A THEORY FOR PRICING NON-FEATURED PRODUCTS IN SUPERMARKETS

by

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Working Paper 931-77 May, 1977

Abstract

Of the 6000 products in the average supermarket, a few receive special displays, advertising, and prices in a given week, but the great majority are assigned prices by simple percentage markups within category, adjusted, if necessary, for competitive conditions and special price endings. Missing is any direct consideration of actual customer price response.

The advent of the Universal Product Code and inexpensive, machine readable sales data by individual item promises to make possible the widespread determination of customer price response by in-store experiment. This in turn, opens up the possiblity of developing and implementing a more adequate theory for setting prices.

Yet, stores should not set price in a simple profit-maximizing way based on in-store measurements of price response. The reason is that the customer may pay a high price once and then not come back to the store, thereby creating a small short term gain and a large long term loss.

A two-stage theory addresses this issue by postulating that customers once in the store purchase goods to maximize their utility.
This determines observable in-store customer price response. The store then maximizes its profit subject to a constraint on customer utility delivered. The level of the constraint becomes a policy parameter that determines, in part, the attractiveness of the store to the customer.

It is shown that under this theory, the store can set prices of non-featured items using a formula containing only empirical measurements and a single policy parameter for all items. It is further shown that the prices are efficient in the sense that for a given level of store profit no other set of prices will permit higher consumer utility and, conversely, for a given level of customer utility, no other prices will permit higher store profit.

1. Introduction

If an OR/MS team from Mars should enter the typical U.S. supermarket and measure sales response to price by varying shelf prices, it would be surprised to find that the store seems to be pricing each item much too low to maximize profit. The store acts as if the customer is considerably more sensitive to prices than customer behavior in the store seems to indicate.

There is a reason for this. The store has a special concern. Even though a customer, once in the store, may buy the item at a higher price, will he come back? A pricing policy that maximizes profits taking into account the problem of bringing the customer into the store may call for much lower prices than would be implied by measurements taken once the customer is there.

Supermarket chains go to great effort to project an overall impression of low prices and good value for the money. They run newspaper ads featuring reduced prices and set up special displays in the stores to emphasize sale items. These are often real bargains with the store not infrequently taking a net loss on an individual item.

However, the great bulk of items are not so featured and it is necessary to have a pricing policy for them. By and large stores use a simple markup over cost. This does not take into account customer price response, and so, in an important sense is ignoring customer preferences. However, since the store does not maximize profit on individual items, the question arises whether individual price measurements have any meaning or usefulness to the store.

The purpose of this paper is to contend that they do and to develop the beginnings of a theory of how to use such information.

2. Notation and definitions

We look at the store's customers in aggregate and suppose a collective utility function. For a fixed time period, utility depends on the quantities of various goods bought by the customer and the total money spent on them. The quantities bought also represent the sales of the store since the customers' spending is the store's revenue. Let

n = number of items

s_i = quantity bought of item i (units)

p_i = price of item i (dol/unit)

$\underline{s} = (s_1, \ldots, s_n)$

$\underline{p} = (p_1, \ldots p_n)$

$t = \underline{p} \cdot \underline{s} = \sum_i p_i s_i$ = total dollars spent in time period (dollars)

We suppose the customers' utility function is

$$u = v(\underline{s}) - w(t)$$

and further that $v(\underline{s})$ is concave and $w(t)$ convex. Then $u(\underline{s},t)$ is concave. For example, we might have $v(\underline{s}) = \sum_i v_i(s_i)$ with the v_i and w functions shaped as shown in Figure 1.

