Information Sharing in a Supply Chain with a Common Retailer

Weixin Shang
Faculty of Business, Lingnan University, shangwx@ln.edu.hk

Albert Y. Ha
School of Business and Management, The Hong Kong University of Science and Technology, imayha@ust.hk

Shilu Tong
Australian School of Business, The University of New South Wales, tongsl@ust.hk

15 December 2012

We consider the problem of sharing retailer’s demand information in a supply chain with two competing manufacturers selling substitutable products through a common retailer. We consider several scenarios with the manufacturers facing either production diseconomy or economy, and the retailer offering information contracts either concurrently or sequentially to the manufacturers. When there is no information contracting, the retailer may have an incentive to share information for free when there is production economy but not diseconomy. A manufacturer always benefits from receiving information except when production economy is large and the rival manufacturer also has the information. When there is information contracting, the retailer has an incentive to share information when either production diseconomy/economy is large or competition is more intense. Information contracting always benefits the retailer, and the benefit is larger when she offers contracts sequentially rather than concurrently to the manufacturers. The manufacturers’ preferences are reversed. The retailer’s profit may increase when production diseconomy becomes larger, and the manufacturers’ total profit may increase when there is production economy and competition becomes more intense. Neither of these is possible when there is no information sharing.

1. Introduction

With the advance of information technology, retailers routinely and efficiently acquire rich market data to obtain information about product demand. Many large retailers have started sharing such information with their suppliers to improve collaboration. One example is the 7-Exchange program offered by SymphonyIRI, a company that specializes in market intelligence services. Under this program, selected manufacturers are provided access to 7-Eleven store level point-of-sale (POS) data. Similarly, Costco offers the CRX data-sharing program through SymphonyIRI to some of its suppliers. Costco maintains a centralized POS data base for all its stores. A supplier who wants to participate in the program needs to be first approved by Costco. An approved supplier can then
get access to selected data (such as the total sales of a product category) by paying an annual subscription fee\(^1\).

Even when information sharing can improve supply chain efficiency, firms do not always have an incentive to share information with their partners because of the concern that these partners may abuse the information and use it for their own benefits, for instance, in future price negotiations. For fee-based data sharing programs, does the retailer have an incentive to share information with some or all the manufacturers? Do the manufacturers have an incentive to participate by paying retailer for the information? How does the incentive depend on the competition between the manufacturers or their production costs? Does it matter whether the retailer offers a data-sharing program concurrently or sequentially to the manufacturers? We hope to shed light on these questions.

We consider the problem of sharing retailer’s demand information in a supply chain with two competing manufacturers selling substitutable products through a common retailer. We focus on the impact of non-linear production cost, with the manufacturers facing either production diseconomy (marginal cost is increasing in production quantity) or production economy (marginal cost is decreasing in production quantity)\(^2\). Before observing a private demand signal, the retailer contracts with the manufacturers on sharing this information. We formulate a multistage game to study the firms’ information sharing, wholesale price and retail price decisions. We consider four versions of the multistage game where the manufacturers face production diseconomy or economy, and information contracting decisions are made concurrently or sequentially. With concurrent information contracting, the retailer makes concurrent and identical offers of selling information for a fixed payment to the manufacturers. With sequential information contracting, the retailer makes sequential offers of selling information for a fixed payment to the first manufacturer, then to the second manufacturer after the first manufacturer’s decision of whether accepting the offer becomes public information.

Our study of information contracting is motivated by the data sharing programs offered by retailers to manufacturers, where a manufacturer typically has to pay a fixed fee for participation\(^3\). Sequential contracting corresponds to the case when a retailer can adopt an approval process to offer the program sequentially to different manufacturers (as in the CRX example) and whether a manufacturer participates in the program is publicly known (e.g., through industry channels or

---

\(^1\) For the details of the data sharing programs offered through SymphonyIRI, see http://www.symphonyiri.com.

\(^2\) Our notions of production diseconomy and production economy are different from diseconomies of scale and economies of scale, which consider how the average cost changes as production volume increases, in the economics literature.

\(^3\) We assume that a retailer offers the program either on her own or through a third-party service provider who takes an insignificant portion of the fixed fee for the service.
public media). Concurrent contracting corresponds to the case when a retailer cannot offer the program sequentially or whether a manufacturer participates in the program is not publicly known.

We solve the four multistage games to obtain the following insights.

1. When there is production diseconomy, the retailer has an incentive to share information if there is information contracting (i.e., side payment) and either production diseconomy is large or competition is intense. This is because (a) an informed manufacturer adjusts his wholesale price to respond positively to the demand signal, which reduces the variability of the production quantity and lowers the production cost, and the cost saving becomes more significant when production diseconomy is larger; (b) information sharing makes the damaging double marginalization effect of linear wholesale price more severe, but more intense competition between the manufacturers suppresses the double marginalization effect. When there is production economy, regardless of having information contracting or not, the retailer has an incentive to share information if either production economy is large or competition is intense. In either case, an informed manufacturer adjusts his wholesale price to respond negatively to the demand signal. Consequently, information sharing not only reduces production cost by increasing the variability of the production quantity but also dampens double marginalization.

2. When there is production diseconomy, regardless of whether information contracting is done concurrently or sequentially, it is possible to have no information sharing or full information sharing (i.e., both manufacturers receive information) in equilibrium. However, partial information sharing (i.e., one manufacturer receives information) is possible only under sequential but not concurrent information contracting. When there is production economy, it is possible to have any of the three information sharing outcomes. Partial information sharing is more likely to occur when information contracting is done sequentially rather than concurrently.

3. Without side payment, a manufacturer always benefits from receiving information. The only exception is when there is production economy, a manufacturer is hurt by receiving information if the rival manufacturer already has the information and production economy is large. In this case, information sharing reduces the production quantity variability faced by the manufacturer and increases his cost.

4. Regardless of having production diseconomy or economy, the retailer prefers to sell information sequentially rather than concurrently to the manufacturers while the manufacturers’ preferences are reversed. This is because sequential information contracting allows the retailer to more easily induce partial information sharing and in this case, a manufacturer is willing to pay more for the information. Our result is consistent with the practice of sequentially approving manufacturers in Costco’s CRX program.
5. With information contracting, the retailer’s profit may increase when production diseconomy becomes larger, and the manufacturers’ total profit may increase when there is production economy and competition becomes more intense. Neither of these is possible when there is no information sharing.

This paper is most related to the literature on incentive for vertical information sharing under different supply chain structures. Most of the papers in the literature consider information sharing in a supply chain with one manufacturer selling to several competing retailers. The focus of this stream of work is on the incentive for the retailers to share demand information with the manufacturer (Li 2002, Li and Zhang 2002, Zhang 2002) and how that incentive depends on confidentiality (Li and Zhang 2008) and the product wholesale contracts (Shin and Tunca 2010, Tang and Girotra 2010). Several papers consider information sharing in a one-to-one supply chain and investigate issues such as signaling unverifiable information (Cachon and Lariviere 2001), dual distribution channels (Yue and Liu 2006) and bilateral information sharing (Mishra et al. 2009). Ha and Tong (2008) and Ha et al. (2011) study information sharing in two competing supply chains. Zhang (2006) examines the issue of sharing inventory information among suppliers who produce different components for a manufacturer in a two-echelon assembly system. Özer et al. (2011) consider the role of trust in information sharing between a supplier and a manufacturer. Zhao et al. (2011) study the issue of information sharing in outsourcing. Kurtuluş et. al. (2012) examine the conditions under which a supplier and a retailer have incentives to combine their information to form a shared demand forecast.

In order to focus on the issue of how information sharing influences the strategic interactions between firms, many models (e.g., Li 2002, Li and Zhang 2008, Gal-Or et al. 2008, Ha et al. 2011) in the above literature ignore the impact of information sharing on operational improvement such as inventory cost saving. Such an impact is not significant when, for instance, production decision is made after demand realizes and therefore there is no cost of mismatch between production and demand. Without considering the effect of operational improvement, it is well known that the value of information sharing is negative for a single supply chain with linear wholesale price and linear production cost (Li and Zhang 2002). When there is production diseconomy, Ha et al. (2011) show that the value of information sharing can be positive in a setting of two competing supply chains and they derive various conditions under which information is shared in a supply chain. To the best of our knowledge, our paper is the first in the literature that considers both production diseconomy and economy. It is also the first that examines information sharing in a supply chain with competing manufacturers selling through a common retailer. Our results extend the literature in several ways. First, by comparing the production diseconomy and economy models, we provide a much more complete understanding of how non-linear production cost impacts the incentive for information
sharing. We show that the information sharing outcomes under these two models are very different, and they can be explained by responsive wholesale pricing and production quantity variabilities in the supply chain. Second, by considering a supply chain with a common retailer, our results lead to interesting contrasts with the existing ones. Ha et al. (2011) show that for two competing supply chains, more intense competition at the retailer level induces less information sharing and the retailers cannot benefit from a larger production diseconomy. We show that for a supply chain with a common retailer, more intense competition at the manufacturer level induces more information sharing and the retailer may benefit from a larger production diseconomy. Third, the existing literature does not emphasize the issue of information contracting because it is relatively straightforward when the contract is between a manufacturer and a retailer. In our model with a common retailer offering information contracts to two competing manufacturers, we show that the information contracting decisions depend strongly on the contracting sequence (concurrent versus sequential offers).

In economics, there is a related literature on information sharing in an oligopoly. See, for example, Vives (1984), Gal-Or (1985) and Li (1985). This body of research focuses on the incentive for a firm to share information with its competitors in an oligopoly and it does not consider the interactions between firms in a vertical chain.

Our paper is also related to the literature on supply chain coordination and competition when there are multiple manufacturers selling through a common retailer. Choi (1991) and Lee and Staelin (1997) consider channel competition under linear wholesale price contracts. More recently, Cachon and Kök (2010) study the impact of other contract forms (quantity discount and two-part pricing) on channel competition and coordination. See Cachon and Kök (2010) for a detailed discussion of this literature. None of the papers in this literature considers the issue of information sharing, which is the focus of this paper.

