A simple proof of the characterization of antipodal graphs

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The antipodal graph \( A(G) \) of a graph \( G \) is defined as the graph on the same vertex set as \( G \) with two vertices being adjacent in \( A(G) \) if the distance between them in \( G \) is the diameter of \( G \). (If \( G \) is disconnected then we define \( \text{diam}(G) = \infty \).)

Aravamudhan and Rajendran [1, 2] gave the following characterization of antipodal graphs.

**Theorem 1.** A graph \( G \) is an antipodal graph if and only if it is the antipodal graph of its complement \( \overline{G} \).

The authors verified this indirectly by describing those graphs which are not antipodal graphs. In addition, they proved the following lemma which we will also use. The proof is straightforward and thus omitted.

**Lemma 2.** \( A(G) = \overline{G} \) if and only if \( \text{diam}(G) = 2 \) or if \( G \) is disconnected and the components of \( G \) are complete graphs.

In [2] these same authors observed that if \( H \) is a connected graph with \( \text{diam}(H) > 2 \), then \( A(H) = A(H') \), where \( H' \) is the graph on the same vertex set such that two vertices...
are adjacent in $H'$ if the distance between them in $H$ is less than $\text{diam}(H)$. This observation is still true when $\text{diam}(H)=2$ (for then $H'=H$) and when $H$ is disconnected. In this case, the components of $H$ and $H'$ consist of the same vertices and the edges of $A(H)$ and $A(H')$ are exactly the edges joining vertices in different components.

This extension leads to another proof of the characterization of antipodal graphs which involves showing that $A(H')=\overline{H'}$.

**Proof of Theorem 1.** First, if $G=A(\overline{G})$ then $G$ is certainly an antipodal graph. For the converse, suppose $G=A(H)$ for some graph $H$. If $\text{diam}(H)=1$, then $H$ is a complete graph, $H=G$ and $G=A(G)$.

If $2 \leq \text{diam}(H) < \infty$, define $H'$ as the graph on the vertex set of $G$ with an edge joining vertices $u$ and $v$ if $d_H(u,v) < \text{diam}(H)$. By our extended observation, $G = A(H) = A(H')$. Note also that if $H$ is connected then $\text{diam}(H') = 2$, while if $H$ is disconnected then $H'$ is disconnected and the components of $H'$ are complete graphs. In either case, $G = A(H') = \overline{H'}$ by lemma 2 so that $G = A(G)$. □

**References**