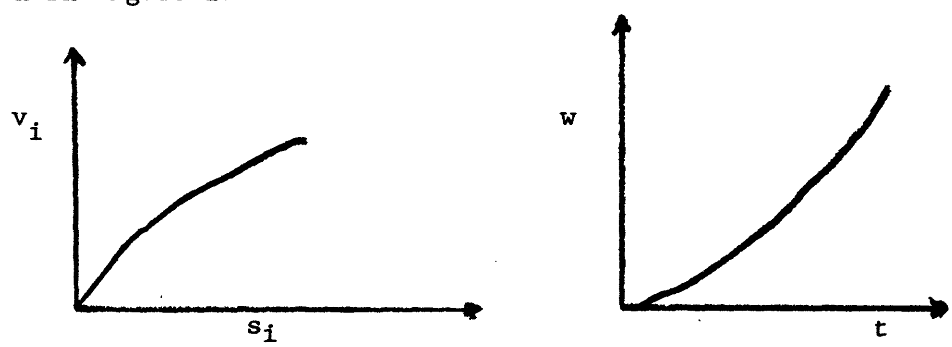


Fig. 1. Components of customer utility function.

Practically, the sketches mean that the more units of any item the customers buy, the less they value an additional unit. In addition, the more money spent in total, the more they prefer not to spend another dollar.

Although virtually all items in a supermarket are sold in discrete units, we take all quantities to be continuous to avoid useless complexities.

3. Customers' problem once in the store

The customers are assumed to make purchases to maximize their aggregate utility, given the prevailing prices in the store. In other words, for fixed $\underline{p} = (p_1 \ldots p_n)$ they solve:

$$\underline{\text{C1}}: \quad \text{Find } \underline{s} \text{ to}$$

$$\max u = v(\underline{s}) - v(\underline{p}\cdot\underline{s})$$

$$\text{subject to } \underline{s} \geqslant 0.$$

The concavity of u guarantees a global maximum for some set of purchased quantities $\underline{s}$.

Since the store will not carry items it cannot sell, we shall assume that the solution of C1 leads to $\underline{s} > 0$. Then the maximum is an interior point and satisfies the necessary conditions

$$\frac{dv}{ds_i} - w'(t)\, p_i = 0 \qquad i = 1, \ldots n \tag{2.1}$$

where $\quad w' = dw/dt$ and $t = \underline{p}\cdot\underline{s}$

Repeated solution of C1 for various prices $\underline{p}$ develops the customers' price response functions:

$s_i = s_i(\underline{p})$ = sales of item i when items are priced $\underline{p}=(p_1, \ldots p_n)$

These in turn imply a total customer spending (store dollar sales) of

$$t(\underline{p}) = \sum_i p_i s_i(\underline{p}).$$

The functions $s_i(\underline{p})$ are those we believe a store might feasibly measure by in-store experiment.

The general shape of $s_i(\underline{p})$ would presumably be:

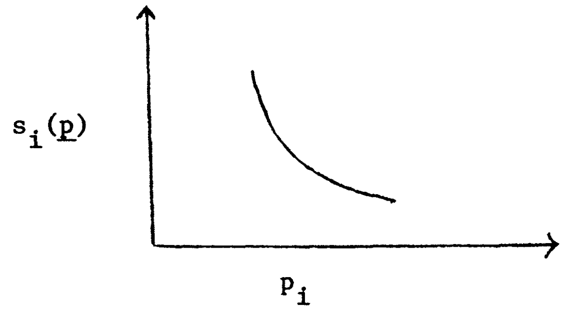


Fig. 2. Price response function.

3. Store's problem given customer in store

The store's problem is, given the way customers respond to price, to set those prices. We investigate several methods for doing this. In this section we again assume the customer is in the store.

3.1 Maximizing Customers' Utility Subject to Profit Constraint:

Suppose the store somehow knows the customers utility function and chooses to maximize it subject to making a fixed profit. Then the store solves:

S1: Find $\underline{p}$ to

$$\max u = v(\underline{s}(\underline{p})) - w(t(\underline{p}))$$

$$\text{s.t.} \quad \sum_i (p_i - c_i)\, s_i(\underline{p}) \geq \pi_o \qquad \text{(S1.1)}$$

$$\underline{p} \geq 0 \qquad \text{(S1.2)}$$

where c_i = incremental cost to the store of item i (dol/item)

$\underline{c} = (c_1,\ldots,c_n)$

π_o = minimum profit for store (dollars).