2. The Model

Consider a supply chain with two identical manufacturers (indexed by 1 or 2) selling substitutable products through a common retailer. The demand function of product $i$ is given by:

$$q_i = a + \theta - (1 + \phi)p_i + \phi p_j,$$

where $p_i$ is the retail price of product $i$, $\phi > 0$ is a parameter for competition intensity (larger $\phi$ means more intense competition), and the random variable $\theta$, with zero mean and variance $\sigma^2$, represents demand uncertainty. Linear demand functions have been extensively used in the economics (e.g., Vives 1999) and operations management literatures (e.g., Li and Zhang 2008, Shin and Tunca 2010).
The retailer has access to a demand signal $Y$, which is an unbiased estimator of $\theta$ (i.e., $E[Y] = E[\theta] = 0$). We assume a linear-expectation information structure: the expectation of $\theta$ conditional on signal $Y$ is a linear function of the signal. This information structure is commonly used in the information sharing literature (e.g., Li 2002, Gal-Or et al. 2008 and Taylor and Xiao 2010) and includes well-known prior-posterior conjugate pairs like normal-normal, beta-binomial, and gamma-Poisson. Define the signal accuracy as $t = 1/E[Var[Y|\theta]]$. It can be shown (Ericson 1969) that $E[\theta|Y]$ is a weighted average of the prior mean $E[\theta]$ and the signal $Y$:

$$E[\theta|Y] = \frac{1}{1+t\sigma^2}E[\theta] + \frac{t\sigma^2}{1+t\sigma^2}Y = \beta(t, \sigma)Y,$$

where $E[\theta] = 0$ as assumed earlier and $\beta(t, \sigma) = t\sigma^2/(1+t\sigma^2)$ is the weight for the signal $Y$. Note that $\beta(t, \sigma)$ is larger when the signal becomes more accurate (larger $t$). The information structure is common knowledge. For more details of the linear-expectation information structure, refer to Vives (1999, §2.7.2).

The retailer has a constant marginal retailing cost, which is normalized to zero. We consider two cases of non-linear production cost. When there is production diseconomy, the cost incurred by a manufacturer for producing $q$ units of his product is given by:

$$bq + c_d q^2.$$

When there is production economy, the cost is given by:

$$\begin{cases} 
 bq - c_e q^2 & \text{if } q < \bar{q}, \\
 b\bar{q} - c_e \bar{q}^2 & \text{otherwise},
\end{cases}$$

where $\bar{q} = b/(2c_e)$. Here $b, c_d$ and $c_e$ are all positive numbers. The use of quadratic functions to model non-linear production costs is quite common in the literature. See, for example, Anand and Mendelson (1997) and Eliashberg and Steinberg (1991).

We consider four versions of a multistage game where the manufacturers face either production diseconomy or economy, and the retailer offers information contracts to the manufacturers either concurrently or sequentially. The sequence of events for the multistage game is given below.

1. Before the retailer observes any demand signal, the retailer and the manufacturers contract on information sharing. With concurrent contracting, the retailer makes concurrent and identical offers to the manufacturers by charging each a side payment $T$ for the information. With sequential contracting, the retailer makes sequential offers by charging the first manufacturer a fixed payment $T_f$ for the information, then charging the second manufacturer a payment $T_s$.  

---

$^4$This is without loss of generalisation because a biased estimator can be transformed into an unbiased one.

$^5$Our results remain valid if we choose $\bar{q}$ to be smaller than $b/(2c_e)$, i.e., the marginal cost decreases to a positive level and remains constant as $q$ increases.
2. The retailer observes a demand signal $Y$ and truthfully discloses it to a manufacturer if an information sharing contract has been signed in the previous stage. A manufacturer who receives the demand signal is said to be *informed*. Otherwise he is said to be *uninformed*. Let $X_i$ be the information status of manufacturer $i$, where $X_i = I$ if manufacturer $i$ is informed (i.e., retailer shares information with him) and $X_i = U$ if he is uninformed (i.e., retailer does not share information with him). Let $n$ be the number of informed manufacturers, where $n = 0, 1, or 2$.

3. Each manufacturer $i$ determines his wholesale price $w_i$, and then the retailer determines retail prices $p_1$ and $p_2$.

4. Market demands $q_1$ and $q_2$ realize, each manufacturer $i$ supplies $q_i$ to the retailer and finally firms receive their payoffs.

In Section 4, we provide more details about how the firms make information sharing decisions under concurrent and sequential contracting. In our models, information sharing contracts are long-term while wholesale contracts are short-term decisions. This is because if firms agree to share information, they have to set up systems for information transmission. After that, they engage in multiple wholesale contract interactions. Therefore the manufacturers and the retailer do not negotiate information sharing contracts and wholesale contracts simultaneously. The demand signal can be interpreted as the information about a product’s potential demand that can be derived from either past sales data of the product category or past consumer demographic data collected by the retailer. The firms use such information, if available, to determine the wholesale and retail prices of the product for the next wholesale contract period. We assume that the manufacturers produce to meet the demand that realizes over the wholesale contract period and there is no demand uncertainty at the time when production occurs. Here $q_i$ is the total production quantity (the same as the realized demand or sales quantity) of product $i$ over the entire wholesale contract period. A manufacturer faces production diseconomy when the marginal cost is increasing in the output volume. This could be the case when the capacity is tight and the production technology is not scalable (Anand and Mendelson 1997). When capacity decision is more long-term than wholesale price decision, a manufacturer does not adjust capacity every time before a wholesale contract is signed. If the capacity is tight, a higher volume means that the manufacturer has to incur a higher marginal cost due to overtime or using production resources that are less efficient. This could also be the case when the manufacturer has to add more expensive suppliers to his supply base or to incur a higher cost in coordinating with more suppliers as output volume increases. See Froeb and McCann (2009) for more discussion about the drivers of marginal cost that is increasing in production volume. A manufacturer faces production economy when the marginal cost is decreasing.

---

6 We can show that, when $\sigma$ and either $c_d$ or $c_e$ are small relative to $a$, it is optimal for manufacturer $i$, with a probability very close to one, to fully meet the demand.
in the output volume. This could happen when there is learning effect in production or a more efficient technology can be used with a larger production volume\(^7\).

For each model, we solve it backward by first solving for the equilibrium wholesale and retail prices, and based on these, computing the ex-ante profits of the firms under different information sharing statuses of the manufacturers. The ex-ante profits are then used to solve for the equilibrium information contracting decisions in the first stage.

3. Pricing Decisions and Firms’ Ex-Ante Profits

In this section, for any given set of the manufacturers’ information statuses \((X_1, X_2)\), we solve for the equilibrium wholesale and retail pricing decisions and then derive the firms’ ex-ante profits before the side payment, if any, due to information contracting. Because the side payment can be regarded as sunk cost, it won’t have any impact on the retailer’s and manufacturers’ pricing decisions. Our analysis here applies to both the production diseconomy and economy models.

3.1. Wholesale and Retail Price Decisions

Knowing the wholesale prices \(w_1\) and \(w_2\) as well as the demand signal \(Y\), the retailer chooses \(p_1\) and \(p_2\) to maximize her expected profit

\[
(p_1 - w_1) (a + E[\theta|Y] - (1 + \phi)p_1 + \phi p_2) + (p_2 - w_2) (a + E[\theta|Y] - (1 + \phi)p_2 + \phi p_1),
\]

where \(E[\theta|Y] = \beta(t, \sigma)Y\). The retailer’s best-response retail price is

\[
\hat{p}_i(w_i, w_j) = \frac{1}{2} (a + \beta(t, \sigma)Y + w_i),
\]

and the resulting demand is

\[
g_i(w_i, w_j) = \frac{1}{2} (a + \beta(t, \sigma)Y - (1 + \phi)w_i + \phi w_j) + (\theta - \beta(t, \sigma)Y).
\]

Next, we show how the manufacturers simultaneously determine their wholesale prices in anticipation of the retailer’s response. We first derive manufacturer \(i\)’s best-response wholesale price to manufacturer \(j\)’s wholesale price \(w_j\), which is a function of \(Y\) if manufacturer \(j\) is informed and does not depend on \(Y\) otherwise. For convenience, we will simply write \(w_j\) in the subsequent analysis, regardless of whether manufacturer \(j\) is informed or not.

To simplify our presentation, let \(c = c_d\) for the production diseconomy model and \(c = -c_e\) for the production economy model. For the production economy model, we assume that \(b\) (\(b < a\)) is large relative to \(c_e\) and \(\sigma\) is small enough so that the probability for the production quantity to be larger

\(^7\)For retailers like 7-Eleven and Costco mentioned in Section 1, some examples of product categories that might exhibit production diseconomy are lunch box, jewellery and wine. Examples of product categories with production economy are furniture, soda, pasta and canned soup.
than \( q = b/(2c_e) \) is negligible\(^8\). We also assume that \( c_e < 2/(1 + \phi) \) because when this is not true, it is optimal for the manufacturers not to produce and the problem is not interesting.

