On the BRST Quantization of the Massless Bosonic Particle in Twistor–Like Formulation.

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Abstract

We study some features of bosonic particle path–integral quantization in a twistor–like approach by use of the BRST–BFV quantization prescription. In the course of the Hamiltonian analysis we observe links between various formulations of the twistor–like particle by performing a conversion of the Hamiltonian constraints of one formulation to another. A particular feature of the conversion procedure applied to turn the second–class constraints into the first–class constraints is that the simplest Lorentz–covariant way to do this is to convert a full mixed set of the initial first– and second–class constraints rather than explicitly extracting and converting only the second–class constraints. Another novel feature of the conversion procedure applied below is that in the case of the $D = 4$ and $D = 6$ twistor–like particle the number of new auxiliary Lorentz–covariant coordinates, which one introduces to get a system of first–class constraints in an extended phase space, exceeds the number of independent second–class constraints of the original dynamical system.

We calculate the twistor–like particle propagator in $D = 3, 4$ and 6 space–time dimensions and show, that it coincides with that of a conventional massless bosonic particle.

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1 Introduction

In the last decades there has been an intensive activity in studying (super)particles and (super) strings by use of different approaches aimed at finding a formulation, which would be the most appropriate for performing the covariant quantization of the models. Almost all of the approaches use twistor variables in one form or another [1] – [18]. This allowed one to better understand the geometrical and group–theoretical structure of the theory and to carry out a covariant Hamiltonian analysis (and in some cases even the covariant quantization) of (super)particle and (super)string dynamics in space–time dimensions $D = 3, 4, 6$ and 10, where conventional twistor relations take place.

It has been shown that twistor–like variables appear in a natural way as superpartners of Grassmann spinor coordinates in a doubly supersymmetric formulation [19] of Casalbuoni–Brink–Schwarz superparticles and Green–Schwarz superstrings [20], the notorious fermionic κ–symmetry [21] of these models being replaced by more fundamental local supersymmetry on the worldsheet supersurface swept by the superparticles and superstrings in target superspace [5]. This has solved the problem of infinite reducibility of the fermionic constraints associated with κ–symmetry [a]. As a result new formulation and methods of quantization of $D = 4$ compactifications of superstrings with manifest target–space supersymmetry have been developed (see [22] for a review). However, the complete and simple solution of the problem of $SO(1, D − 1)$ covariant quantization of twistor–like superparticles and superstrings in $D > 4$ is still lacking.

To advance in solving this problem one has to learn more on how to deal with twistor–like variables when performing the Hamiltonian analysis and the quantization of the models. In this respect a bosonic relativistic particle in a twistor–like formulation may serve as the simplest but rather nontrivial toy model.

The covariant quantization of the bosonic particle has been under intensive study with both the operator and path–integral method [2, 23, 24, 25, 26, 7, 12, 13]. In the twistor–like approach the bosonic particle has been mainly quantized by use of the operator formalism. For that different but classically equivalent twistor–like particle actions have been considered [2, 3, 7, 12, 13].

The aim of the present paper is to study some features of bosonic particle path–integral quantization in the twistor–like approach by use of the BRST–BFV quantization prescription [27] – [29]. In the course of the Hamiltonian analysis we shall observe links between various formulations of the twistor–like particle [2, 3, 5] by performing a conversion of the Hamiltonian constraints of one formulation to another. A particular feature of the conversion procedure [30] applied to turn the second–class constraints into the first–class constraints is that the simplest Lorentz–covariant way to do this is to convert a full mixed set of the initial first– and second–class constraints rather than explicitly extracting and

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[a]A comprehensive list of references on the subject the reader may find in [18]

converting only the second–class constraints. Another novel feature of the conversion procedure applied below (in comparison with the conventional one [29, 30]) is that in the case of the $D = 4$ and $D = 6$ twistor–like particle the number of new auxiliary Lorentz–covariant coordinates, which one introduces to get a system of first–class constraints in an extended phase space, exceeds the number of independent second–class constraints of the original dynamical system, (but because of an appropriate amount of the first–class constraints we finally get, the number of physical degrees of freedom remains the same).

In Section 2 the classical mechanics of a twistor–like bosonic particle in D=3,4 and 6 is considered. The Hamiltonian analysis of the constraints accompanied by the conversion procedure is carried out and a classical BRST charge is constructed by introducing ghosts corresponding to a set of the first–class constraints obtained as a result of conversion.

In Section 3 the problem of admissible gauge choice for variables describing the matter–ghost system of the model is discussed.

In Section 4 we perform the path–integral quantization of the model in $D = 3, 4$ and 6 space–time dimensions using the extended BRST scheme [28]. We calculate the particle propagator and show, that it coincides with that of the massless bosonic particle. At the end of this Section we make a comment on problems of the D=10 case.

Notation. We use the following signature for the space-time metrics: $(+, -, ..., -)$.

