy of the grade of membership of the vague value.

Based on the accuracy function, the degree of accuracy in the grades of membership of the alternative $A_i$, which satisfies the decision-maker’s requirements, can be measured as follows:

$$H(E(A_i)) = t_{A_i} + 1 - t_{A_i}^m,$$  \hspace{1cm} (7)

where $H(E(A_i)) \in [0, 1]$. The larger the value of $H(E(A_i))$, the greater the degree of accuracy in the grades of membership of the alternative $A_i$.

In Example 2.1, we have obtained $E(A_1) = [0.7, 0.5, 0.5]$ and $E(A_2) = [0.5, 0.5, 0.5]$, with $S(E(A_1)) = S(E(A_2)) = 0$. Then, by applying Eq. (7), we get

$$H(E(A_1)) = 0 + 1 - 1 = 0,$$

$$H(E(A_2)) = 0.5 + 1 - 0.5 = 1.$$

Therefore, the alternative $A_2$ has a greater degree of accuracy than the alternative $A_1$ in terms of grades of membership. Alternative $A_2$ is thus the better choice.

Example 2.2. Let $A_1$ and $A_2$ be two alternatives, and let $C_1$, $C_2$, and $C_3$ be three criteria. Assume that the characteristics of the alternatives are represented by the vague sets

$$A_1 = \{[(C_1, [0.2, 0.7]), (C_2, [0.2, 0.7]), (C_3, [0.2, 0.7])],$$

$$A_2 = \{[(C_1, [0.3, 0.8]), (C_2, [0.3, 0.8]), (C_3, [0.3, 0.8])],$$

and assume that the decision-maker wants to choose an alternative that satisfies the criteria $C_1$ and $C_2$, or which satisfies the criterion $C_3$; then by applying Eq. (1), we obtain

$$E(A_1) = \{[0.2, 0.7] \land \{[0.2, 0.7] \lor \{[0.2, 0.7] = [0.2, 0.7],$$

$$E(A_2) = \{[0.3, 0.8] \land \{[0.3, 0.8] \lor \{[0.3, 0.8] = [0.3, 0.8].$$

By applying Eq. (7), we obtain

$$H(E(A_1)) = 0.2 + 1 - 0.7 = 0.5,$$

$$H(E(A_2)) = 0.3 + 1 - 0.8 = 0.5.$$

In this case, we still do not know which alternative is better.

3. Improved decision-making method

From the examples in Section 2, we can see that these functions are not reasonable in some cases. This situation can be problematic in a practical application. To overcome this problem, we propose an improved score function based on the unknown degrees of the vague values.

**Definition 3.** Let $x$ be a vague value, $x = [t_x, 1 - f_x]$, where $t_x \in [0, 1]$, $f_x \in [0, 1]$, $t_x + f_x \leq 1$. The unknown degree, or hesitancy degree, of $x$ is denoted by $m_x$, and is defined by $m_x = 1 - t_x - f_x$, and $0 \leq m_x \leq 1$.

For example, let $A$ be a vague set with truth-membership function, $t_A$, and false-membership function, $f_A$. If $[t_A, 1 - f_A] = [0.5, 0.7]$, then we can see that $t_A = 0.5$, $f_A = 0.3$, and $m_A = 0.2$. This result can be interpreted as “the vote for resolution is 5 in favor, 3 against, and 2 abstentions”.

In the following, we propose an improved score function.

Let $x = [t_x, 1 - f_x]$ be a vague value, where $t_x \in [0, 1]$, $f_x \in [0, 1]$, $m_x \in [0, 1]$, and $t_x + f_x \leq 1$. The score of $x$ can be evaluated by the modified score function, $J$, as follows:

$$J(x) = t_x - f_x + m_x + t_x (1 - m_x) - 1,$$  \hspace{1cm} (8)

where $J(x) \in [-1, +1]$, and $\mu \in [-1, +1]$. The parameter $\mu$ is introduced by taking into account the effect of the unknown degree $m_x$ on the score of $x$, and can be chosen according to actual cases. In Eq. (8), if $\mu > 0$, the value of the third term $m_x$ of $J(x)$ is positive, and it tends to increase the score of $x$ due to its addition in the first term $t_x$; if $\mu < 0$, the value of the third term $m_x$ is negative, then it tends to decrease the score of $x$ due to its addition to the second item $f_x$; if $\mu = 0$, then the improved score function $J(x)$ is the same score function as $S(x)$ proposed by Chen et al. [10]. In the renewed vote for resolution, $m_x$ can also be interpreted as: most abstentions may be in favor of $\mu > 0$, most of them for $\mu < 0$, or they still keep abstentions with $\mu = 0$.

Based on Eqs. (1) and (8), the degree of suitability to which the alternative $A_i$ satisfies the decision-maker’s requirements can be measured as follows:

$$J(E(A_i)) = t_{A_i} (1 - \mu_i) + t_{A_i}^m (1 + \mu_i) - 1,$$  \hspace{1cm} (9)

where $J(E(A_i)) \in [-1, +1]$ and $\mu_i \in [-1, +1]$. The larger the value of $J(E(A_i))$, the higher the degree of suitability to which the alternative $A_i$ satisfies the decision-maker’s requirements.