When manufacturer \( i \) is informed, his expected profit is given by

\[
(w_i - b)E[q_i(w_i, w_j)|Y] - cE[q_i^2(w_i, w_j)|Y].
\]

When manufacturer \( i \) is uninformed, his expected profit becomes

\[
(w_i - b)E[q_i(w_i, w_j)] - cE[q_i^2(w_i, w_j)].
\]

It can be shown that the manufacturer’s expected profit function is concave\(^9\). An informed manufacturer \( i \) maximizes his expected profit by choosing the best-response wholesale price

\[
\hat{w}_i(w_j) = \frac{(1 + (1 + \phi)c)(a + \beta(t, \sigma)Y) + (1 + \phi)b + \phi(1 + (1 + \phi)c)w_j}{(1 + \phi)(2 + (1 + \phi)c)}.
\]

An uninformed manufacturer \( i \) maximizes his expected profit by choosing the best-response wholesale price

\[
\hat{w}_i(w_j) = \frac{(1 + (1 + \phi)c) a + (1 + \phi)b + \phi(1 + (1 + \phi)c) E[w_j]}{(1 + \phi)(2 + (1 + \phi)c)}.
\]

Here we use Bayesian Nash equilibrium as the solution concept. Manufacturer \( i \) conjectures about manufacturer \( j \)'s wholesale price \( w_j \) (which is a function of \( Y \) if manufacturer \( j \) is informed and does not depend on \( Y \) otherwise). When manufacturer \( i \) is informed, he knows exactly the demand signal \( Y \) observed by an informed manufacturer \( j \) and therefore his best-response wholesale price is a function of \( w_j \). When manufacturer \( i \) is uninformed, he does not know the demand signal \( Y \) observed by an informed manufacturer \( j \) and therefore his best-response wholesale price is a function of \( E[w_j] \), where the expectation is taken over the distribution of \( Y \). When manufacturer \( j \) is uninformed, \( w_j \) does not depend on \( Y \) and \( E[w_j] \) is interpreted simply as \( w_j \). An equilibrium \((w_1^*, w_2^*, p_1^*, p_2^*)\) can be found by solving \( w_i^* = \hat{w}_i(w_j^*) \) and \( p_i^* = \hat{p}_i(w_i^*, w_j^*) \). Let \( \bar{w} \) and \( \bar{p} \) be the deterministic solutions when \( \sigma = 0 \), where \( \bar{w} = [(1 + (1 + \phi)c)a + (1 + \phi)b]/(2 + \phi + (1 + \phi)c) \) and \( \bar{p} = [3 + \phi + 2(1 + \phi)c]a + (1 + \phi)b]/[2(2 + \phi + (1 + \phi)c)] \).

**Lemma 1.** Given \( n \), the number of informed manufacturers, there exists a unique equilibrium such that

\[
\begin{align*}
  w_i^* &= \begin{cases} 
  \bar{w} + \alpha_{iW}(n)Y & \text{if } n = 0 \text{ or } 2, \\
  \bar{w} + \alpha_{iW}^0_Y(1)Y & \text{if } n = 1,
  \end{cases} \\
  p_i^* &= \begin{cases} 
  \bar{p} + \alpha_{iP}(n)Y & \text{if } n = 0 \text{ or } 2, \\
  \bar{p} + \alpha_{iP}^0_Y(1)Y & \text{if } n = 1,
  \end{cases}
\end{align*}
\]

\(^8\)As we can see in the subsequent analysis, all the main results such as the equilibrium solution do not depend on \( b \) and \( \sigma \).

\(^9\)For the production economy model, the expected profit is concave when \( 0 \le c_e < 2/(1 + \phi) \).
where

\[ \alpha_{w}(0) = \alpha_{w}(1) = 0, \alpha_{w}(2) = \frac{1 + (1 + \phi)c}{(1 + \phi)(2 + (1 + \phi)c)} \beta(t,\sigma), \]
\[ \alpha_{p}(0) = \frac{\beta(t,\sigma)}{2}, \alpha_{p}(1) = \frac{3 + 2\phi + (1 + \phi)(2 + \phi)c}{2(1 + \phi)(2 + (1 + \phi)c)} \beta(t,\sigma), \]
\[ \alpha_{p}(2) = \frac{3 + \phi + 2(1 + \phi)c}{2(2 + \phi + (1 + \phi)c)} \beta(t,\sigma). \]

The above lemma shows that, in equilibrium, the retailer and an informed manufacturer adjust, respectively, the retail prices and the wholesale price in response to the demand signal by following a linear strategy. An uninformed manufacturer obviously does not respond to the demand signal.

For the production diseconomy model \((c = c_d)\), a larger variability in the production quantity increases cost. When a manufacturer’s wholesale price responds positively to the demand signal, it induces the retailer’s price to respond more strongly to the demand signal, the production quantity becomes less variable and therefore the manufacturer’s cost becomes lower. It also makes the double marginalization effect stronger which increases the manufacturer’s revenue. An informed manufacturer and the retailer always respond positively to the demand signal.

For the production economy model \((c = -c_e)\), a larger variability in the production quantity decreases cost. When a manufacturer’s wholesale price responds positively to the demand signal, it increases his revenue due to the stronger double marginalization effect but also increases his production cost because production quantity becomes less variable. The reverse is true when the wholesale price responds negatively to the demand signal. Because the production quantity variability effect is more significant when production economy is larger, an informed manufacturer’s wholesale price responds positively to the demand signal when production economy is small \((c_e < 1/(1 + \phi))\) and negatively otherwise.

3.2. Firms’ Ex-Ante Profits

Based on the equilibrium pricing decisions, for a given \(n\), we take expectations with respect to the demand signal \(Y\) to obtain the firms’ ex-ante profits before the demand signal is observed. Let manufacturer \(i\)’s profit be denoted by \(\pi_{M}(n)\) when \(n = 0\) or \(2\), and by \(\pi_{M}^{X}(1)\) when \(n = 1\). Let the retailer’s profit be denoted by \(\pi_{R}(n)\).

\[ \pi_{M}(0) = \pi_{M} - \frac{c\beta(t,\sigma)\sigma^2}{4} - c[1 - \beta(t,\sigma)]\sigma^2, \]
\[ \pi_{M}^{U}(1) = \pi_{M} - \frac{c}{4} \left[ \frac{2 + 3\phi + (1 + \phi)(1 + 2\phi)c}{(1 + \phi)(2 + (1 + \phi)c)} \right]^2 \beta(t,\sigma)\sigma^2 - c[1 - \beta(t,\sigma)]\sigma^2, \]
\[ \pi_{M}^{I}(1) = \pi_{M} + \frac{1}{4(1 + \phi)(2 + (1 + \phi)c)} \beta(t,\sigma)\sigma^2 - c[1 - \beta(t,\sigma)]\sigma^2, \]
\[ \pi_{M}(2) = \pi_{M} + \frac{1 + \phi}{4(2 + \phi + (1 + \phi)c)} \beta(t,\sigma)\sigma^2 - c[1 - \beta(t,\sigma)]\sigma^2, \]
\[ \pi_R(0) = \pi_R + \frac{\beta(t, \sigma)\sigma^2}{2}, \]
\[ \pi_R(1) = \pi_R + \left[ \frac{1 + 2\phi}{1 + \phi} + \frac{1}{(1 + \phi)(2 + (1 + \phi)c)^2} \right] \frac{\beta(t, \sigma)\sigma^2}{4}, \]
\[ \pi_R(2) = \pi_R + \frac{(1 + \phi)^2}{2(2 + \phi + (1 + \phi)c)^2} \beta(t, \sigma)\sigma^2, \]
where \( \pi_M = (1 + \phi)(2 + (1 + \phi)c)(a - b)^2/[4(2 + \phi + (1 + \phi)c)^2] \) and \( \pi_R = (1 + \phi)^2(a - b)^2/[2(2 + \phi + (1 + \phi)c)^2] \) are the profits in the deterministic model. Note that \( \beta(t, \sigma) = 0 \) when \( t = 0 \) (demand signal has no information) and \( \beta(t, \sigma) \) approaches to one when \( t \) approaches infinity (demand signal becomes perfect). We may interpret \( \beta(t, \sigma) \) as the fraction of demand uncertainty predictable by the demand signal. In the profit function of a firm, the second term accounts for the effect of predictable demand uncertainty. For a manufacturer, the third term accounts for the effect of residual demand uncertainty that cannot be predicted by the demand signal. Consider the case of production diseconomy. An informed firm benefits from predictable demand uncertainty because he/she can adjust either the wholesale price or the retailer prices in response to the demand signal. An uninformed firm is hurt by the predictable demand uncertainty because of the resulting larger production quantity variability. After the retailer has determined the retail prices, the residual demand uncertainty creates additional production quantity variability which hurts a manufacturer. Now consider the case of production economy. A firm, whether informed or not, benefits from both the predictable and residual demand uncertainties because a larger production quantity variability is beneficial in this case.

### 4. The Production Diseconomy Model

#### 4.1. Effect of Information Sharing

**Proposition 1.** If information is shared between a manufacturer and the retailer,

(a) it benefits the manufacturer but hurts the retailer;

(b) it benefits the rival manufacturer when he is informed but hurts him otherwise.

When the retailer discloses information to manufacturer \( i \), wholesale price \( w_i \) responds positively to the demand signal. Consequently, (1) double marginalization of \( w_i \) becomes stronger\(^{10}\); (2) because \( w_i \) and \( w_j \) are strategic complements, double marginalization of \( w_j \) becomes stronger if manufacturer \( j \) is informed and remains the same if he is uninformed\(^{11}\); (3) the production

\(^{10}\) From Lemma 1, under different information sharing statuses, the average wholesale price is the same and equals \( \bar{w} \), the wholesale price of the deterministic model. When \( w_i \) responds positively to the demand signal, it becomes higher when the signal is high and lower when it is low. On average, this allows manufacturer \( i \) to capture a larger share of the total revenue but also has a more damaging effect in distorting retail decisions. We say that the double marginalization of \( w_i \) becomes stronger (on average).

\(^{11}\) From Lemma 1, if manufacturer \( j \) is uninformed, he charges the same wholesale price \( \bar{w} \) regardless of whether manufacturer \( i \) is informed or not. If manufacturer \( j \) is informed, when manufacturer \( i \) receives information and responds positively to the demand signal, because \( w_i \) and \( w_j \) are strategic complements, it induces manufacturer \( j \) to respond more strongly to the demand signal, resulting in a stronger double marginalization effect of \( w_j \).
quantity variability faced by manufacturer $i$ becomes smaller while that faced by manufacturer $j$ becomes larger\(^{12}\). Because stronger double marginalization hurts the retailer but benefits both manufacturers, and smaller production quantity variability benefits the manufacturer, Proposition 1 follows\(^{13}\).