$t(\underline{p}) = \underline{p}\cdot\underline{s}(\underline{p})$

Since customer utility can always be increased by decreasing prices, maximum utility will be reached only when the profit constraint is satisfied by equality and so, without loss of optimality, we can replace the inequality with equality in (S1.1). Furthermore a solution with any prices zero would mean the store should not stock the product. Therefore, we suppose in any real solution $\underline{p} > 0$ and use this fact as convenient.

3.2 Maximizing Profit Subject to Utility Constraint.

In this case the store would solve

S2: Find $\underline{p}$ to

$$\max \pi = \sum_i (p_i - c_i)s_i$$

$$\text{s.t.} \quad v(\underline{s}(\underline{p})) - w(t(\underline{p}) \geq u_o \qquad (S2.1)$$

$$\underline{p} \geq 0 \qquad (S2.2)$$

where u_o = minimal customer utility.

We shall assume that $\sum_i (p_i - c_i)s_i(\underline{p})$ is concave in $\underline{p}$. Also we can again assume without loss of optimality that the utility constraint is satisfied by an equality in (S2.1) and real situtations lead to $\underline{p} > 0$.

3.3 Equivalence of S1 and S2

Suppose u_o is the customer utility achieved in S1 by maximizing utility subject to the profit constraint $\pi = \pi_o$. We show that, if the same value of u_o is made the utility constraint in maximizing store profit in S2, then the same sets of prices maximize customer utility in S1 and store profit in S2. Furthermore, the maximal profit in S2 is π_o.

Theorem: If u_o in S2 equals max u from S1, the set of $\underline{p}$ maximizing π in S2 is the same as the set maximizing u in S1 and max $\pi = \pi_o$.

Proof: Consider S1. Since at optimum $\underline{p} > o$, constraints (S1.2) can be ignored. Treating (S1.1) as an equality and introducing a Lagrange multiplier, λ, we create the unconstrained problem

$$\text{S1A:} \quad \max_{\underline{p}} \; L_1(p, \lambda) = v(s(\underline{p})) - w(t(\underline{p})) + \lambda\left[\sum_i (p_i - c_i) s_i(\underline{p})\right]$$

$$= u(\underline{p}) + \lambda\pi(\underline{p})$$

We solve S1A for $\underline{p}(\lambda)$. This solves S1 if λ is chosen such that $\pi(\underline{p}) = \pi_o$. Call such λ, λ_o.

A similar argument transforms S2 into the unconstrained problem

$$\text{S2A:} \quad \max_{\underline{p}} \; L_2(\underline{p}, \mu) = \sum_i (p_i - c_i)\, s_i(\underline{p}) + \mu[v(s(\underline{p}) - w(t(\underline{p}))]$$

$$= \pi(\underline{p}) + \mu\, u(\underline{p})$$

Solving S2A solves S2 when μ is chosen to make $u(\underline{p}) = u_o$.

It can be seen that S1A and S2A are the same since λ and μ are constants as far as the maximization is concerned. In other words, S2A will be solved by any $\underline{p}$ that solves

$$\text{S2B:} \quad \max_{\underline{p}} \; (1/\mu)L_2(\underline{p}, u) = (1/\mu)\pi(\underline{p}) + u(\underline{p})$$

The RHS of S2B is just S1A and so if $\underline{p}$ solves S1A for λ, it solves S2A for $\mu = 1/\lambda$ and conversely.

Therefore the specific values $\underline{p}(\lambda_o)$ and λ_o, solve S2A with

$$\max \quad L_2 = \pi(p(\lambda_o)) + (1/\lambda_o)\ u(p(\lambda_o))$$

But $u(p(\lambda_o)) = \max u$ and $\max u = u_o$ by assumption of the theorem. Thus $1/\lambda_o$ is the required value of μ that makes $u(\underline{p}) = u_o$ and S2 is solved by $\underline{p}(\lambda_o)$. Since we have already seen $\pi(p(\lambda_o)) = \pi_o$ we now know that any $\underline{p}$ maximizing S1 maximizes S2 with $\max \pi = \pi_o$. A similar argument works the converse: any $\underline{p}$ maximizing π subject to $u = u_o$ will max u subject to $\pi = \pi_o$ and will give $\max u = u_o$.