2 Classical Hamiltonian dynamics and the BRST-charge.

2.1 Preliminaries

The dynamics of a massless bosonic particle in D=3,4,6 and 10 space–time can be described by the action [2]

$$S = \frac{1}{2} \int d\tau \, \dot{x}^m (\bar{\lambda} \gamma_m \lambda), \tag{1}$$

where $x^m(\tau)$ is a particle space–time coordinate, $\lambda^\alpha(\tau)$ is an auxiliary bosonic spinor variable, the dot stands for the time derivative $\frac{\partial}{\partial \tau}$ and γ^m are the Dirac matrices.

The derivation of the canonical momenta [b] $P_m^{(x)} = \frac{\partial L}{\partial \dot{x}^m}$, $P_\alpha^{(\lambda)} = \frac{\partial L}{\partial \dot{\lambda}^\alpha}$ results in a set of primary constraints

$$\Psi_m = P_m^{(x)} - \frac{1}{2}(\bar{\lambda} \gamma_m \lambda) \approx 0,$$

$$P_\alpha^{(\lambda)} \approx 0. \tag{2}$$

They form the following algebra with respect to the Poisson brackets[c]

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[b] In what follows $P^{(..)}$ denotes the momentum conjugate to the variable in the brackets

[c] The canonical Poisson brackets are

$$[P_m^{(x)}, x^n]_P = \delta_m^n; \qquad [P_\alpha^{(\lambda)}, \lambda^\beta]_P = \delta_\alpha^\beta$$

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$$[\Psi_m, \Psi_n]_P = 0, \quad [P^{(\lambda)}_\alpha, P^{(\lambda)}_\beta]_P = 0, \quad [\Psi_m, P^{(\lambda)}_\alpha]_P = (\gamma_m \lambda)_\alpha. \tag{3}$$

One can check that new independent secondary constraints do not appear in the model. In general, Eqs. (2) are a mixture of first– and second–class constraints. The operator quantization of this dynamical system in $D = 4$ (considered previously in [7, 12]) was based on the Lorentz–covariant splitting of the first– and second–class constraints and on the subsequent reduction of the phase space (either by explicit solution of the second–class constraints [12] or, implicitly, by use of the Dirac brackets [7]), while in [11, 13] a conversion prescription [29, 30] was used. The latter consists in the extension of the phase space of the particle coordinates and momenta with auxiliary variables in such a way, that new first–class constraints replace the original second–class ones. Then the initial system with the second–class constraints is treated as a gauge fixing of a "virtual" [29] gauge symmetry generated by the additional first–class constraints of the extended system [29, 30]. This is achieved by taking the auxiliary conversion degrees of freedom to be zero or expressed in terms of initial variables of the model.

The direct application of this procedure can encounter some technical problems for systems, where the first– and second–class constraints form a complicated algebra (see, for example, constraints of the $D = 10$ superstring in a Lorentz–harmonic formulation [14]). Moreover, in order to perform the covariant separation of the first– and second–class constraints in the system under consideration it is necessary either to introduce one more independent auxiliary bosonic spinor μ_α (the second component of a twistor $Z^A = (\lambda^\alpha, \mu_\alpha)$ [1]) or to construct the second twistor component from the variables at hand by use of a Penrose relation [1] $\bar{\mu}^{\dot\alpha} = i x^{\alpha\dot\alpha} \lambda_\alpha$ $(D = 4)$, $\mu^\alpha = x^{\alpha\beta} \lambda_\beta$ $(D = 3)$. In the latter case the structure of the algebra of the first– and second–class constraints separated this way [6, 7] makes the conversion procedure rather cumbersome. To elude this one can try to simplify the procedure by converting into the first class the whole set (2) of the mixed constraints. The analogous trick was used to convert fermionic constraints in superparticle models [3, 31].

Upon carrying out the conversion procedure we get a system characterized by the set of first–class constraints T_i that form (at least on the mass shell) a closed algebra with respect to the Poisson brackets defined for all the variables of the modified phase space. In order to perform the BRST–BFV quantization procedure we associate with each constraint of Grassmann parity ϵ the pair of canonical conjugate auxiliary variables (ghosts) η_i, $P_i^{(\eta)}$ with Grassmann parity $\epsilon + 1$ [d]. The resulting system is required to be invariant under gauge transformations generated by a nilpotent fermionic BRST charge Ω.

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[d]If the extended BRST–BFV method is used, with each constraint associated are also a Lagrange multiplier, its conjugate momentum of Grassmann parity ϵ and an antighost and its momentum of Grassmann parity $\epsilon + 1$ (see [27, 28] for details).

This invariance substitutes the gauge symmetry, generated by the first class constraints in the initial phase space. The generator Ω is found as a series in powers of ghosts

$$\Omega = \eta_i T_i + higher\ order\ terms,$$

where the structure of higher–order terms reflects the noncommutative algebraic structure of the constraint algebra [28]. Being the generator of the BRST symmetry Ω must be a dynamical invariant:

$$\dot{\Omega} = [\Omega, H]_P = 0,$$

where H is a total Hamiltonian of the system, which has the form

$$H = H_0 + [\chi, \Omega]_P. \tag{4}$$

In (4) H_0 is the initial Hamiltonian of the model and χ is a gauge fixing fermionic function whose form is determined by admissible gauge choices [23, 25, 26, 32, 13] (see Section 3 for the discussion of this point).