### 4.2. Equilibrium Information Sharing Decisions

With concurrent information contracting, the retailer makes concurrent and identical offers to the manufacturers by charging each a side payment $T$ for the information. The manufacturers then simultaneously decide whether to accept the offers. We solve the game backward by solving the manufacturer game for a given payment $T$. Denote a manufacturer’s decision by $X_i = I$ if he agrees to pay the retailer for sharing information and $X_i = U$ otherwise. Based on the ex-ante profits in Section 3 and the side payment $T$, we construct the payoff matrix of the manufacturer game (see Table 1 in the Appendix) and then solve for the equilibrium of the game, $(X_1^*, X_2^*)$, as a function of $T$. When there are multiple equilibria, we can show that a Pareto-optimal equilibrium always exists and we assume that it is the outcome of the manufacturer game. We can then compute the retailer’s profit in the equilibrium for a given $T$, and hence solve the retailer’s problem of finding $T$ that maximizes her profit.

With sequential information contracting, the retailer makes sequential offers to the manufacturers for selling information. First, the retailer randomly picks one of the two manufacturers and offers to charge him a payment $T_f$ for the information. Next, the first manufacturer decides whether to accept the offer. Let $X_f = I$ if the first manufacturer agrees to accept the offer and $X_f = U$ otherwise. Then the retailer makes an offer to the second manufacturer, who has observed $X_f$, by charging another payment $T_s$ for the information. Finally, the second manufacturer decides whether to accept the offer. Here we assume that the retailer cannot credibly commit on charging the second manufacturer the same payment $T_f$, which is reasonable because the retailer gives these offers at different times. Such an assumption is not critical because it can be shown that when it is optimal for the retailer to sell information to both manufacturers, she charges them the same payments in equilibrium. We solve the model backward by considering the optimal decision of a firm in each stage, given the decisions made in the previous stages and in anticipation of other firms’ best responses in the subsequent stages. The firms’ payoffs are determined based on the ex-ante profits given in Section 3 and the side payments $T_f$ and $T_s$.

Let $n^*_Z$ be the optimal number of informed manufacturers that maximizes the retailer’s profit, where $Z = C$ if information contracts are offered concurrently and $Z = S$ if they are offered sequentially.

\(^{12}\)For the effect of information sharing on the production quantity variability, see Lemma 2 in the Appendix.

\(^{13}\)For an informed manufacturer $j$, he benefits because the positive double marginalization effect dominates the negative production quantity variability effect.
**Proposition 2.**

(a) There exists \( c_d^C \) such that \( n_d^C = 0 \) if \( 0 < c_d < c_d^C \) and \( n_d^C = 2 \) if \( c_d^C \leq c_d \).

(b) There exists \( c_d^{S1} \) and \( c_d^{S2} \) such that \( n_d^S = 0 \) if \( 0 < c_d < c_d^{S1} \), \( n_d^S = 1 \) if \( c_d^{S1} \leq c_d < c_d^{S2} \), and \( n_d^S = 2 \) if \( c_d^{S2} \leq c_d \).

(c) \( c_d^{S1} < c_d^C < c_d^{S2} \).

(d) \( c_d^C, c_d^{S1} \) and \( c_d^{S2} \) are decreasing in \( \phi \).

We illustrate the equilibrium information sharing decisions in Figure 1. Note that because the threshold values \( c_d^{S1}, c_d^C \) and \( c_d^{S2} \) depend only on \( \phi \), Figure 1 fully characterizes the information sharing regions.

![Equilibrium Information Sharing Decisions with Production Diseconomy](image)

**Proposition 3.**

(a) Without information contracting, the retailer does not share any information.

(b) With information contracting, (i) a larger production diseconomy or more intense competition induces more information sharing; (ii) the retailer’s profit is higher under sequential information contracting while the total profit of the manufacturers is higher under concurrent information contracting.

Ha et al. (2011) show that when a retailer shares private demand information with a manufacturer who faces production diseconomy, it makes the double marginalization effect of linear wholesale price more severe (which distorts retail price decision and lowers supply chain’s revenue) and reduces production quantity variability (which lowers supply chain’s cost). They further show that
a larger production diseconomy together with less intense retail competition between two supply chains induces more information sharing\textsuperscript{14}. In our setting, a larger production diseconomy still induces more information sharing because the cost saving due to a smaller production quantity variability becomes more significant. However, unlike the case of two supply chains engaging in retail competition, more intense competition at the manufacturer level induces more information sharing. This is because more intense wholesale price competition mitigates the damaging double marginalization effect and makes information sharing more valuable.

It can be shown that the equilibrium information sharing decisions under concurrent information contracting maximize the total profit of the three firms. From the system’s perspective, when compared with partial information sharing, full information sharing induces a stronger double marginalization effect and a smaller total production quantity variability. It turns out that the second effect dominates the first one and therefore partial information sharing is never optimal to the system. However, it may occur under sequential information contracting when production diseconomy is neither too small nor too large ($c_d^{S1} \leq c_d < c_d^{S2}$). In this case, the value of sharing information to the retailer and the second manufacturer is negative, after the retailer has agreed to share information with the first manufacturer. Because neither manufacturer wants to be the only uninformed firm, the retailer can charge a high payment to extract profit from the manufacturers at the expense of a less efficient outcome. Therefore the retailer prefers sequential information contracting while the manufacturers’ preferences are reversed.

Let $\Pi^{SR}_R$ be the retailer’s ex-ante profit after accounting for the side payment under sequential information contracting.

**Proposition 4.**

(a) $\pi_R(0)$ is decreasing in $c_d$.

(b) There exists $\delta_d^{S1} > 0$ such that $\Pi^{SR}_R(c_d^{S1} + \delta_d^{S1}) - \Pi^{SR}_R(c_d^{S1} - \delta_d^{S1}) > 0$.

(c) There exists $\delta_d^{S2} > 0$ such that $\Pi^{SR}_R(c_d^{S2} + \delta_d^{S2}) - \Pi^{SR}_R(c_d^{S2} - \delta_d^{S2}) > 0$.

Without information contracting, a larger production diseconomy increases the production cost and softens the manufacturers’ competition. Because both effects hurt the retailer, she is worse off. With sequential information contracting, however, there exists a neighborhood around $c_d^{S1}$ or $c_d^{S2}$ such that when an increase in production diseconomy induces a change in the information sharing equilibrium, the retailer is strictly better off. We can show that $\Pi^{SR}_R$ has positive jumps at $c_d^{S1}$ and $c_d^{S2}$, and is continuous elsewhere. Similarly, the total ex-ante profit of the manufacturers has a negative jump at $c_d^{S1}$. Besides these cases, the ex-ante profit functions are all continuous in $c_d$\textsuperscript{15}.

\textsuperscript{14}This is because when information is shared in a supply chain, it triggers a negative competitive reaction from the rival chain but such a reaction is weaker when competition is less intense.

\textsuperscript{15}Similarly, we can show that there are cases where the retailer is strictly better off or the manufacturers are strictly worse off when an increase in $\phi$ induces more information sharing. In our model, a change in $\sigma$ or $t$ does not affect the information sharing equilibrium outcome $n$, though it affects the firms’ profits.
We have performed an extensive numerical study to investigate, with information contracting, how the retailer’s profit depends on $c_d$ when the information sharing equilibrium does not change. Our results show that the retailer’s profit may increase in $c_d$ for $n_d^C = 2$, or $n_d^S = 1$ or 2, when production diseconomy is large, competition is less intense, demand uncertainty is large and the signal is accurate.

Proposition 4 and the numerical results together show that when there is information contracting, production diseconomy has an additional effect on the retailer. A larger production diseconomy increases the value of information sharing, which benefits the retailer because she can then sell information to more manufacturers or charge a higher payment for the information. Note that for the case of two competing supply chains studied by Ha et al. (2011), a retailer cannot benefit from a larger production diseconomy when the manufacturers are leaders in information contracting and they do not compete in buying information from the same retailer.

5. Production Economy Model

5.1. Effect of Information Sharing

Proposition 5. Suppose information is shared between a manufacturer and the retailer.

(a) If $0 < c_e < 1/(1 + \phi)$, it benefits the manufacturer, hurts the retailer and benefits the rival manufacturer.

(b) If $1/(1 + \phi) < c_e < 2/(1 + \phi)$, there exist $c_{ae}$, $c_{be}$, and $c_{Ne}$ such that (i) it benefits the manufacturer except when the rival manufacturer is informed and $c_e > c_{ae}$; (ii) it benefits the retailer except when the rival manufacturer is informed and $c_e > c_{Ne}$; (iii) it hurts the informed rival manufacturer; (iv) it hurts the uninformed rival manufacturer except when $c_e > c_{be}$.

Suppose manufacturer $i$ receives the information. If $c_e < 1/(1 + \phi)$, $w_i$ responds positively to the demand signal. The effects are the same as those in the production diseconomy model except a larger production quantity variability is now beneficial because it reduces cost. This explains why information sharing benefits the rival manufacturer in the production diseconomy model but may hurt him in the production economy model. If $c_e > 1/(1 + \phi)$, $w_i$ responds negatively to the demand signal. As a result, (1) double marginalization of $w_i$ becomes weaker, (2) because $w_i$ and $w_j$ are strategic substitutes\(^{16}\), double marginalization of $w_j$ becomes stronger if manufacturer $j$ is informed and does not change otherwise; (3) production quantity variability faced by manufacturer $i$ becomes larger except when manufacturer $j$ is informed and production economy is large enough, it becomes smaller; and (4) production quantity variability faced by manufacturer $j$ becomes smaller except when manufacturer $j$ is uninformed and production economy is large enough, it becomes larger\(^{17}\).

\(^{16}\)They are strategic complements if $c_e < 1/(1 + \phi)$ and strategic substitutes otherwise.

\(^{17}\)For the effect of information sharing on production quantity variability, see Lemma 4 in the Appendix.
The production quantity variability effect in (3) and (4) explains why information sharing could hurt the manufacturer and the rival manufacturer (parts b(i), b(iii) and b(iv) of the proposition) when $c_e > 1/(1 + \phi)$. For part b(ii), the retailer benefits from the weaker double marginalization effect of $w_i$ except when manufacturer $j$ is informed and production economy becomes so large that the stronger double marginalization effect of $w_j$ dominates.