The above results display the tradeoff between the objectives of store profit and customer utility and show the existance of efficient prices which permit these objectives to be as high as possible up to the point of tradeoff between the two. However, this development supposes the customers are already in the store and their sales response to price $s(p)$ is measured there. As discussed in the introduction, a major issue is whether an individual customer comes back to the store after shopping there or shops at the store in the first place.

4. Customers' propensity to shop at store

We now hypothesize that although there are many ways to bring customers to the store and keep them coming (e.g., special features, product variety, cleanliness, pleasant surroundings, and friendly service) one important way is to provide low prices throughout the store, or better yet provide high value of customer utility. Thus the store might choose to maximize customer utility subject to a profit constraint or, as we have seen to be equivalent, to maximize profit subject to a utility constraint. In the last form, the value of utility, if it could be made operational, would become a parameter which the store could adjust. Therefore, we proceed to find the prices a store should charge according to the following assumptions: (1) the customer is buying to maximize utility (i.e., behaving according to $s_i = s_i(\underline{p})$ from C1) (2) the store is pricing to maximize profit subject to a customer utility constraint as in S2.

Starting from the lagrangian for S2,

$$L_2(\underline{p}, \mu) = \sum_i (p_i - c_i) s_i(\underline{p}) + \mu[v(\underline{s}(\underline{p})) - w(\underline{s} \cdot \underline{p})]$$

necessary conditions for a maximum include

$$\frac{\partial L_2}{\partial p_j} = s_j + \sum_i (p_i - c_i) \frac{\partial s_i}{\partial p_j}$$

$$+ \mu' \{ \sum_i \frac{\partial v}{\partial s_i} \frac{\partial s_i}{\partial p_j} - w'[s_j + \sum_i p_i \frac{\partial s_i}{\partial p_j}] \} = 0$$

$$j = 1, \ldots . n \tag{4.1}$$

However, from C1

$$\frac{\partial v}{\partial s_i} - w' p_i = 0 \qquad i = 1, \ldots n \tag{2.1}$$

Define elasticity and cross-elasticities:

$$\eta_j(\underline{p}) = -\frac{p_i}{s_i} \frac{\partial s_j}{\partial p_j} \tag{4.2}$$

$$\eta_{ij}(\underline{p}) = \frac{p_j}{s_i} \frac{\partial s_i}{\partial p_j} \tag{4.3}$$

With appropriate manipulations (4.1) becomes

$$p_j = \frac{\eta_j + \sum\limits_{i \neq j} \left(\frac{p_i - c_i}{c_j}\right) \frac{s_i}{s_j} \eta_{ij}}{\eta_j - 1 + uw'} c_j \tag{4.4}$$

This is not really solved for p_i since p_i appears on the right hand side in $\eta_j(\underline{p})$, $s_j(\underline{p})$, η_{ij} $(\underline{p})$, w' $(\underline{p} \cdot \underline{s}(\underline{p}))$, and $\mu(\underline{p})$. However, if we suppose that these quantities vary slowly with p_i or, more relevant, that we will measure η_i, η_{ij}, and s_i at the current $\underline{p}$ and make changes from current $\underline{p}$ rather gradually, then (4.4) is an appropriate calculation. If a p_i as calculated does not differ too much from current practice then the calculated value can be expected to be very close. If the calculated p_i is quite different, a step in the direction of the new value is indicated, at which point new measurements would be taken.

All quantities in (4.4) are presumed measurable and known except $\mu w'$. Notice, however, that $\mu w'$ is independent of the particular item being priced (i.e. of i). Thus, given the measurements, <u>one constant sets all prices.</u> This constant expresses the store's desire to lower its prices to keep the customers' satisfaction high and maintain store loyalty.