Upon quantization Ω and H become operators acting on quantum state vectors. The physical sector of the model is singled out by the requirement that the physical states are BRST invariant and vanish under the action of Ω. Another words, we deal with a *quantum gauge theory.*

When the gauge is fixed, we remain only with physically nonequivalent states, and the Hamiltonian H is argued to reproduce the correct physical spectrum of the quantum theory.

When the model is quantized by the path–integral method, we also deal with a quantum gauge theory. The Hamiltonian (4) is used to construct an effective action and a corresponding BRST-invariant generating functional which allows one to get transition amplitudes between physical states of the theory.

Below we consider the conversion procedure and construct the BRST charge for the twistor–like particle model in dimensions $D = 3, 4$ and 6.

2.2 D=3

In $D = 3$ the action (1) is rewritten as

$$S = \frac{1}{2} \int d\tau \lambda^\alpha \dot{x}_{\alpha\beta} \lambda^\beta, \tag{5}$$

where λ^α is a real two-component commuting spinor (spinor indices are risen and lowered by the unit antisymmetric tensor $\epsilon_{\alpha\beta}$) and $x_{\alpha\beta} = x_m \gamma^m_{\alpha\beta}$.

The system of primary constraints (2)

$$\Psi_{\alpha\beta} = P^{(x)}_{\alpha\beta} - \lambda_\alpha \lambda_\beta \approx 0,$$
$$P^{(\lambda)}_\alpha \approx 0, \tag{6}$$

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is a mixture of a first–class constraint generating the τ–reparametrization transformations of x

$$\phi = \lambda^\alpha P^{(x)}_{\alpha\beta} \lambda^\beta$$

and four second–class constraints

$$(\lambda P^{(\lambda)}), \quad (\mu P^{(x)} \mu) - (\lambda\mu)^2, \quad (\mu P^{(\lambda)}), \quad (\lambda P^{(x)} \mu), \tag{7}$$

where $\mu^\alpha = x^{\alpha\beta} \lambda_\beta$ (see [7] for details).

In order to perform a conversion of (6) into a system of first–class constraints we introduce a pair of canonical conjugate bosonic spinors $(\zeta^\alpha, P^{(\zeta)}_\beta)$, $[P^{(\zeta)}_\beta, \zeta^\alpha]_P = \delta^\alpha_\beta$, and take the modified system of constraints, which is of the first class, in the following form:

$$\Psi'_{\alpha\beta} = P^{(x)}_{\alpha\beta} - (\lambda_\alpha - \zeta_\alpha)(\lambda_\beta - \zeta_\beta) \approx 0,$$
$$\Phi'_\alpha = P^{(\lambda)}_\alpha + P^{(\zeta)}_\alpha \approx 0. \tag{8}$$

Eqs. (8) reduce to (6) by putting the auxiliary variables ζ^α and $P^{(\zeta)}_\alpha$ equal to zero. This reflects the appearance in the model of a new gauge symmetry with respect to which ζ^α and $P^{(\zeta)}_\alpha$ are pure gauge degrees of freedom.

It is convenient to choose the following phase–space variables as independent ones:

$$v^\alpha = \lambda^\alpha - \zeta^\alpha, \qquad P^{(v)}_\alpha = \frac{1}{2}(P^{(\lambda)}_\alpha - P^{(\zeta)}_\alpha),$$
$$w^\alpha = \lambda^\alpha + \zeta^\alpha, \qquad P^{(w)}_\alpha = \frac{1}{2}(P^{(\lambda)}_\alpha + P^{(\zeta)}_\alpha), \tag{9}$$

Then Eqs. (8) take the following form

$$\Psi'_{\alpha\beta} = P^{(x)}_{\alpha\beta} - v_\alpha v_\beta \approx 0,$$
$$P^{(w)}_\alpha \approx 0. \tag{10}$$

These constraints form an Abelian algebra.

One can see that w^α variables do not enter the constraint relations, and their conjugate momenta are zero. Hence, the quantum physical states of the model will not depend on w^α.

Enlarging the modified phase space with ghosts, antighosts and Lagrange multipliers in accordance with the following table

Constraint	Ghost	Antighost	Lagrange multiplier
$\Psi'_{\alpha\beta}$	$c^{\alpha\beta}$	$\tilde{c}^{\alpha\beta}$	$e^{\alpha\beta}$
$P^{(w)}_\alpha$	b^α	$\tilde{b}^\alpha$	f^α

we write the classical BRST charges [27, 28] of the model in the minimal and extended BRST–BFV version as follows

$$\Omega_{min} = c^{\alpha\beta} \Psi'_{\beta\alpha} + b^\alpha P^{(w)}_\alpha, \tag{11}$$

$$\Omega = P^{(\tilde{c})}_{\alpha\beta} P^{(e)\beta\alpha} + P^{(\tilde{b})\alpha} P^{(f)}_\alpha + \Omega_{min}. \tag{12}$$