5.2. Equilibrium Information Sharing Decisions

Unlike the case of production diseconomy, the retailer may have an incentive to share information either when there is no information contracting or after a manufacturer rejects the information contract. Obviously the retailer would like to avoid the latter case, which could be a subgame perfect equilibrium (SPE), because she does not want to share information for free. We assume that the retailer can commit on not sharing information for free after a manufacturer rejects the information contract, which can be justified by either repeated interactions or reputation. By not deviating from the no-free-sharing commitment, the retailer can avoid the SPE in the long run which improves her profit. See Liu and van Ryzin (2008) for a similar assumption on price commitment and the relevant discussion. In Section 6, we relax this assumption and show that the information sharing equilibrium has the same threshold structure, though the firms’ profits are different.

Let $Z$ denote the contracting arrangement, where $Z = N$ if there is no information contracting, $Z = C$ if information contracts are offered concurrently, and $Z = S$ if they are offered sequentially. Let $n^Z_e$ be the number of informed manufacturers in equilibrium.

**Proposition 6.**

(a) There exist $c^N_e$, $c^C_e$ and $c^S_e$ such that $n^N_e = 0$ if $c_e < 1/(1 + \phi)$, $n^S_e = 2$ if $1/(1 + \phi) \leq c_e < c^S_e$, and $n^C_e = 1$ if $c^C_e \leq c_e < 2/(1 + \phi)$.

(b) $c^N_e < c^S_e \leq c^C_e$.

(c) $c^N_e$, $c^C_e$ and $c^S_e$ are decreasing in $\phi$.

We illustrate the equilibrium information sharing decisions in Figure 2. Note that region D is very narrow and because the threshold values of $c_e$ depend only on $\phi$, Figure 2 fully characterizes the information sharing regions.

**Proposition 7.**

(a) Regardless of having information contracting or not, a larger production economy or more intense competition induces the retailer to share information. She shares with one manufacturer when either production economy or competition intensity is sufficiently large, and shares with both manufacturers otherwise.
The retailer’s profit is higher under sequential information contracting while the total profit of the manufacturers is higher under concurrent information contracting.

Information sharing is beneficial only when either production economy or competition intensity is large enough so that wholesale price responds negatively to the demand signal. Unlike the case of production diseconomy, full information sharing does not always generate a higher value to the system when compared with partial information sharing. From Proposition 5 and the discussion after it, when production economy is large enough, all the three firms could be worse off when the number of informed manufacturers increases from one to two. Sequential information contracting leads to more partial information sharing because the retailer can make the manufacturers pay more for information.

Let $\Pi_M$ be the total ex-ante profit of the two manufacturers after accounting for the side payment for information sharing, where $Z = C$ or $S$. Let $\phi^C$ and $\phi^S$ be respectively the inverse functions of $c_e^C(\phi)$ and $c_e^S(\phi)$.

**Proposition 8.**

(a) $\pi_M(0)$ is decreasing in $\phi$.

(b) There exists $\delta^C > 0$ such that $\Pi_M^C(\phi^C + \delta^C) - \Pi_M^C(\phi^C - \delta^C) > 0$.

(c) There exist $\phi^C > 0$ and $\delta^S > 0$ such that $\Pi_M^S(\phi^S + \delta^S) - \Pi_M^S(\phi^S - \delta^S) > 0$ if $\phi^S < \phi^C$.

Without information sharing, the manufacturers’ total profit decreases as competition becomes more intense\(^{18}\). When there is production economy and information contracting, however, more

\(^{18}\)This is true also for the production diseconomy model.
intense competition can increase the manufacturers’ total profit if it induces the retailer to stop sharing information with one of the two informed manufacturers. This is because when competition intensity is large enough and the number of informed manufacturers is reduced from two to one, both manufacturers’ production quantities become more variable which lower their costs. In this case, the retailer has an incentive to share information with less manufacturers because she can share the efficiency gain via information contracting.

6. Extensions

6.1. Vertical Nash in the Production Diseconomy Model

We consider the case when wholesale prices and retail prices are determined under Vertical Nash. The sequence of events is the same as that of the basic model given in Section 2 except for event 3. In event 3 of the basic model, the manufacturers are Stackelberg leaders who simultaneously offer wholesale prices to the retailer before she determines the retail prices. For the case of Vertical Nash, each manufacturer determines a wholesale price for his own product, the retailer determines the retail margins for both products, and all these decisions are made simultaneously. This corresponds to the case when the manufacturers and the retailer have similar power in determining the wholesale prices.

As shown in Figure 3, the equilibrium information sharing decisions exhibit the same threshold structure as before. The results of Proposition 2 remain qualitatively the same (with the thresholds having different values) except that the threshold \( c_{d}^{S2} \) may not be decreasing in \( \phi \).

**Proposition 9.** A larger production diseconomy or more intense competition induces more information sharing. The only exception is under sequential contracting, there exists \( \phi^{b} \) such that if \( \phi \leq \phi^{b} \) and \( c_{d}^{S2}(\phi) \leq c_{d}^{S2}(\phi^{b}) \), more intense competition induces less information sharing.

Here \( c_{d}^{S2}(\phi) \) is defined as in Proposition 2 and we show explicitly its dependence on \( \phi \). The above proposition shows that the qualitative results under Vertical Nash are almost the same as before except when \( \phi \) is small and \( c_{d} \) takes some intermediate values. This can be explained as follows. With sequential contracting, information sharing between the retailer and manufacturer \( i \) leads to a revenue loss (due to the double marginalization effect) for the retailer and a saving in production cost (due to the production quantity variability effect) for manufacturer \( i \). When manufacturer \( j \) is informed, more intense competition increases both the cost saving and the revenue loss.

\[ \text{Refer to Lemma 4 in the Appendix. If } \phi \text{ increases, the threshold } [(5 + 10\phi + 3\phi^2) - (1 + \phi)\sqrt{1 + 2\phi + 9\phi^2}] / [2(1 + \phi)(1 + 2\phi)] \text{ decreases and when it becomes smaller than } c_{e}, \text{ we have } V_{Q}(2) \leq V_{Q}^{I}(1) < V_{Q}(1). \]

\[ \text{We abuse notations by using the same set of notations as before for the case of Vertical Nash.} \]

\[ \text{With concurrent contracting, more intense competition always reduces the revenue loss caused by information sharing.} \]
Under the conditions in Proposition 9, the impact of competition intensity on revenue loss is more significant than that on cost saving\(^{22}\). As a result, the value of information sharing between the retailer and manufacturer \(i\) decreases when competition becomes more intense.

### 6.2. Information Contracting with Voluntary Sharing in the Production Economy Model

Here we relax the assumption that the retailer can commit on not sharing information for free. The sequence of events is the same as that given in Section 2, except for the information contracting event (i.e., event 1). After the manufacturers decide whether to accept the information contract, the retailer can decide whether to share information for free with a manufacturer who does not accept the information contract. As before, let \(Z = C\) denote concurrent contracting and \(Z = S\) denote sequential contracting.

**Proposition 10.**

- **(a)** There exist \(\tilde{c}_C^e\) and \(\tilde{c}_S^e\) such that \(n_Z^Z = 0\) if \(c_e < 1/(1 + \phi)\), \(n_Z^Z = 2\) if \(1/(1 + \phi) \leq c_e < \tilde{c}_C^e\), and \(n_Z^Z = 1\) if \(\tilde{c}_C^e \leq c_e < 2/(1 + \phi)\).
- **(b)** There exist \(c^I_e < \tilde{c}_C^e\) and \(c^I_e > \tilde{c}_S^e\) such that if \(1/(1 + \phi) \leq c_e \leq c^I_e\) or \(c^I_e < c_e < 2/(1 + \phi)\), the retailer does not receive any payment for sharing information.
- **(c)** \(\tilde{c}_C^e < \tilde{c}_C^e\).
- **(d)** \(\tilde{c}_C^e\) and \(\tilde{c}_S^e\) are decreasing in \(\phi\).

\(^{22}\)This is not true for the basic model because wholesale prices are more sensitive to competition intensity under Vertical Nash when compared with the basic model.
The equilibrium information sharing decisions have the same threshold structure as that of the basic model. If both manufacturers reject the information contracts and the retailer shares information for free with one of them, there are two possible pure-strategy equilibria depending on which manufacturer is chosen to receive the information. With concurrent contracting, $\tilde{c}_C^e$ is the same for both subgame equilibria. It follows from part (c) that when the retailer cannot commit on not sharing information for free, it is more likely to have partial information sharing. With sequential contracting, however, $\tilde{c}_S^e$ is smaller than $c_S^e$ when the manufacturer who receives the information contract first is chosen to receive information for free (after both manufacturers have rejected the information contract) but larger than $c_S^e$ otherwise. When the retailer cannot commit on not sharing information for free, it is more likely to have partial information sharing under the first subgame equilibrium but less likely under the second subgame equilibrium.

7. Concluding Remarks

In this paper, we have examined the issues of information sharing and information contracting in a supply chain with two competing manufacturers selling through a common retailer. We consider the case where the retailer possesses verifiable information (such as consumer demographic data or POS data) and offers ex-ante information sharing contracts to the manufacturers. Another interesting case is ex-post information sharing where the retailer decides whether to share information after observing the demand signal. If the information is verifiable, this corresponds to strategic information sharing and it has been shown that the retailer uses a threshold policy to disclose information in a single supply chain (Guo 2009). If the information is not verifiable, the retailer could communicate her information via cheap talk. Chu et al. (2011) show that in a single supply chain where the manufacturer’s only decision is wholesale price, the retailer always has an incentive to deflate demand information to induce a lower wholesale price. Because of this, no information can be shared credibly. Chu et al. (2011) also show that when the manufacturer uses the shared information to make both capacity and wholesale price decisions, it is possible for the retailer to share unverifiable information via cheap talk. It would be interesting to consider strategic information sharing or cheap talk for the supply chain structure studied in this paper.