5. Measurement Issues

Is it practical to measure price response as required by (4.4)? Today, no. Although many people have made in-store price response measurements, the task requires much effort, particularly to collect accurate sales data. This usually means taking inventory at the start, tallying all quantities restocked on the shelves , and taking inventory at the end.

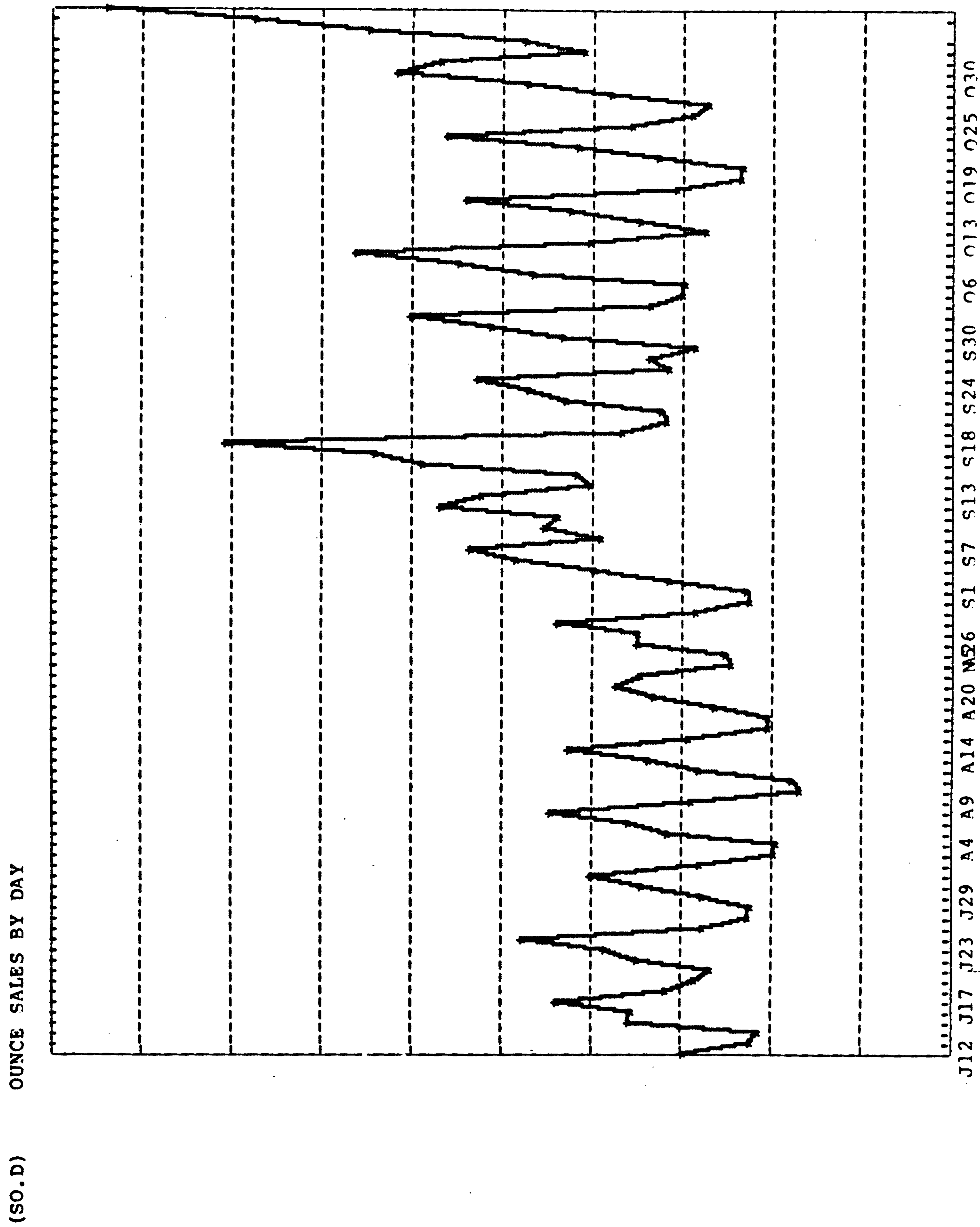
By contrast the introduction of point-of-purchase sales recording equipment drastically changes the situation. Sales in machine readable form in great time and item detail are possible. The quantity of data is, in fact, a little overwhelming and much needs to be learned about handling it effectively for analytic purposes.