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2.3 D=4

In this dimension we use two–component $SL(2, C)$ spinors ($\lambda^\alpha = \epsilon^{\alpha\beta}\lambda_\beta$; $\bar{\lambda}^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar{\lambda}_{\dot\beta}$; $\alpha, \dot\alpha = 1, 2$; $\epsilon^{12} = -\epsilon_{21} = 1$). Other notation coincides with that of the $D = 3$ case. Then in $D = 4$ the action (1) can be written as following

$$S = \frac{1}{2} \int d\tau \lambda^\alpha \dot{x}_{\alpha\dot\alpha} \bar{\lambda}^{\dot\alpha}, \tag{13}$$

where $x_{\alpha\dot\alpha} = x_m \sigma^m_{\alpha\dot\alpha}$, and $\sigma^m_{\alpha\dot\alpha}$ are the relativistic Pauli matrices. The set of the primary constraints (2) in this dimension

$$\Psi_{\alpha\dot\alpha} = P^{(x)}_{\alpha\dot\alpha} - \bar{\lambda}_{\dot\alpha}\lambda_\alpha \approx 0,$$

$$P^{(\lambda)}{}_\alpha \approx 0, \tag{14}$$

$$\bar{P}^{(\bar{\lambda})}_{\dot\alpha} \approx 0$$

contains two first–class constraints and three pairs of conjugate second–class constraints [7, 6]. One of the first class constraints generates the τ-reparametrization transformations of $x^{\dot\alpha\alpha}$

$$\phi = \lambda^\alpha P^{(x)}_{\alpha\dot\alpha} \bar{\lambda}^{\dot\alpha}$$

and another one generates $U(1)$ rotations of the complex spinor variables

$$U = i(\lambda^\alpha P^{(\lambda)}_\alpha - \bar{\lambda}^{\dot\alpha} \bar{P}^{(\lambda)}_{\dot\alpha}). \tag{15}$$

The form of the second–class constraints is analogous to that in the D=3 case (see Eq. (7) and [7]), and we do not present it explicitly since it is not used below.

To convert the mixed system of the constraints (14) into first–class constraints one should introduce at least three pairs of canonical conjugate auxiliary bosonic variables, their number is to be equal to the number of the second–class constraints in (14). However, since we do not want to violate the manifest Lorentz invariance, and the $D = 4$ Lorentz group does not have three–dimensional representations, we are to find a way round. We introduce two pairs of canonical conjugate conversion spinors $(\zeta^\alpha, P^{(\zeta)}_\alpha)$, $[\zeta^\alpha, P^{(\zeta)}_\beta]_P = -\delta^\alpha_\beta$, $[\bar{\zeta}^{\dot\alpha}, \bar{P}^{(\bar{\zeta})}_{\dot\beta}]_P = -\delta^{\dot\alpha}_{\dot\beta}$, (i.e. four pairs of real auxiliary variables) and modify the constraints (14) and the $U(1)$ generator, which becomes an independent first–class constraint in the enlarged phase space. Thus we get the following system of the first–class constraints:

$$\Psi'_{\alpha\dot\alpha} = P^{(x)}_{\alpha\dot\alpha} - (\bar{\lambda} - \bar{\zeta})_{\dot\alpha}(\lambda - \zeta)_\alpha \approx 0,$$

$$\Phi_\alpha = P^{(\lambda)}_\alpha + P^{(\zeta)}_\alpha \approx 0, \tag{16}$$

$$\bar{\Phi}_{\dot\alpha} = \bar{P}^{(\bar{\lambda})}_{\dot\alpha} + \bar{P}^{(\bar{\zeta})}_{\dot\alpha} \approx 0,$$

$$U = i(\lambda^\alpha P^{(\lambda)}_\alpha + \zeta^\alpha P^{(\zeta)}_\alpha - \bar{\lambda}^{\dot\alpha} \bar{P}^{(\bar{\lambda})}_{\dot\alpha} - \bar{\zeta}^{\dot\alpha} \bar{P}^{(\bar{\zeta})}_{\dot\alpha}) \approx 0.$$

One can see (by direct counting), that the number of independent physical degrees of freedom of the particle in the enlarged phase space is the same as in the initial one. The latter is recovered by imposing gauge fixing conditions on the new auxiliary variables

$$\zeta^\alpha = 0, \qquad \bar\zeta^{\dot\alpha} = 0, \qquad P_\alpha^{(\zeta)} = 0, \qquad P_{\dot\alpha}^{(\bar\zeta)} = 0. \qquad (17)$$