In Example 2.1, we have obtained $E(A_1) = [0.7, 0.5, 0.5]$ and $E(A_2) = [0.5, 0.5, 0.5]$, with $S(E(A_1)) = S(E(A_2)) = 0$. If $\mu_1$ and $\mu_2$ are 0.5 and 0.5, respectively, then by applying Eq. (9), we get

$$J(E(A_1)) = 0(1 - 0.5) + 1(1 + 0.5) - 1 = 0.5,$$

$$J(E(A_2)) = 0.5(1 - 0.5) + 0.5(1 + 0.5) - 1 = 0.$$
Therefore, the alternative $A_1$ is better than the alternative $A_2$.

In Example 2.2, we have obtained $E(A_1) = [0.2, 0.7]$ and $E(A_2) = [0.3, 0.8]$, with $H(E(A_1)) = H(E(A_2)) = 0.5$. Similarly, choosing $\mu_1 = \mu_2 = 0.5$ and using Eq. (9), we can get

$J(E(A_1)) = 0.2(1 - 0.5) + 0.7(1 + 0.5) - 1 = 0.15$,

$J(E(A_2)) = 0.3(1 - 0.5) + 0.8(1 + 0.5) - 1 = 0.35$.

Obviously, the alternative $A_2$ has a greater degree of suitability than the alternative $A_1$. Therefore, the alternative $A_2$ is the better choice.

This measurement method provides additional useful information to efficiently help the decision-maker. The following five theorems can be introduced according to the proposed measure function.

Theorem 1. Let $a = [a_1, a_2]$ be a vague value, and $J$ be the score function. Then $J(a) = 0$ if and only if $a_1(1 - \mu_a) + a_2(1 + \mu_a) = 1$, where $\mu_a \in [-1, +1]$.

Theorem 2. Let $a = [a_1, a_2]$ be a vague value, and $J$ be the score function. Then $J(a) = 1$ if and only if $a_1(1 - \mu_a) + a_2(1 + \mu_a) = 2$, where $\mu_a \in [-1, +1]$.

Theorem 3. Let $a = [a_1, a_2]$ and $b = [b_1, b_2]$ be two vague values, and $J$ be the score function. Then $J(a \land b) \geq J(b)$ if and only if $a_1(1 - \mu_a) + b_2(1 + \mu_b) \geq b_1(1 - \mu_b) + a_2(1 + \mu_a)$, where $\mu_a, \mu_b \in [-1, +1]$.

Theorem 4. Let $a = [a_1, a_2]$, $b = [b_1, b_2]$, and $c = [c_1, c_2]$ be three vague values, and $J$ be the score function. Then $J(a \land b \land c) \geq J(b \land c)$ if and only if $\min(a_1(1 - \mu_a), c_1(1 - \mu_c)) + \min(b_2(1 + \mu_b), c_2(1 + \mu_c)) \geq \min(b_1(1 - \mu_b), c_1(1 - \mu_c)) + \min(a_2(1 + \mu_a), c_2(1 + \mu_c))$, where $\mu_a, \mu_b, \mu_c \in [-1, +1]$.

Theorem 5. Let $a = [a_1, a_2]$, $b = [b_1, b_2]$ and $c = [c_1, c_2]$ be three vague values, and $J$ be the score function. Then $J(a \lor b \lor c) \geq J(b \lor c)$ if and only if $\max(a_1(1 - \mu_a), c_1(1 - \mu_c)) + \max(b_2(1 + \mu_b), c_2(1 + \mu_c)) \geq \max(b_1(1 - \mu_b), c_1(1 - \mu_c)) + \max(a_2(1 + \mu_a), c_2(1 + \mu_c))$, where $\mu_a, \mu_b, \mu_c \in [-1, +1]$.

The above theorems are easy to verify. The proofs are similar to those of the theorems in [10] (omitted).

In the following, we present a weighted technique for handling multi-criteria fuzzy decision-making problems. Assuming that there is a decision-maker who wants to choose an alternative that satisfies the criteria $C = \{C_1, C_2, \ldots, C_k\}$, or which satisfies the criteria $D = \{D_1, D_2, \ldots, D_l\}$, the decision-maker’s requirements can be represented by the following expression:

$$
\{C_1 \land C_2 \land \ldots \land C_k\} \lor \{D_1 \land D_2 \land \ldots \land D_l\}
$$

Assume that the weights of the criteria $C = \{C_p | p = 1, 2, \ldots, k\}$ and $D = \{D_q | q = 1, 2, \ldots, t\}$, entered by the decision-maker, are $W = \{w_p | p = 1, 2, \ldots, k\}$ and $V = \{v_q | q = 1, 2, \ldots, t\}$, respectively, where $w_p \in [0, 1]$, $\sum_{p=1}^{k} w_p = 1$, $v_q \in [0, 1]$, and $\sum_{q=1}^{t} v_q = 1$; and assume that the decision-maker enters the parameters $\mu_{CP} (p = 1, 2, \ldots, k)$ and $\mu_{DP} (q = 1, 2, \ldots, t)$, where $\mu_{CP} \in [-1, +1]$ and $\mu_{DP} \in [-1, +1]$. Then the degree to which the alternative $A_i$ satisfies the decision-maker’s requirements can be measured by the weighting function $R$.