In supermarkets optical scanning equipment that reads the Universal Product Codes (UPC) offers the prospect of automatic sales recording. As of this writing, however, only 100 stores in the U.S. are equipped with scanners so that we are not talking about an immediate revolution. Yet, many stores have electronic check-out equipment on which it is possible, with moderate special effort, to record sales by item. Therefore, to investigate the measurement potential of UPC, we ran a 16 week, 8 store experiment in a Boston supermarket chain in which daily sales and price data were collected for the frozen orange juice category (16 items).

It is not our purpose here to do a full scale evaluation of the potential of UPC data for marketing measurements, but we wish to present preliminary indications that price measurements may be feasible and so give a flavor of the basic data. Figure 3 shows frozen orange juice sales in ounces for the 8 stores over the period July 12 to November 6, 1976.

Fig. 3

CATEGORY SALES



Prominent is day-of-week variation, Saturday being the peak day. Also evident is seasonal variation, (e.g., down in August because of vacations), and at least two noticeable special feature weeks (weeks ending Sept. 18 and November 6).

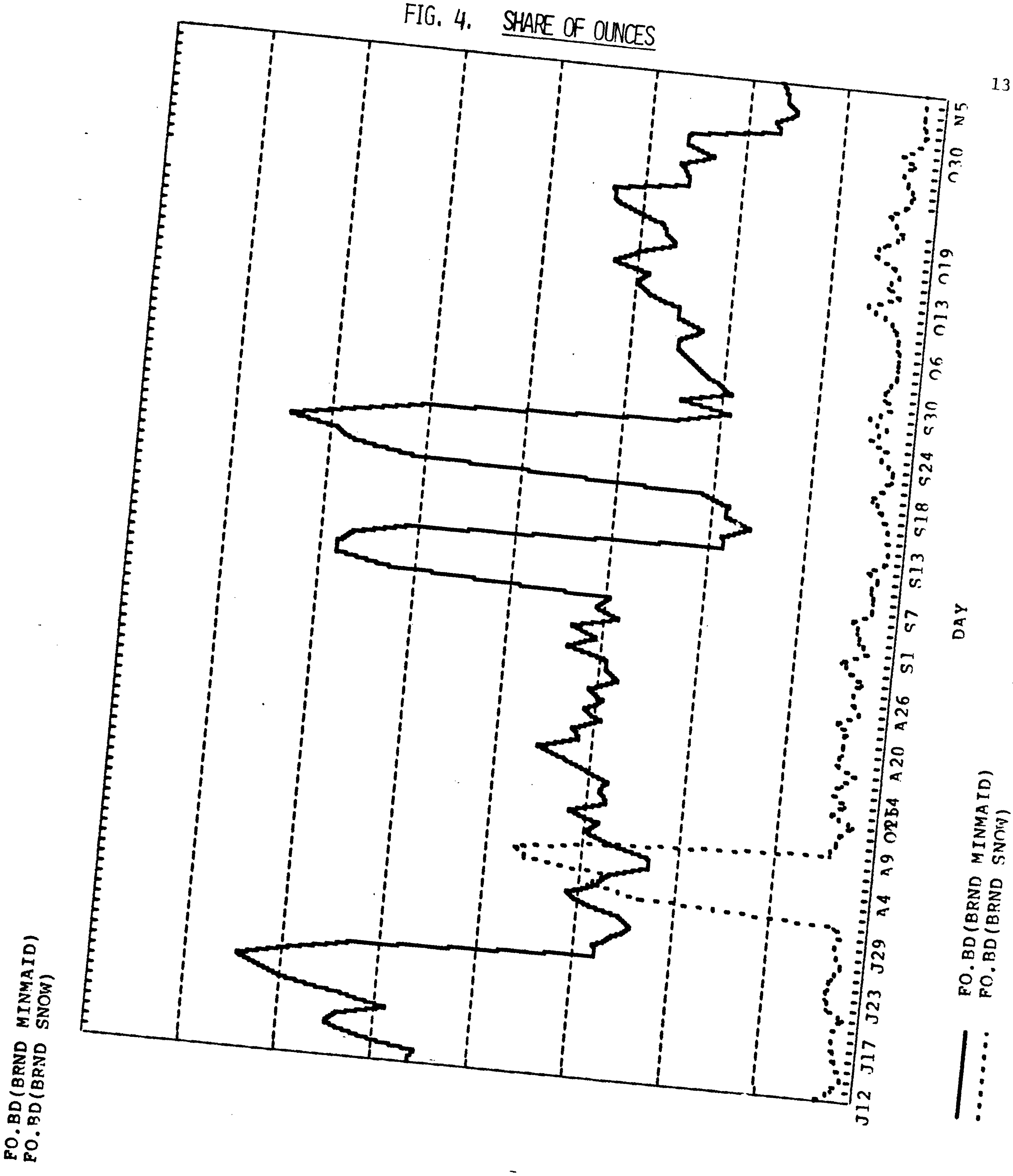
Figure 4 shows daily market share for two national brands: Minute Maid and Snow Crop. Share lacks any substantial day-of-week variation but now weekly special features jump out of the data as bumps in share. The power of the data to support the analyses of features is clear.