By introducing a new set of the independent spinor variables analogous to that in (9) one rewrites Eqs. (16) as follows

$$\Psi'_{\alpha\dot\alpha} = P_{\alpha\dot\alpha}^{(x)} - v_\alpha \bar v_{\dot\alpha} \approx 0,$$

$$U = i(P_\alpha^{(v)} v^\alpha - P_{\dot\alpha}^{(\bar v)} \bar v^{\dot\alpha}) \approx 0,$$

$$P^{(w)}{}_\alpha \approx 0, \qquad (18)$$

$$P_{\dot\alpha}^{(\bar w)} \approx 0.$$

Again, as in the $D = 3$ case, w_α, $\bar w_{\dot\alpha}$ and their momenta decouple from the first pair of the constraints (18), and can be completely excluded from the number of the dynamical degrees of freedom by putting

$$w_\alpha = \lambda_\alpha + \zeta_\alpha = 0, \qquad P^{(w)}{}_\alpha = \frac{1}{2}(P^{(\lambda)}{}_\alpha + P^{(\zeta)}{}_\alpha) = 0 \qquad (19)$$

in the strong sense. This gauge choice, which differs from (17), reduces the phase space of the model to that of a version of the twistor–like particle dynamics, subject to the first pair of the first–class constraints in (18), considered by Eisenberg and Solomon [3]. The constraints (18) form an abelian algebra, as in the $D = 3$ case. In compliance with the BRST–BFV prescription we introduce ghosts, antighosts and Lagrange multipliers associated with the constraints (18) as follows

Constraint	Ghost	Antighost	Lagrange multiplier
$\Psi'_{\alpha\dot\alpha}$	$c^{\dot\alpha\alpha}$	$\tilde c^{\dot\alpha\alpha}$	$e^{\dot\alpha\alpha}$
U	a	$\tilde a$	g
$P_\alpha^{(w)}$	b^α	$\tilde b^\alpha$	f^α
$P_{\dot\alpha}^{(\bar w)}$	$\bar b^{\dot\alpha}$	$\tilde{\bar b}^{\dot\alpha}$	$\bar f^{\dot\alpha}$

Then the BRST–charges of the $D = 4$ model have the form

$$\Omega_{min} = c^{\dot\alpha\alpha}\Psi_{\alpha\dot\alpha} + b^\alpha P_\alpha^{(w)} + \bar b^{\dot\alpha} P_{\dot\alpha}^{(\bar w)} + aU, \qquad (20)$$

$$\Omega = P_{\alpha\dot\alpha}^{(\tilde c)} P^{(e)\dot\alpha\alpha} + P_\alpha^{(\tilde b)} P^{(f)\alpha} + P_{\dot\alpha}^{(\tilde{\bar b})} P^{(\bar f)\dot\alpha} + P^{(\tilde a)} P^{(g)} + \Omega_{min}. \qquad (21)$$

2.4 D=6

In $D = 6$ a light–like vector V^m can be represented in terms of commuting spinors as follows

$$V^m = \lambda_i^\alpha \gamma_{\alpha\beta}^m \lambda^{\beta i},$$

where λ_i^α is an $SU(2)$–Majorana–Weyl spinor which has the $SU^*(4)$ index $\alpha = 1, 2, 3, 4$ and the $SU(2)$ index $i = 1, 2$. $\gamma_{\alpha\beta}^m$ are $D = 6$ analogs of the Pauli matrices (see [33, 8]). $SU(2)$ indices are risen and lowered by the unit antisymmetric tensors ϵ_{ij}, ϵ^{ij}. As to the $SU^*(4)$ indices, they can be risen and lowered only in pairs by the totally antisymmetric tensors $\epsilon_{\alpha\beta\gamma\delta}$, $\epsilon^{\alpha\beta\gamma\delta}$ ($\epsilon_{1234} = 1$).

Rewriting the action (1) in terms of $SU(2)$–Majorana–Weyl spinors, one gets

$$S = \frac{1}{2} \int d\tau \dot{x}^m \lambda_i^\alpha (\gamma_m)_{\alpha\beta} \lambda^{\beta i}. \tag{22}$$

The system of the primary constraints (2) takes the form

$$\Psi_{\alpha\beta} = P_{\alpha\beta}^{(x)} - \epsilon_{\alpha\beta\gamma\delta} \lambda_i^\gamma \lambda^{\delta i} \approx 0,$$

$$P^{(\lambda)i}{}_\alpha \approx 0, \tag{23}$$

where $P_{\alpha\beta}^{(x)} = P_m^{(x)} \gamma_{\alpha\beta}^m$. $\Psi_{\alpha\beta}$ is antisymmetric in α and β and contains six independent components. (To get (23) we used the relation $(\gamma_m)_{\alpha\beta} \gamma_{\gamma\delta}^m \sim \epsilon_{\alpha\beta\gamma\delta}$).

From Eqs. (23) one can separate four first–class constraints by projecting (23) onto λ_i^α [6, 8]. One of the first–class constraints generates the τ–reparametrizations of $x^{\alpha\beta}$

$$\phi = \lambda_i^\alpha P_{\alpha\beta}^{(x)} \lambda^{\beta i},$$

and another three ones form an $SU(2)$ algebra

$$T_{ij} = \lambda_{(i}^\alpha P_{\alpha j)}^{(\lambda)},$$

Braces denote the symmetrization of i and j. All other constraints in (23) are of the second class.