We focus on investigating how information sharing influences strategic interactions between firms and ignore its impact on operational improvements such as inventory cost reduction. To address the latter issue, we need to consider the case where the manufacturers make production decisions before demand realizes so that information sharing allows them to make these decisions with less uncertainty (and hence reduces the cost of mismatch between production and demand). This would be an interesting case to consider but, because it requires a very different mode of analysis, is left for further research.
Appendix: Lemmas and Proofs

We first present four lemmas that are useful for showing the effect of information sharing on a manufacturer’s production quantity variability or for the proof of the propositions.

Consider the production diseconomy model. Let the variance of a manufacturer’s production quantity be denoted by $V_q(n)$ for $n = 0$ or $2$, and by $V_q^X(n)$ for $n = 1$, where $X = U$ if the manufacturer is uninformed and $X = I$ if he is informed. The following lemma shows the effect of information sharing on production quantity variability. For example, the production quantity variability of an uninformed manufacturer becomes larger ($V_q(0) < V_q^U(1)$) when the rival manufacturer receives information from the retailer.

**Lemma 2.** $V_I^q(1) < V_q(2) < V_q(0) < V_q^U(1)$.

The following lemma is useful for the proofs of Propositions 1, 3 and 4. Note that the profit functions in the lemma are given in Section 3.2 with $c = c_d$.

**Lemma 3.**
(a) $\pi_M(2) > \pi_M^I(1) > \pi_M(0) > \pi_M^U(1)$.
(b) $\pi_R(0) > \pi_R(1) > \pi_R(2)$.
(c) $\pi_R(1) - \pi_R(2) > \pi_R(0) - \pi_R(1)$.

Now consider the production economy model. Using the same notations as the production diseconomy model, the following lemma shows the effect of information sharing on production quantity variability.

**Lemma 4.**
(a) If $0 < c_e < 1/(1 + \phi), V_q^I(1) < V_q(2) < V_q(0) < V_q^U(1)$.
(b) If $1/(1 + \phi) < c_e < (4 + 5\phi)/[(2 + 3\phi)(1 + \phi)], V_q^U(1) < V_q(0) < V_q(2) < V_q^I(1)$.
(c) If $(4 + 5\phi)/[(2 + 3\phi)(1 + \phi)] \leq c_e < [(5 + 10\phi + 3\phi^2) - (1 + \phi)\sqrt{1 + 2\phi + 9\phi^2}]/[2(1 + \phi)(1 + 2\phi)], V_q(0) \leq V_q^U(1) < V_q(2) < V_q^I(1)$.
(d) If $[(5 + 10\phi + 3\phi^2) - (1 + \phi)\sqrt{1 + 2\phi + 9\phi^2}]/[2(1 + \phi)(1 + 2\phi)] \leq c_e < 2/(1 + \phi), V_q(0) < V_q(2) \leq V_q^U(1) < V_q^I(1)$.

The following lemma is useful for the proof of Proposition 5. Note that the profit functions in the lemma are given in Section 3.2 with $c = -c_e$.

**Lemma 5.**
(a) $\pi_M^I(1) \geq \pi_M(0)$. There exists $c_e^* \in (1/(1 + \phi), 2/(1 + \phi))$ such that $\pi_M(2) \geq \pi_M^I(1)$ if $c_e \leq c_e^*$ and $\pi_M(2) < \pi_M^U(1)$ otherwise.
(b) $\pi_M(0) \geq \pi_M^I(1)$ if $1/(1 + \phi) \leq c_e \leq (4 + 5\phi)/[(2 + 3\phi)(1 + \phi)]$ and $\pi_M(0) < \pi_M^I(1)$ otherwise.
such that \( \forall c_e \leq 1/(1+\phi) \) and \( \pi_M(2) < \pi_M^U(1) \) otherwise.

(d) There exists \( c_e^N \in (1/(1+\phi), 2/(1+\phi)) \) such that \( \pi_R(0) > \pi_R(1) > \pi_R(2) \) if \( c_e < 1/(1+\phi) \), \( \pi_R(2) \geq \pi_R(1) \geq \pi_R(0) \) if \( 1/(1+\phi) \leq c_e \leq c_e^N \), and \( \pi_R(1) > \pi_R(2) > \pi_R(0) \) if \( c_e^N < c_e < 2/(1+\phi) \).

We are now ready to present the proofs of the lemmas and the propositions.

Proof of Lemma 1. It is easy to verify that \( w_i^* \) and \( p_i^* \) satisfy \( w_i^* = \hat{w}_i(w_i^*) \) and \( p_i^* = \hat{p}_i(w_i^*, w_j^*) \), and hence they are an equilibrium. To show the uniqueness, it suffices to show the solution to equations \( w_i^* = \hat{w}_i(w_i^*) \) (for \( i = 1, 2 \)) is unique, because the retailer’s best-response retail prices are uniquely determined by the wholesale prices. When \( n = 0 \) or \( 2 \), we have \( E[w_j] = w_j \), and it is easy to check that the linear equations \( w_i^* = \hat{w}_i(w_i^*) \) have a unique solution. When \( n = 1 \), without loss of generality, assume \( X_i = I \). Take expectation of \( \hat{w}_i(w_i) \) with respect to \( Y \) and substitute it into \( \hat{w}_j(w_j) \), we can find a unique solution \( w_i^* \). Then \( w_i^* = \hat{w}_i(w_i^*) \) is also unique.

Proof of Proposition 1. The results follow directly from Lemma 3.

Proof of Proposition 2. For part (a), notice that \( \lfloor \pi_R(2) + 2\pi_M(2) \rfloor - \lfloor \pi_R(0) + 2\pi_M(0) \rfloor > 0 \) if \( c_d > c_d^C = (\sqrt{8 + 8\phi + \phi^2} - 2 - \phi)/2(1 + \phi) \). Given any side payment \( T \), the manufacturers’ payoff matrix of the information sharing game is given in Table 1. When \( T \leq \pi_M^U(1) - \pi_M^U(0) \), the dominant-strategy equilibrium is \((I, I)\). When \( T \geq \pi_M(2) - \pi_M^U(1) \), the dominant-strategy equilibrium is \((U, U)\). When \( \pi_M^U(1) - \pi_M(0) \leq T \leq \pi_M(2) - \pi_M^U(1) \), there are two equilibria \((I, I)\) and \((U, U)\). In this case, the manufacturers choose the Pareto-optimal equilibrium, which is \((I, I)\) if \( T \leq \pi_M(2) - \pi_M(0) \), and \((U, U)\) otherwise. Therefore, the optimal decision for the retailer is either \( n = 0 \) or \( n = 2 \) with \( T = \pi_M(2) - \pi_M(0) \). The retailer prefers \( n = 2 \) to \( n = 0 \) if \( \pi_R(2) + 2|\pi_M(2) - \pi_M(0)| \geq \pi_R(0) \).

Therefore, \( n_d^C = 2 \) if \( c_d \geq c_d^C \), and \( n_d^C = 0 \) otherwise.

For part (b), we define

\[
\begin{align*}
V_{R+M}^U &= \lfloor \pi_M^U(1) + \pi_R(1) \rfloor - \lfloor \pi_M(0) + \pi_R(0) \rfloor, \\
V_{R+M}^I &= \lfloor \pi_M(2) + \pi_R(2) \rfloor - \lfloor \pi_M^U(1) + \pi_R(1) \rfloor.
\end{align*}
\]

We can show that \( V_{R+M}^U \geq 0 \) if \( c_d \geq c_d^S1 = (\sqrt{2} - 1)/(1 + \phi) \). It can be proved that \( V_{R+M}^I \) is convex in \( c_d \), \( V_{R+M}^I|_{c_d=0} < 0 \), \( V_{R+M}^I|_{c_d=c_d^S1} < 0 \), and \( V_{R+M}^I|_{c_d=-\infty} > 0 \). Hence, there exists a unique \( c_d^S2 \) (\( c_d^S2 > c_d^S1 \)) such that \( V_{R+M}^I \geq 0 \) if \( c_d \geq c_d^S2 \).
Without loss of generality, we assume that the retailer offers a contract to manufacturer 1 first. Now we consider the contracting outcome with manufacturer 2, given $X_1$. Suppose $X_1 = I$: with $n = 2$, the payoffs of the three firms equal $(\pi_R(2) + T_f + \pi_M(2) - T_f, \pi_M(2) - T_f)$; with $n = 1$, their payoffs equal $(\pi_R(1) + T_f, \pi_M(1) - T_f, \pi_M(1))$: manufacturer 2 agrees to buy information iff $T_f \leq \pi_M(2) - \pi_M'(1)$, and hence the retailer induces $n = 2$ iff $V_{R+M}^I \geq 0$. Suppose $X_1 = U$: with $n = 1$, the payoffs of the three firms equal $(\pi_R(1) + T_f, \pi_M(1) - T_f, \pi_M(1))$: with $n = 0$, their payoffs equal $(\pi_R(0), \pi_M(0), \pi_M(0))$: manufacturer 2 agrees to buy information iff $T_f \leq \pi_M(1) - \pi_M(0)$, and the retailer induces $n = 1$ iff $V_{R+M}^U \geq 0$.

Then we consider the contracting outcome with manufacturer 1, in anticipation of its impact on manufacturer 2’s decision. (i) If $V_{R+M}^U < 0$ (i.e., $0 < c_d < c_d^{S1}$), manufacturer 2 will not buy information, and manufacturer 1 will buy information iff $T_f \leq \pi_M(1) - \pi_M(0)$. Notice that $\pi_R(1) + T_f \leq \pi_R(0) + V_{R+M}^U < \pi_R(0)$, it is optimal for the retailer not to share information. (ii) If $V_{R+M}^U \geq 0$ and $V_{R+M}^I < 0$ (i.e., $c_d^{S1} \leq c_d < c_d^{S2}$), whatever decision manufacturer 1 makes, the retailer will induce a different information sharing outcome with manufacturer 2. Hence, manufacturer 1 agrees to buy information iff $T_f \leq \pi_M(1) - \pi_M'(1)$. It is clear that the optimal decision of the retailer is to induce $n = 1$ with $T_f = \pi_M(1) - \pi_M'(1)$. (iii) If $V_{R+M}^I \geq 0$ (i.e., $c_d \geq c_d^{S2}$), the retailer will share information with manufacturer 2 for sure. Manufacturer 1 agrees to buy information iff $T_f \leq \pi_M(2) - \pi_M'(1)$. Notice that $\pi_R(2) + 2[\pi_M(2) - \pi_M'(1)] = \pi_R(1) + [\pi_M(2) - \pi_M'(1)] + V_{R+M}^I > \pi_R(1) + [\pi_M(1) - \pi_M(0)]$, the retailer will induce $n = 2$ by charging $T_f = T_s = \pi_M(2) - \pi_M'(1)$.