Insofar as price measurement goes, the experimental design examined the effects of changing the spread of price between national brands and private labels (normally about 16%). Three treatments considered were normal, larger spread and smaller spread. For a given treatment prices were held constant for 4 weeks. Figure 5 shows what happened in two pairs of stores. Store pair A received treatments: Normal spread, large spread, normal spread, small spread, in that order. Store pair B received the same except large and small interchanged. Plotted in the ratio, national brand sales/private label sales, for each store pair over the experimental period, scaled to make the normal treatment periods average to 1.0.

Price effects are evident. In the second 4 weeks the small spread stores show a substantial gain for national brands relative to the large spread stores (solid line lies above dashed line). In the third period, the stores come back together as both use normal spreads. In the fourth period the treatments are reversed and so are the sales results (dashed line above solid line).

Several remarks can be made. First, the price effects are

FIG. 4. SHARE OF OUNCES



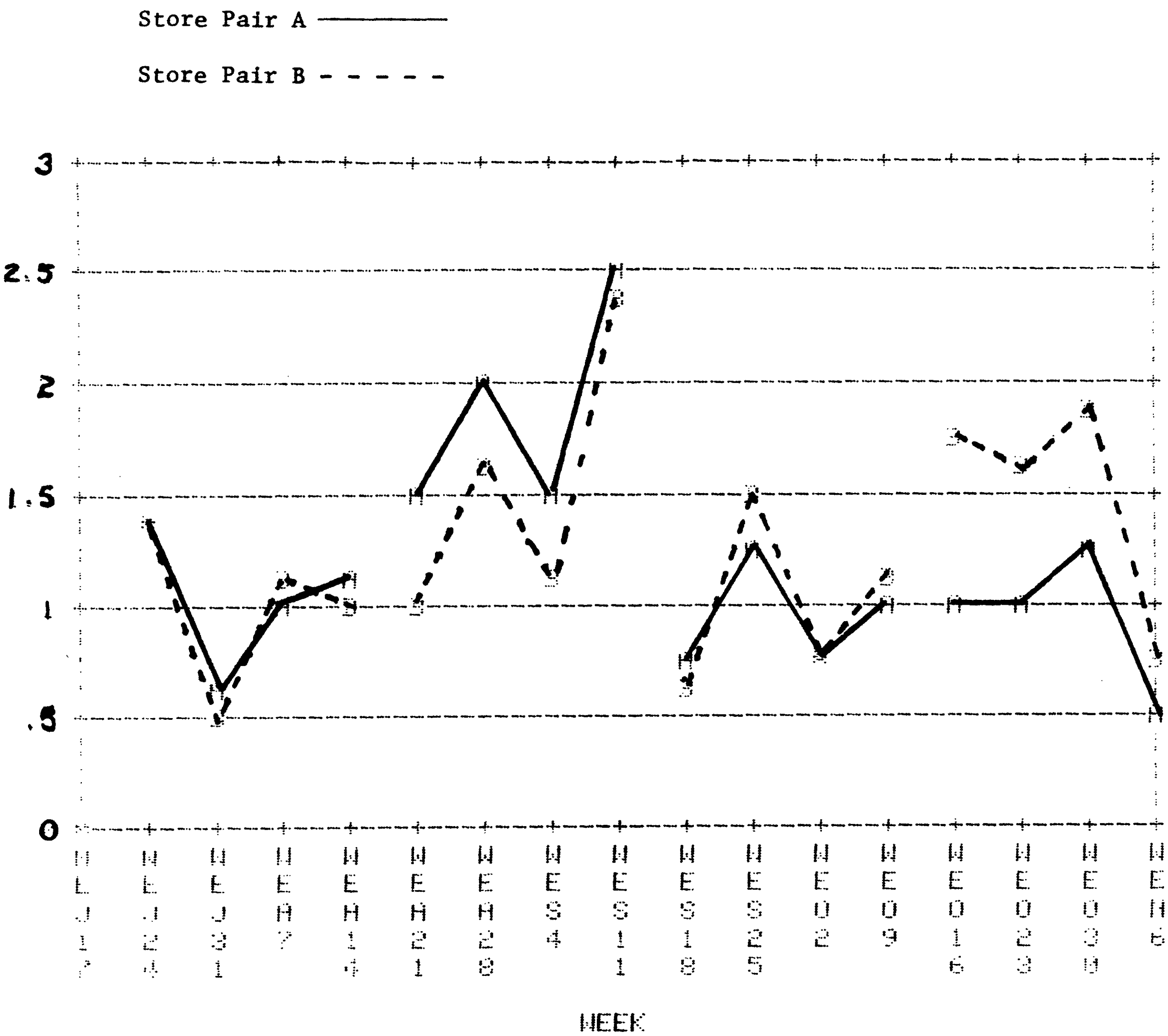


Fig. 5. Effect of price changes on sales of selected frozen orange juice items. In the first and third 4-week periods normal prices were in effect. In the second and fourth periods the two pairs of stores had different prices, switching roles between the two cases.

detectable, at least in this instance. Secondly, response appears to be quick, i.e. it shows up in the first week. Thirdly, the effect also appears to be reversible: it disappears in the normal third period and is reversed in the fourth period. Detectibility, speed of response, and reversibility are obviously all key issues in designing on-going systems. We certainly do not claim to have resolved all these issues in general; much work will have to be done but these initial results are encouraging.

6. Discussion

The theory developed here focuses on a central idea: a two-step process for relating in-store response measurements to price-setting. A variety of other considerations also enter pricing. These include competitive prices, price ending effects, price-quality perceptions, and the concept of price-quality categories to meet the needs of different market segments (e.g., good, better, best). Some of these effects might be incorporated in the theory, others are probably best left for management adjustment.

A number of theoretical issues deserve further investigation. One of these is disaggregation of the customers. Another is the explicit consideration of competition: as competing stores vary their customer utility constant, what happens to share and profit?

However, the main results have already identified a theory which shows promise of providing useful pricing calculations based on empirical measurements and a policy parameter.