The conversion of (23) into first–class constraints is carried out by analogy with the $D = 4$ case. According to the conventional conversion prescription we had to introduce five pairs of canonical conjugate bosonic variables. Instead, in order to preserve Lorentz invariance, we introduce the canonical conjugate pair of bosonic spinors ζ_j^β, $P_\alpha^{(\zeta)i}$ ($[P_\alpha^{(\zeta)i}, \zeta_j^\beta]_P = \delta_\alpha^\beta \delta_j^i$,) modify the constraints (23) and the $SU(2)$ generators. This results in the set of independent first–class constraints

$$\Psi'_{\alpha\beta} = P_{\alpha\beta}^{(x)} - \epsilon_{\alpha\beta\gamma\delta} (\lambda_i^\gamma - \zeta_i^\gamma)(\lambda^{\delta i} - \zeta^{\delta i}) \approx 0,$$

$$\Phi_\alpha{}^i = P_\alpha^{(\lambda)i} + P_\alpha^{(\zeta)i} \approx 0, \tag{24}$$

$$T_{ij} = \lambda^{\alpha}_{(i} P^{(\lambda)}_{\alpha j)} - \zeta^{\alpha}_{(i} P^{(\zeta)}_{\alpha j)} \approx 0.$$

In terms of spinors v^{α}_i and w^{α}_i, and their momenta, defined as in the $D = 3$ case (9), they take the following form

$$\Psi'_{\alpha\beta} = P^{(x)}_{\alpha\beta} - \epsilon_{\alpha\beta\gamma\delta} v^{\gamma}_i v^{\delta i} \approx 0,$$

$$T_{ij} = v^{\alpha}_{(i} P^{(v)}_{\alpha j)} \approx 0, \tag{25}$$

$$P^{(w)i}_{\alpha} \approx 0.$$

These constraints form a closed algebra with respect to the Poisson brackets. The only nontrivial brackets in this algebra are

$$[T_{ij}, T_{kl}]_p = \epsilon_{jk} T_{il} + \epsilon_{il} T_{jk} + \epsilon_{ik} T_{jl} + \epsilon_{jl} T_{ik}, \tag{26}$$

which generate the $SU(2)$ algebra.

We introduce ghosts, antighosts and Lagrange multipliers related to the constraints (26)

Constraint	Ghost	Antighost	Lagrange multiplier
$\Psi'_{\alpha\beta}$	$c^{\alpha\beta}$	$\tilde{c}_{\alpha\beta}$	$e^{\alpha\beta}$
T_{ij}	a^{ij}	$\tilde{a}_{ij}$	g^{ij}
Φ^i_{α}	b^{α}_i	$\tilde{b}^i_{\alpha}$	f^{α}_i

and construct the BRST charges corresponding respectively, to the minimal and extended BRST–BFV version, as follows

$$\Omega_{min} = c^{\alpha\beta} \Psi'_{\beta\alpha} + b^{\alpha}_i P^{(w)i}_{\alpha} + a^{ij} T_{ji} + \tag{27}$$

$$(\epsilon_{jk} P^{(a)}_{il} + \epsilon_{il} P^{(a)}_{jk} + \epsilon_{ik} P^{(a)}_{jl} + \epsilon_{jl} P^{(a)}_{ik}) a^{ij} a^{kl}.$$

$$\Omega = P^{(\tilde{c})}_{\alpha\beta} P^{(e)\beta\alpha} + P^{(\tilde{b})\alpha}_i P^{(f)i}_{\alpha} + P^{(\tilde{a})ij} P^{(g)}_{ji} + \Omega_{min}, \tag{28}$$

Higher order terms in ghost powers appear in (27) and (28) owing to the noncommutative $SU(2)$ algebra of the T_{ij} constraints (26).

3   Admissible gauge choice.

One of the important problems in the quantization of gauge systems is a correct gauge choice. In the frame of the BRST–BFV quantization scheme gauge fixing is made by an appropriate choice of the gauge fermion that determines the structure of the quantum Hamiltonian. The Batalin and Vilkovisky theorem [27, 28] reads that the result of path integration does not depend on the choice of the gauge fermions if they belong to the same equivalence class with respect to the BRST–transformations. An analogous theorem

takes place in the operator BRST–BFV quantization scheme [26]. Further analysis of this problem for systems possessing the reparametrization invariance showed that the result of path integration does not depend on the choice of the gauge fermion if only appropriate gauge conditions are compatible with the boundary conditions for the parameters of the corresponding gauge transformations [24, 25, 26, 32, 13]. In particular, it was shown that the so–called "canonical gauge", when the worldline gauge field of the reparametrization symmetry of the bosonic particle is fixed to be a constant, is not admissible in this sense. (see [25, 13] for details). Anyway one can use the canonical gauge as a consistent limit of an admissible gauge [26].

Making the analysis of the twistor–like model one can show that admissible are the following gauge conditions on Lagrange multipliers from the corresponding Tables of the previous section in the dimensions $D = 3$, 4 and 6 of space–time, respectively,

$$D = 3 : \qquad \dot{e}^{\alpha\beta} = 0; \qquad f^{\alpha} = 0; \tag{29}$$

$$D = 4 : \qquad \dot{e}^{\alpha\dot{\beta}} = 0; \qquad f^{\alpha} = 0; \qquad f^{\dot{\alpha}} = 0; \qquad g = 0; \tag{30}$$

$$D = 6 : \qquad \dot{e}^{\alpha\beta} = 0; \qquad f_i^{\alpha} = 0; \qquad g^{ij} = 0; \tag{31}$$

The canonical gauge

$$e = constant, \tag{32}$$

can be considered as a limit of more general admissible gauge $e - \varepsilon\dot{e} = constant$ (at $\varepsilon \to 0$) [26]. Then the use of the gauge condition (32) does not lead to any problems with the operator BRST–BFV quantization.