$$
R(A_i) = \max\{J((i_{1C_1}, i_{1C_2})\ast w_1 + J((i_{2C_1}, i_{2C_2}))\ast w_2 + \cdots + J((i_{kC_1}, i_{kC_2}))\ast w_k,
J((i_{1D_1}, i_{1D_2})\ast v_1 + J((i_{2D_1}, i_{2D_2}))\ast v_2 + \cdots + J((i_{lD_1}, i_{lD_2}))\ast v_l). \}
$$

By applying Eq. (9), Eq. (10) can be rewritten into

$$
R(A_i) = \max\{\mu_{iC_1}(1 - \mu_{C_1}) + \mu_{iC_2}(1 + \mu_{C_2}) - 1\ast w_1 + (\mu_{iC_1}(1 - \mu_{C_1}) + \mu_{iC_2}(1 + \mu_{C_2}) - 1)\ast w_2 + \cdots + (\mu_{iC_1}(1 - \mu_{C_1}) + \mu_{iC_2}(1 + \mu_{C_2}) - 1)\ast w_k,
\mu_{iD_1}(1 - \mu_{D_1}) + \mu_{iD_2}(1 + \mu_{D_2}) - 1\ast v_1 + (\mu_{iD_1}(1 - \mu_{D_1}) + \mu_{iD_2}(1 + \mu_{D_2}) - 1)\ast v_2 + \cdots + (\mu_{iD_1}(1 - \mu_{D_1}) + \mu_{iD_2}(1 + \mu_{D_2}) - 1)\ast v_l). \}
$$

where $R(A_i) \in [-1, +1]$ and $1 \leq i \leq m$. Let

$R(A_1) = p_1$,

$R(A_2) = p_2$,

$: \ldots$,

$R(A_m) = p_m$.

If $R(A_i) = p_i$ is the largest value among the values $p_1, p_2, \ldots, p_m$, then the alternative $A_i$ is the best choice.

The following example is shown in [10, 11].

**Example 3.1.** Let $A_1$, $A_2$, $A_3$, $A_4$, and $A_5$ be alternatives, and let $C_1$, $C_2$, and $D_1$ be three criteria. Assuming that the characterisitcs of the alternatives are represented by the vague sets:

$A_1 = ((C_1, [0.5, 0.7]), (C_2, [0.8, 0.9]), (D_1, [0.3, 0.4]))$.

$A_2 = ((C_1, [1, 1]), (C_2, [0.7, 0.8]), (D_1, [0.1, 0.2]))$.

$A_3 = ((C_1, [0, 0]), (C_2, [0.4, 0.5]), (D_1, [0.8, 0.9]))$.

$A_4 = ((C_1, [0.8, 0.9]), (C_2, [0.1, 0.2]), (D_1, [0.5, 0.6]))$.

$A_5 = ((C_1, [0.7, 0.8]), (C_2, [0.5, 0.6]), (D_1, [0.1, 0.2]))$.

Assume that the decision-maker wants to choose an alternative that satisfies the criterion $C_1$ AND $C_2$, or which satisfies the criterion $D_1$, where the weights of the criteria $C_1$ and $C_2$ entered by the decision-maker are 0.7 and 0.3, respectively, and $\mu_{iC_1}, \mu_{iC_2}$, and $\mu_{iD_1}$ are 0.5, 0.5 and 0.5 ($i = 1, 2, 3, 4, 5$), respectively.

By applying Eq. (8), we get

$J[0.5, 0.7] = 0.3000$, $J[0.8, 0.9] = 0.7500$, $J[0.3, 0.4] = -0.2500$. 

By applying Eqs. (10) and (11), we obtain
\[ R(A_1) = \max(J[0.5, 0.7] \ast 0.7 + J[0.8, 0.9] \ast 0.3, J[0.3, 0.4]) = 0.4350. \]

By the same calculation, we get
\[ R(A_2) = 0.8650, \quad R(A_3) = 0.7500, \]
\[ R(A_4) = 0.3300, \quad R(A_5) = 0.4300. \]

Obviously, the order of quality is \( R(A_2), R(A_3), R(A_1), R(A_5), \) and \( R(A_4). \)

Therefore, we can see that the alternative \( A_2 \) is the best choice. This result is the same as in [10,11].

4. Conclusions and further research

In this paper, we have provided improved score functions to measure the degree of suitability of each alternative, with respect to a set of criteria to be represented by vague values for handling multi-criteria fuzzy decision-making problems. We also used examples to illustrate the application of the proposed function. The improved method can be used to rank the decision alternatives according to the decision criteria, and then the techniques proposed in this paper can provide more useful approaches than those of Cheng and Tan [10], and Hong and Choi [11], to efficiently help the decision-maker. But in the improved method, a new problem is how to choose a reasonable parameter \( \mu \) for the improved score function. This issue will be investigated in future work.

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References