Source Visualization by Using Statistically Optimized Near-Field Acoustical Holography in Conical Coordinates

J Stuart Bolton
Purdue University, bolton@purdue.edu

Yong T. Cho
Kyungil University

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SOURCE VISUALIZATION BY USING STATISTICALLY OPTIMIZED NEAR-FIELD ACOUSTICAL HOLOGRAPHY IN CONICAL COORDINATES

Yong T. Cho* and J. Stuart Bolton

Ray W. Herrick Laboratories
Purdue University

*Currently working at
School of Mechanical and Automotive Engineering,
Kyungil University, Korea
INTRODUCTION

• NAH is a useful tool for visualizing noise sources throughout a 3D space.
  - Fast since implemented using spatial Fourier transform.
  - Needs zero padding of measurement results to avoid wrap-around error.
  - Meaningless velocity results close to measurement edge due to discontinuity.

• Statistically Optimized Nearfield Acoustical Holography
  - First introduced by Jørgen Hald in planar coordinates
  - No spatial Fourier transform involved.
  - More accurate result over entire measurement area.
SONAH in Conical Coordinates

- Aeroacoustic sources are more closely defined in conical geometry
- NAH: non-regular geometries X
  SONAH: conical geometry O
- Wave functions in conical geometry are formulated by modifying cylindrical wave functions
SONAH in Conical Coordinates

- **Conical geometry definition**

Conical array parallel to surface of diverging flow

Conical array parallel to surface of diverging flow

Cylindrical array intersects flow

Rigid Boundary, $z=0$

\[ r = r_0 + z \tan \alpha \]

Reconstruction surface

Hologram

$\rho \theta \phi \alpha r z y x$
The sound pressure, $p(r)$, can be expressed as a linear combination of the measured sound pressure $p(r_n)$,

$$p(r) \approx \sum_{n=1}^{N} c_n(r) p(r_n)$$

If a good representation of the sound field can be obtained by using a finite subset of wave functions, the coefficients $c_n$ can be determined.

$$\Phi_{Km}(r) \approx \sum_{n=1}^{N} c_n(r) \Phi_{Km}(r_n), \quad m = 1 \ldots M$$
SONAH in Conical Coordinates

- Conical SONAH formulation (2)

\[ p(r, \phi, z) = \sum_{m=-\infty}^{\infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{H_m^{(1)}(k_r r)}{H_m^{(1)}(k_r r_s)} P_m(r_s, k_z) e^{im\phi} e^{ik_z z} dk_z \]

• Defining wave function \( \Phi_{kz,m}(r) \) in conical coordinates,

\[ \Phi_{kz,m}(r) \equiv \frac{H_m^{(1)}(k_r r)}{H_m^{(1)}(k_r r_s)} e^{im\phi} e^{ik_z z}, \quad k_r = \begin{cases} \sqrt{k^2 - k_z^2} & \text{for } |k| \geq |k_z| \\ i\sqrt{k_z^2 - k^2} & \text{for } |k| < |k_z| \end{cases} \]

\[ \Phi_{kz,m}(r) = \Phi_{kz,m}(r, \phi, z) \equiv \frac{H_m^{(1)}(k_r (r_o + z \tan \alpha))}{H_m^{(1)}(k_r r_s)} e^{im\phi} e^{ik_z z}, \quad r \geq r_s. \]
SONAH in Conical Coordinates

- Conical SONAH formulation (3)

\[
\begin{bmatrix}
A^+ A
\end{bmatrix}_{ji} = \sum_{m=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \Phi^*_{k_{zq},m}(r_{h,j}) \Phi_{k_{zq},m}(r_{h,i})
\]

\[
= \sum_{m=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \left\{ \frac{H^{* (1)}_m(k_r (r_{ho} + z_{h,j}, \tan \alpha)) H^{(1)}_m(k_r (r_{ho} + z_{h,i}, \tan \alpha))}{|H^{(1)}_m(k_r r_s)|^2} \right\}
\]

\[
e^{i(m(\phi_{h,i} - \phi_{h,j}) + k_{zq}(z_{h,i} - z_{h,j}))}
\]

\[
\begin{bmatrix}
A^+ \alpha
\end{bmatrix}_j = \sum_{m=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \Phi^*_{k_{zq},m}(r_{h,j}) \Phi_{k_{zq},m}(r)
\]

\[
= \sum_{m=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \left\{ \frac{H^{* (1)}_m(k_r (r_{ho} + z_{h,j}, \tan \alpha)) H^{(1)}_m(k_r (r_o + z_i, \tan \alpha))}{|H^{(1)}_m(k_r r_s)|^2} \right\}
\]

\[
e^{i(m(\phi_{h,j} - \phi_{h,j}) + k_{zq}(z_i - z_{h,j}))}
\]
SONAH in Conical Coordinates

- Conical SONAH formulation (4)

  • Estimated pressure \( p(r) \) is,
    \[
    p(r) \approx \sum_{n=1}^{N} c_n(r) \omega(r_n) = p^T c(r) = p^T \left( A^+ A + \theta^2 I \right)^{-1} A^+ \quad (1)
    \]
    where, \( p^T \) is measured pressure vector at \( r_n \)