Below we shall use the "relativistic" gauge conditions (29), (30) and (31) for the path–integral quantization. The use of the canonical gauge (32) in this case would lead to a wrong form of the particle propagator.

4 Path–integral BRST quantization.

In this section we shall use the extended version of the BRST–BFV quantization procedure [28, 29] and fix the gauge by applying the conditions (29), (30), (31). The gauge fermion, corresponding to this gauge choice, is

$$\chi_D = \frac{1}{2} P_m^{(c)} e^m, \quad D = 3, 4, 6, \tag{33}$$

The Hamiltonians constructed with (33) are [27, 28]

$$H_D = [\Omega_D, \chi_D], \qquad D = 3, 4, 6$$

$$H_3 = e^m (P_m^{(x)} - \frac{1}{2} v^{\alpha} (\gamma_m)_{\alpha\beta} v^{\beta}) - P_m^{(c)} P^{(\tilde{c})m}, \tag{34}$$

$$H_4 = e^m(P_m^{(x)} - \frac{1}{2}\bar{v}^{\dot{\alpha}}(\sigma_m)_{\dot{\alpha}\alpha}v^{\alpha}) - P_m^{(c)}P^{(\tilde{c})m}, \qquad (35)$$

$$H_6 = e^m(P_m^{(x)} - \frac{1}{2}v_i^{\alpha}(\gamma_m)_{\alpha\beta}v^{\beta i}) - P_m^{(c)}P^{(\tilde{c})m}, \qquad (36)$$

We shall calculate the coordinate propagator $Z = \langle x_1^m \mid U_0 \mid x_2^m \rangle$ (where $U_0 = expiH(T_1 - T_2)$ is the evolution operator), therefore boundary conditions for the phase space variables are fixed as follows:

$$x^m(T_1) = x_1^m, \qquad x^m(T_2) = x_2^m, \qquad (37)$$

the boundary values of the ghosts, antighosts and canonical momenta of the Lagrange multipliers are put equal to zero (which is required by the BRST invariance of the boundary conditions [28]), and we sum up over all possible values of the particle momentum and the twistor variables.

The standard expression for the matrix element of the evolution operator is

$$Z_D = \int [D\mu DP^{\mu}]_D exp(i\int_{T_1}^{T_2} d\tau([P^{\mu}\dot{\mu}]_D - H_D)), \qquad D = 3, 4, 6. \qquad (38)$$

$[D\mu DP^{\mu}]_D$ contains functional Liouville measures of all the canonical variables of the BFV extended phase space [27]. $[P^{\mu}\dot{\mu}]_D$ contains a sum of products of the canonical momenta with the velocities.

For instance, an explicit expression for the path–integral measure in the $D = 3$ case is

$$[D\mu DP^{\mu}] = DxDP^{(x)}DvDP^{(v)}DwDP^{(w)}DeDP^{(e)}DfDP^{(f)}$$

$$DbDP^{(b)}DcDP^{(c)}D\tilde{b}DP^{(\tilde{b})}D\tilde{c}DP^{(\tilde{c})}.$$

We can perform straightforward integration over the all variables that are not present in the Hamiltonians (34), (35), (36) [e]. Then (38) reduces to the product of two terms

$$Z_D = I_D G_D, \qquad (39)$$

where

$$G_D = \int DcDP^{(c)}D\tilde{c}DP^{(\tilde{c})}exp(i\int_{T_1}^{T_2} d\tau(P_m^{(c)}\dot{c}^m + P_m^{(\tilde{c})}\dot{\tilde{c}}^m - \frac{1}{2}P_m^{(\tilde{c})}P^{(c)m})), \qquad (40)$$

and I_D includes the integrals over bosonic variables entering (34), (35), (36) together with their conjugated momenta. We use the method analogous to that in [34] for computing these integrals.

The calculation of the ghost integral G_D results in

$$G_D = (\Delta T)^D, \qquad \Delta T = T_2 - T_1, \qquad D = 3, 4, 6. \qquad (41)$$

---

[e] All calculations are done up to a multiplication constant, which can always be absorbed by the integration measure.