For part (c), it is easy to check the ordering of $c_d^{S1} < c_d^{S2}$. For the ordering of $c_d^{S1} < c_d^{S2}$, it follows from the convexity of $V_{R+M}^I$ with respect to $c_d$ and $V_{R+M}^U|_{c_d = c_d^{S2}} < 0$.

For part (d), it is obvious that $c_d^{S1}$ and $c_d^{S2}$ are decreasing in $\phi$. Let $\phi(c_d^{S2})$ denote the inverse function of $c_d^{S2}(\phi)$. We can show $dV_{R+M}^U/dc_d|_{c_d = c_d^{S2}} > 0$ and $dV_{R+M}^I/d\phi|_{\phi = \phi(c_d^{S2})} > 0$. Taking derivative of $\phi$ on both sides of the equation $V_{R+M}^I = 0$, we have

$$
\frac{dV_{R+M}^I}{dc_d}|_{c_d = c_d^{S2}} \frac{dc_d^{S2}}{d\phi} + \frac{dV_{R+M}^I}{d\phi}|_{\phi = \phi(c_d^{S2})} = 0,
$$

which implies $dc_d^{S2}/d\phi < 0$. 

**Proof of Proposition 3.** Part (a) follows from Lemma 3(b). Part (b)(i) follows from Proposition 2. For part (b)(ii), the conclusion is obvious when $0 < c_d < c_d^{S1}$ and $c_d \geq c_d^{S2}$, because the retailer sells information to the same number of manufacturers under both models and the side payment under sequential information contracting is higher. When $c_d^{S1} \leq c_d < c_d^{S2}$, the retailer prefers sequential information contracting because $(\pi_R(1) + [\pi_M(1) - \pi_M'(1)]) - \pi_R(0) = [\pi_M(0) - \pi_M'(1)] + V_{R+M}^U > 0$. When $c_d^{S1} \leq c_d < c_d^{S2}$, again the retailer prefers sequential information contracting because $\pi_R'(1) + 2\pi_M(0) - 2\pi_M'(1) - \pi_M(2) > 0$ when $c_d \geq c_d^{S2}$, and hence $(\pi_R(1) + [\pi_M(1) - \pi_M'(1)]) - (\pi_R(2) + 2[\pi_M(2) - \pi_M(0)]) = [\pi_M(1) + 2\pi_M(0) - 2\pi_M'(1) - \pi_M(2)] - V_{R+M}^I > 0$. 

\[\Box\]
Proof of Proposition 4. Part (a) can be obtained by taking derivative of $\pi_R(0)$ with respect to $c_d$. For parts (b) and (c), By definition, $V_{R+M}^{U} = [\pi_M(1) + \pi_R(1)] - [\pi_M(0) + \pi_R(0)] = 0$ at $c_d = c_d^S$ and $V_{R+M}^{I} = [\pi_M(2) + \pi_R(2)] - [\pi_M(1) + \pi_R(1)] = 0$ at $c_d = c_d^S$. Based on these two equations, Proposition 2 and Lemma 3, it is straightforward to prove the results. □

Proof of Proposition 5. The results follow directly from Lemma 5.

Proof of Proposition 6. For part (a), first consider $Z = N$. We can show that $c_e^* > c_e^N$ and the result follows directly from parts (a), (bi) and (bii) of Proposition 5.

Now consider $Z = C$. We can show that $\pi_M(2) - \pi_M(1) \geq \pi_M(1) - \pi_M(0)$ iff $c_e \leq c_{e1}$, for some $c_{e1} \in \left(\frac{1}{1+\sigma}, \frac{2}{1+\sigma}\right)$. (i) When $c_e < c_{e1}$; if $T > \pi_M(2) - \pi_M(1)$, the unique equilibrium is $(U, U)$; if $T \leq \pi_M(1) - \pi_M(0)$, the unique equilibrium is $(I, I)$; when $\pi_M(1) - \pi_M(0) < T \leq \pi_M(2) - \pi_M(1)$, there are two equilibria $(U, U)$ and $(I, I)$, and the manufacturers pick up the Pareto-optimal equilibrium characterized as follows. If $c_e < \frac{1}{1+\sigma}$, then $\pi_M(2) - \pi_M(0) \geq \pi_M(2) - \pi_M(1)$ and the Pareto-optimal equilibrium is $(I, I)$ (with the optimal side payment for $n = 2$ equals $T = \pi_M(2) - \pi_M(1)$); if $\frac{1}{1+\sigma} \leq c_e < c_{e1}$, then the Pareto-optimal equilibrium is $(U, U)$ (in this case, the optimal side payment for $n = 2$ is $T = \pi_M(1) - \pi_M(0)$). Then for $c_e < \frac{1}{1+\sigma}$, we have $\pi_R(2) + 2[\pi_M(2) - \pi_M(1)] < \pi_R(0)$, and hence the retailer induces $(U, U)$. For $\frac{1}{1+\sigma} \leq c_e < c_{e1}$, since $\pi_R(2) + 2[\pi_M(1) - \pi_M(0)] > \pi_R(0)$, the retailer induces $n = 2$ with $T = \pi_M(1) - \pi_M(0)$. (ii) When $c_e < c_e < \frac{2}{1+\sigma}$, i.e., $\pi_M(2) < \pi_M(1)$, clearly $\pi_M(2) - T < \pi_M(1)$ for all $T \geq 0$. Hence, if the other manufacturer is informed already, a manufacturer would not buy information. The retailer prefers $n = 1$ with $T = \pi_M(1) - \pi_M(0)$ iff

$$V_{R+M}^{U} = \frac{(1 + \phi)c_e - 1}{2(1 + \phi)c_e + 1} - \frac{(1 + \phi)^2c_e^2 + 2(1 + \phi)c_e + 1}{4(1 + \phi)(2 - (1 + \phi)c_e)^2} \beta(t, \sigma)\sigma^2$$

is non-negative, which is always true for $c_e^* < c_e < \frac{2}{1+\phi}$. (iii) $c_{e1} \leq c_e \leq c_e$, i.e., $0 \leq \pi_M(2) - \pi_M(1) < \pi_M(1) - \pi_M(0)$. When $T \leq \pi_M(2) - \pi_M(1)$, the dominant-strategy equilibrium is $(I, I)$. When $T > \pi_M(1) - \pi_M(0)$, the dominant-strategy equilibrium is $(U, U)$. When $\pi_M(2) - \pi_M(1) < T \leq \pi_M(1) - \pi_M(0)$, the equilibria are $(I, U)$ and $(U, I)$. Therefore, the optimal decision for the retailer is either $n = 0$, or $n = 1$ with $T = \pi_M(1) - \pi_M(0)$, or $n = 2$ with $T = \pi_M(2) - \pi_M(1)$. The retailer prefers $n = 1$ to $n = 0$ iff $V_{R+M}^{U} \geq 0$, which is always true for $c_e < \frac{1}{1+\phi}$. She prefers $n = 2$ to $n = 1$ iff $(\pi_R(2) + 2[\pi_M(2) - \pi_M(1)]) - (\pi_R(1) + [\pi_M(1) - \pi_M(0)]) > 0$. For $c_e \in \left(\frac{1}{1+\phi}, \frac{2}{1+\phi}\right)$, we define $c_e^S = \max\{c_{e1}, c_{e2}\}$, then the optimal information sharing outcome is as follows. If $c_e < \frac{1}{1+\phi}$, $n_e^C = 0$; if $\frac{1}{1+\phi} \leq c_e < c_e^S$, $n_e^C = 2$ with $T = \min\{\pi_M(1) - \pi_M(0), \pi_M(2) - \pi_M(1)\}$; if $c_e^S \leq c_e < \frac{2}{1+\phi}$, $n_e^C = 1$ with $T = \pi_M(1) - \pi_M(0)$.

Finally consider $Z = S$. We can show that $V_{R+M}^{U} \geq 0$ if $\frac{1}{1+\phi} \leq c_e \leq c_{e3}$ for some $c_{e3} \in \left(\frac{1}{1+\phi}, \frac{2}{1+\phi}\right)$. Again, we assume that the retailer offers a contract to manufacturer 1 first. The firm profits
and manufacturer 2’s best response remain the same as in the diseconomy case. (i) If \( V_{R+M}^U < 0 \) (i.e., \( 0 < c_e < \frac{1}{1 + \phi} \)), manufacturer 2 will not buy information, and manufacturer 1 will buy information iff \( T_f \leq \pi_M^f(1) - \pi_M(0) \). Notice that \( \pi_R(1) + T_f \leq \pi_R(0) + V_{R+M}^U < \pi_R(0) \), it is optimal for the retailer not to share information. (ii) If \( V_{R+M}^I \geq 0 \) and \( V_{R+M}^I < 0 \) (i.e., \( c_{e3} < c_e < \frac{2}{1 + \phi} \)), whatever decision manufacturer 1 makes, the retailer will induce a different information sharing outcome with manufacturer 2. Manufacturer 1 agrees to buy information iff \( T_f \leq \pi_M^f(1) - \pi_M^U(1) \), and manufacturer 2 will buy iff \( X_1 = U \) and \( T_s \leq \pi_M^f(1) - \pi_M(0) \). Hence the retailer will induce \( n = 1 \) with \( T = \pi_M^f(1) - \min \{ \pi_M^U(1), \pi_M(0) \} \). (iii) If \( V_{R+M}^I \geq 0 \) (i.e., \( \frac{1}{1 + \phi} \leq c_e \leq c_{e3} \)), the retailer will share information with manufacturer 2 for sure. Manufacturer 1 agrees to buy information iff \( T_f \leq \pi_M(2) - \pi_M^f(1) \). Hence the retailer chooses between sharing with both \( (T_f = T_s = \pi_M(2) - \pi_M^U(1)) \) and sharing with manufacturer 2 only \( (T_s = \pi_M^f(1) - \pi_M(0)) \), and she prefers \( n = 2 \) iff \( c_e < c_{e2} \).