  • Estimated radial particle velocity \( u_r(r) \) is,
    \[
    u_r(r) \approx p^T \left( \beta A^+ A + \theta^2 I \right)^{-1} A^+ \quad (2)
    \]
    where, \( A^+ \beta(r) \) is a correlation vector that relates measured pressure and particle velocity.
SONAH in Conical Coordinates

- Dipole numerical simulation

\[ r_{ho} = 14.15 \text{ cm} \]
\[ r_o = 9 \text{ cm} \]
\[ N_\phi = 32 \]
\[ N_z = 17 \]
\[ \alpha = 15^\circ \]
- Dipole numerical simulation (1000 Hz)
  Directly measured and backward projected pressure

Directly measured $p$

$r_{ho} = 14.15$ cm

Directly measured $p$

$r_{ho} = 9$ cm

Back projected $p$

$r_o = 9$ cm

(MSE : 0.074 %)
SONAH in Conical Coordinates

- Dipole numerical simulation (1000 Hz)
  Directly measured and backward projected particle velocity

Directly measured $u_r$

$r_{ho} = 9$ cm

Back projected $u_r$

$r_o = 9$ cm

(MSE : 0.17 %)
SONAH in Conical Coordinates

- Conical SONAH measurement

Microphone arrays and loudspeakers

\((r_o=10.6 \text{ cm}, 15.6 \text{ cm}, \alpha =15^\circ)\)

\[z_{inc} = 2 \cos \alpha \text{ cm}, \ N_\phi = 32, \ N_z = 25\]

Spatially averaged pressure

\((r_o=10.6 \text{ cm})\)
SONAH in Conical Coordinates

- Conical SONAH measurement result (1), 684 Hz
  Measured $p$, $r_o = 10.6 \text{ cm}$
  Measured $p$, $r_o = 15.6 \text{ cm}$

Back projected $p$

Back projected $u_r$

Back Projected $p$, cylindrical

Back Projected $u_r$, cylindrical
SONAH in Conical Coordinates

- Conical SONAH measurement result (2), 2648 Hz

Measured $p$, $r_0 = 10.6$ cm  
Measured $p$, $r_0 = 15.6$ cm  
Back projected $p$

Back projected $u_r$

Back Projected $p$, cylindrical

Back Projected $u_r$, cylindrical
SONAH in Conical Coordinates

- Conical SONAH mean square error
  - Directly measured and back projected pressure
    Two loudspeakers
    \( r_0 = 15.6 \text{ cm to } 10.6 \text{ cm} \) \((\alpha = 15^\circ)\)

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>684</th>
<th>976</th>
<th>1764</th>
<th>2128</th>
<th>2648</th>
<th>3288</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE (%)</td>
<td>12.44</td>
<td>13.83</td>
<td>14.81</td>
<td>15.72</td>
<td>15.72</td>
<td>12.74</td>
</tr>
</tbody>
</table>

- Directly measured and back projected pressure, velocity
  Numerical dipole simulation
  \( r_0 = 14.15 \text{ cm to } 9 \text{ cm} \) \((\alpha = 15^\circ, 1000 \text{ Hz})\)

<table>
<thead>
<tr>
<th></th>
<th>Pressure</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE (%)</td>
<td>0.074</td>
<td>0.17</td>
</tr>
</tbody>
</table>
SONAH in Conical Coordinates

- Computation effort required

Estimate $2N_\phi N_z$ by $2N_\phi N_z$ square matrices $[A^+ A]_{ji}$, $[A^+]_j$

number of elements in SONAH matrices

<table>
<thead>
<tr>
<th>Hologram</th>
<th>Square</th>
<th>Cylindrical</th>
<th>Conical</th>
<th>Arbitrary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elements</td>
<td>$N_\phi N_z$</td>
<td>$2N_\phi N_z$</td>
<td>$4N_\phi N_z^2$</td>
<td>$4N_\phi^2 N_z^2$</td>
</tr>
<tr>
<td>$N_\phi=32$, $N_z=17$</td>
<td>544*</td>
<td>1088</td>
<td>36992</td>
<td>1183744</td>
</tr>
<tr>
<td>$N_\phi=32$, $N_z=25$</td>
<td>800*</td>
<td>1600</td>
<td>80000</td>
<td>2560000</td>
</tr>
<tr>
<td>$N_\phi=32$, $N_z=34$</td>
<td>1088* (0.5)</td>
<td>2176 (1)</td>
<td>147968 (68)</td>
<td>4734976 (2176)</td>
</tr>
</tbody>
</table>
SONAH in Conical Coordinates

- Conclusions

• Conical SONAH accuracy confirmed
  Numerical simulation and loudspeaker measurement
• Reasonable to use cylindrical wave functions for conical geometry
• More detailed visualization of sources by back projection from conical to cylindrical surfaces
• Cylindrical and conical SONAH matrix quite different computation time
  conical SONAH >> cylindrical SONAH
• Finite difference calculation would be required to calculate particle velocity normal to conical surface (and hence intensity and sound power)