Let us demonstrate main steps of the I_D calculation in the $D = 3$ case

$$
\begin{aligned}
I_3 &= \int DxDP^{(x)}DeDP^{(e)}DvDP^{(v)}exp(i\int_{T_1}^{T_2} d\tau(P_m^{(x)}\dot{x}^m + P_m^{(e)}\dot{e}^m + P_\alpha^{(v)}\dot{v}^\alpha \\
&\quad -e^m(P_m^{(x)} - \frac{1}{2}v^\alpha(\gamma_m)_{\alpha\beta}v^\beta))
\end{aligned}
\tag{42}
$$

Integration over $P_m^{(e)}$ and $P_m^{(v)}$ results in the functional δ-functions $\delta(\dot{e})$, $\delta(\dot{v})$ which reduce functional integrals over e^m and v^α to ordinary ones:

$$
I_3 = \int DxDP^{(x)}d^3ed^2v \; exp(ip_m\Delta x^m - i\int_{T_1}^{T_2} d\tau(x^m\dot{P}_m^{(x)} + e^m(P_m^{(x)} - \frac{1}{2}v^\alpha(\gamma_m)_{\alpha\beta}v^\beta)),
\tag{43}
$$

where $\Delta x^m = x_2^m - x_1^m$ (37). Since the integral over v^α is a usual Gauss integral after integrating over x^m and v^α one obtains

$$
I_3 = \int d^3pd^3e\frac{1}{\sqrt{e^me_m - i0}}exp(i(p_m\Delta x^m - e^mp_m\Delta T)).
\tag{44}
$$

In general case of $D = 3$, 4 and 6 dimensions, one obtains

$$
I_D = \int d^Dpd^De\frac{1}{(e^me_m - i0)^{\frac{D-2}{2}}}exp(i(p_m\Delta x^m - e^mp_m\Delta T)),
\tag{45}
$$

that can be rewritten as

$$
I_D = \int d^Dpd^De\int_0^\infty dc \; exp(i(p_m\Delta x^m - e^mp_m\Delta T + (e^me_m - i0)c^{\frac{2}{D-2}})),
\tag{46}
$$

where c is an auxiliary variable.

Integrating over p^m and e^m one gets

$$
Z_D = \int_0^\infty dc\frac{1}{c^{D/2}}exp(i\frac{\Delta x^m\Delta x_m}{2c} - c0), \qquad D = 3, 4, 6,
\tag{47}
$$

or

$$
Z_D = \frac{1}{(\Delta x^m\Delta x_m - i0)^{\frac{D-2}{2}}},
$$

which coincides with the coordinate propagator for the massless bosonic particle in the standard formulation [25].

On the other hand integrating (45) only over e^m we get the massless bosonic particle causal propagator in the form

$$
Z_D = \int d^Dp\frac{1}{p^mp_m + i0}exp(ip_m\Delta x^m).
$$

4.1   Comment on the $D = 10$ case

Above we have restricted our consideration to the space–time dimensions 3, 4 and 6. The case of a bosonic twistor–like particle in $D = 10$ is much more sophisticated. The

Cartan–Penrose representation of a $D = 10$ light–like momentum vector is constructed out of a Majorana–Weyl spinor λ^α which has 16 independent components

$$P^m = \lambda \Gamma^m \lambda. \tag{48}$$

Transformations of λ^α which leave (48) invariant take values on an S^7– sphere (see [3, 4, 13] and references therein). In contrast to the $D = 4$ and $D = 6$ case, where such transformations belong to the group $U(1) \sim S^1$ (18) and $SU(2) \sim S^3$ (24), respectively, S^7 is not a Lie group and its corresponding algebra contains structure functions instead of structure constants. Moreover, among the 10 constraints (48) and 16 constraints $P_\alpha^{(\lambda)} = 0$ on the momenta conjugate to x^m and λ^α $18 = 10 + 16 - 1 - 7$ (where 7 comes from S^7 and 1 corresponds to local τ–reparametrization) are of the second class. They do not form a representation of the Lorentz group and cause the problem for covariant Hamiltonian analysis.

One can overcome these problems in the framework of the Lorentz–harmonic formalism (see [14, 18] and references therein), where to construct a light–like vector one introduces eight Majorana–Weyl spinors instead of one λ^α. Such a spinor matrix takes values in a spinor representation of the double covering group $Spin(1, 9)$ of $SO(1, 9)$ and satisfies second–class harmonic conditions. The algebra of the constraints in this "multi–twistor" case is easier to analyze than that with only one commuting spinor involved. The path–integral BRST quantization of the $D = 10$ twistor–like particle is in progress.

5   Conclusion

In the present paper the BRST–BFV quantization of the dynamics of massless bosonic particle in $D = 3, 4, 6$ was performed in the twistor–like formulation. To this end the initially mixed system of the first– and second–class constraints was converted into the system of first–class constraints by extending the initial phase space of the model with auxiliary variables in a Lorentz–covariant way. The conversion procedure (rather than having been a formal trick) was shown to have a meaning of a symmetry transformation which relates different twistor–like formulations of the bosonic particle, corresponding to different gauge choices in the extended phase space.

We quantized the model by use of the extended BRST–BFV scheme for the path–integral quantization. As a result we have presented one of the numerous proofs of the equivalence between the twistor–like and conventional formulation of the bosonic particle mechanics.

This example demonstrates peculiar features of treating the twistor–like variables within the course of the covariant Hamiltonian analysis and the BRST quantization, which one should take into account when studying more complicated twistor–like systems, such as superparticles and superstrings.

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