To summarize, if \( 0 < c_e < \frac{1}{1 + \phi} \), \( n_e^S = 0 \); if \( \frac{1}{1 + \phi} \leq c_e \leq c_{e3} \), \( n_e^S = 2 \) with \( T_f = T_s = \pi_M(2) - \pi_M^U(1) \); if \( c_{e3} \leq c_e < \frac{2}{1 + \phi} \), \( n_e^S = 1 \), with \( T = \pi_M(1) - \min \{ \pi_M^U(1), \pi_M(0) \} \) if \( c_{e3} < c_e < \frac{2}{1 + \phi} \), and \( T_s = \pi_M^f(1) - \pi_M(0) \) otherwise.

For part (b), we can show that \( c_e^N, c_e^C, c_e^S \) are all decreasing in \( \phi \) using the approach in the proof of part (d) of Proposition 2. Then the results follow immediately.

For part (c), we can show \( c_e^N < c_e^C \) and \( c_e^N < c_e^S \), then \( c_e^N < c_e^S = \min \{ c_{e2}, c_{e3} \} \leq c_e^C = \max \{ c_{e1}, c_{e2} \} \).

**Proof of Proposition 7.** Part (a) follows from Proposition 6. For part (b), we can show that \( c_{e2} \) is between \( c_{e1} \) and \( c_{e3} \). (i) If \( c_{e1} \leq c_{e3} \), \( c_e^S = c_e^C = c_{e2} \), the information sharing outcome is the same under the two models, and the side payment under sequential information contracting is higher. Then the results follow immediately. (ii) If \( c_{e1} > c_{e3} \), then \( c_e^C = c_e^S = c_{e3} \). Concurrent information contracting induces more information sharing. When the information sharing outcome is the same under the two models, clearly the side payment under sequential information contracting is higher; otherwise \( c_e^S < c_e < c_e^C \), \( n_e^S = 1 \) with \( T_s = \pi_M^f(1) - \pi_M^U(1) \) and \( n_e^C = 2 \) with \( T = \pi_M^f(1) - \pi_M(0) \). The retailer prefers sequential information contracting because \( (\pi_R(1) + [\pi_M^f(1) - \pi_M^U(1)]) - (\pi_R(2) + 2[\pi_M^f(1) - \pi_M(0)]) = -V_{R+M}^I + [\pi_M^f(1) - \pi_M(0)] - [\pi_M^f(1) - \pi_M(0)] + [\pi_M(0) - \pi_M^U(1)] \geq 0 \). The manufacturer’s total profit under concurrent information contracting is \( 2[\pi_M(2) + \pi_M(0) - \pi_M^f(1)] \), higher than their total profit under sequential information contracting \( 2\pi_M^U(1) \).

**Proof of Proposition 8.** Part (a) can be obtained by taking derivative of \( \pi_M(0) \) with respect to \( \phi \).

For part (b), we need to show there is a positive jump in the total profit of the manufacturers at \( \phi_e^C \). If \( c_{e1} < c_{e2} \), the total profit of the manufacturers increases from \( 2\pi_M^U(1) \) to \( \pi_M(0) + \pi_M^U(1) \) as
φ increases at φ^c_i; otherwise, the total profit increases from 2[π_M(2) + π_M(0) − π'_M(1)] to π_M(0) + π'_M(1) because π_M(2) < π'_M(1) at c_c = c_c1.

For part (c), we can show that if c_c2 < c_c3, the total profit of the manufacturers increases from 2π'_M(1) to π_M(0) + π'_M(1) at c_c = c_c2. The results follow because we can show that c_c2 < c_c3 is equivalent to φ^c < φ^c = 1.7602.

**Proof of Proposition 9.** We follow the approach in Choi (1991) to get the firms’ expected profits in the second stage. For convenience, we abuse notations by using the same set of notations as in the basic model where the manufacturers are the leaders in offering wholesale prices. Given w_1 and w_2, the retailer maximizes her expected profit \( (p_1 - w_1)(a + E[\theta|Y] - (1 + \phi)p_1 + \phi p_2) + (p_2 - w_2)(a + E[\theta|Y] - (1 + \phi)p_2 + \phi p_1) \) by setting the retail prices to \( \hat{p}_i(w_1, w_2) = \frac{1}{2}(a + \beta(t, \sigma)Y + w_i) \).

Given the profit margins charged by the retailer, an informed manufacturer maximizes his expected profit \( (w_i - b)E[q_i(p_i, p_j)|Y] - cE[q_i^2(p_i, p_j)|Y] \) by setting the wholesale price to \( \hat{w}_i(p_i, p_j) = (2c + \frac{1}{1 + \sigma})(a + \beta(t, \sigma)Y - (1 + \phi)p_i + \phi p_j) + b \); an uninformed manufacturer maximizes his expected profit \( (w_i - b)E[q_i(p_i, p_j)|Y] - cE[q_i^2(p_i, p_j)] \) by setting the wholesale price to \( \hat{w}_i(p_i, p_j) = (2c + \frac{1}{1 + \sigma})(a - (1 + \phi)E[p_i] + \phi E[p_j]) + b \). Following the proof of Lemma 1, we can find a unique equilibrium

\[
\hat{w}_i^* = \begin{cases} \bar{w} + \alpha_w(n)Y & \text{if } n = 0 \text{ or } 2, \\ \bar{w} + \alpha_w^i(1)Y & \text{if } n = 1, \end{cases}
\]

\[
\hat{p}_i^* = \begin{cases} \bar{p} + \alpha_p(n)Y & \text{if } n = 0 \text{ or } 2, \\ \bar{p} + \alpha_p^i(1)Y & \text{if } n = 1, \end{cases}
\]

where

\[
\alpha_w(0) = \alpha_w^U(1) = 0, \alpha_w^L(1) = \frac{1 + 2(1 + \phi)c}{(1 + \phi)(3 + 2(1 + \phi)c)}\beta(t, \sigma), \alpha_w(2) = \frac{1 + 2(1 + \phi)c}{3 + 2\phi + 2(1 + \phi)c}\beta(t, \sigma),
\]

\[
\alpha_p(0) = \alpha_p^U(1) = \frac{\beta(t, \sigma)}{2}, \alpha_p^L(1) = \frac{4 + 3\phi + 2(1 + \phi)(2 + \phi)c}{2(1 + \phi)(3 + 2(1 + \phi)c)}\beta(t, \sigma),
\]

\[
\alpha_p(2) = \frac{2 + \phi + 2(1 + \phi)c}{3 + 2\phi + 2(1 + \phi)c}\beta(t, \sigma).
\]

Here \( \bar{w} \) and \( \bar{p} \) are the deterministic solutions when \( \sigma = 0 \), where \( \bar{w} = [(1 + 2(1 + \phi)c)a + 2(1 + \phi)b]/[3 + 2\phi + 2(1 + \phi)c] \) and \( \bar{p} = [(2 + \phi + 2(1 + \phi)c)a + (1 + \phi)b]/[3 + 2\phi + 2(1 + \phi)c] \). Based on the equilibrium pricing decisions, we obtain the firms’ ex-ante profits as follows.

\[
\pi_M(0) = \bar{\pi}_M - \frac{c\beta(t, \sigma)\sigma^2}{4} - c[1 - \beta(t, \sigma)]\sigma^2,
\]

\[
\pi_M^U(1) = \bar{\pi}_M - \frac{c}{4} \left[ 3 + 4\phi + 2(1 + \phi)(1 + 2\phi)c \right] \beta(t, \sigma)\sigma^2 - c[1 - \beta(t, \sigma)]\sigma^2,
\]

\[
\pi_M^L(1) = \bar{\pi}_M + \frac{1 + (1 + \phi)c}{(1 + \phi)(3 + 2(1 + \phi)c)}\beta(t, \sigma)\sigma^2 - c[1 - \beta(t, \sigma)]\sigma^2,
\]
\[
\pi_M(2) = \bar{\pi}_M + \frac{(1 + \phi)(1 + (1 + \phi)c)}{(3 + 2\phi + 2(1 + \phi)c)}\beta(t, \sigma)\sigma^2 - c[1 - \beta(t, \sigma)]\sigma^2,
\]

\[
\pi_R(0) = \bar{\pi}_R + \frac{\beta(t, \sigma)\sigma^2}{2},
\]

\[
\pi_R(1) = \bar{\pi}_R + \left[\frac{1 + 2\phi}{1 + \phi} + \frac{4}{(1 + \phi)(3 + 2(1 + \phi)c)^2}\right] \frac{\beta(t, \sigma)\sigma^2}{4},
\]

\[
\pi_R(2) = \bar{\pi}_R + \frac{2(1 + \phi)^2}{(3 + 2\phi + 2(1 + \phi)c)^2} \beta(t, \sigma)\sigma^2,
\]

where \(\bar{\pi}_M = (1 + \phi)(1 + (1 + \phi)c)(a - b)^2/(3 + 2\phi + 2(1 + \phi)c)^2\), \(\bar{\pi}_R = 2(1 + \phi)^2(a - b)^2/(3 + 2\phi + 2(1 + \phi)c)^2\). The rest of the proof is similar to those for Propositions 2 and 3, and we omit the details.

PROOF OF Proposition 10. It is similar to that of Proposition 6 and we omit the details.

References


