Distance functions in multidimensional image processing applications

András Hajdu[1], Tamás Tóth[2], Krisztián Veréb[3], and Zoltán Zörgő[4]

[1] Institute of Informatics, University of Debrecen, H-4010 Debrecen PO Box 12.
hajdua@inf.unideb.hu
[2] Institute of Informatics, University of Debrecen, H-4010 Debrecen PO Box 12.
tothtam@delfin.klte.hu
[3] Institute of Informatics, University of Debrecen, H-4010 Debrecen PO Box 12.
sparrow@inf.unideb.hu
[4] Institute of Informatics, University of Debrecen, H-4010 Debrecen PO Box 12.
zorgoz@inf.unideb.hu

Abstract. In this paper we show how we can take advantage of using different distance functions in image processing applications. The proposed methods are based on well-known algorithms that use distance measurement. We focus on multidimensional image indexing and segmentation procedures, and also show examples for the extraction of such feature vectors that can be used in image retrieval. As a special family, we perform a detailed analysis using distance functions generated by neighbourhood sequences. The application of such distance functions is quite natural and descriptive for images, since e.g. the colour coordinates of the pixels are non-negative integers. An additional interesting property of neighbourhood sequences is that they do not generate metrics in general, so we can obtain many distance functions in this way. Our final purpose is to find distance functions that provide the "best" results for a given problem, so we present some tools that help with finding them.

1 Introduction

Indexing and segmenting colour images is a very important field in digital image processing with growing interest. Many of these applications are based on some distance function to calculate the difference between the colours of the pixels. Usually some classic metrics are used, though a special distance function might be more suitable according to the nature of the problem. For example, several image representations use integer colour coordinate values, thus digital distance functions also might be considered.

This observation has led to the investigation of the applicability of a family of digital distance functions in [11]. In that paper the authors showed how colour image indexing and segmentation can be achieved by applying distance functions generated by neighbourhood sequences in simple colour distance measurement, region growing, and clustering. Now, we continue the work started in [11] by introducing some analytical tools that help with choosing the optimal distance functions for the given task. However, for the sake of the reader, first we briefly summarize the main points presented in [11].

2 Neighbourhood sequences

To get familiar with neighbourhood sequences the reader should overview the results presented in [5, 8, 10, 15]. Now we recall those concepts of this theory which are essential in our investigations. Though we will consider the RGB image representation in details, we give the notions for arbitrary dimension, since our procedures can be applied to arbitrary dimensional integer image representations. From now on, let n be an arbitrary positive integer. Let q and r be two points in $\mathbb{Z}^n$. The i-th coordinate of the point q is indicated by $\mathrm{Pr}_i(q)$. Let m be an integer with $1 \leq m \leq n$. The points q and r are m-neighbours, if the following two conditions hold:

- $|\mathrm{Pr}_i(q) - \mathrm{Pr}_i(r)| \leq 1 \quad (1 \leq i \leq n)$,
- $\sum_{i=1}^{n} |\,\mathrm{Pr}_i(p) - \mathrm{Pr}_i(q)| \leq m$.

The sequence $A = (A(i))_{i=1}^{\infty}$, where $A(i) \in \{1, \ldots, n\}$ for all $i \in \mathbb{N}$, is called an n-dimensional (shortly nD) neighbourhood sequence. If for some $l \in \mathbb{N}$, $A(i + l) = A(i)$ $(i \in \mathbb{N})$, then A is periodic with period l. In this case we briefly write $A = \{A(1)A(2)\ldots A(l)\}$. For example, we write $\{12\}$ for the neighbourhood sequence 1,2,1,2,1,2,.... The point sequence $q = q_0, q_1, \ldots, q_m = r$, where q_{i-1} and q_i are $A(i)$-neighbours in $\mathbb{Z}^n$ $(1 \leq i \leq m)$, is called an A-path from q to r of length m. The A-distance $d(q, r; A)$ of q and r is defined as the length of the shortest A-path(s) between them. The distance functions generated by neighbourhood sequences are not metrics in general and the existence of this property can be checked by a simple criterion [14]. Nevertheless, in some cases non-metric distance functions also may provide nice results, thus it is not recommended to exclude them from analysis. Moreover, this way we have a lot more distance functions to choose from to refine our results.

3 Applications based on measuring distance in the RGB cube by neighbourhood sequences

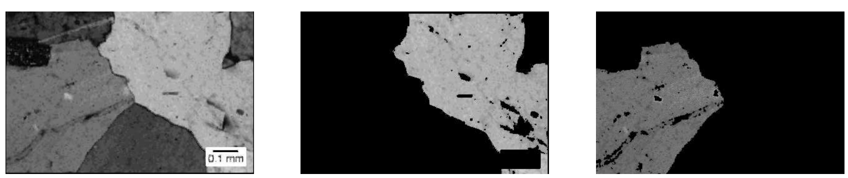
Colour image indexing and segmentation procedures are based on the comparison of the colour of the pixels. We use the 24-bit RGB cube (that is the domain is between black=$(0, 0, 0)$ and white=$(255, 255, 255)$) to illustrate the descriptive behaviour of neighbourhood sequences in measuring the distance of two colours. As it is shown in [11], reasonable differences may occur according to the chosen neighbourhood sequence, thus it must be selected carefully to achieve the desired result. Now we present some applications. We start with the generalisation of the "fuzziness" method referring to Adobe Photoshop terminology, then consider colour image segmentation with region growing, finally present an indexing method using cluster analysis.

Fuzziness. This indexing procedure [9] selects those pixels which are within a given distance to one or more initially fixed seed colours. The implementation of this method for a fixed distance function also can be found in Adobe Photoshop, where it is referred to as the "Fuzziness" option [1]. Figure 1 shows that the results of the fuzziness procedure highly depend on the chosen seeds, threshold and neighbourhood sequence.

(a) (b) (c)

Fig. 1. Fuzziness from the indicated seed colour with threshold=45, using neighbourhood sequences; (a) original image, (b) {1}, (c) {2}.

Region growing. To obtain a connected region, we can insert the distance functions used at fuzziness into a region growing algorithm (see [9, 16]). In Figure 2 we show some results.



(a) (b) (c)

Fig. 2. Region growing of stone parts; (a) original image, (b) using one seed colour, (c) using two seed colours.

Clustering. We recall an algorithm for indexing colour images based on cluster analysis [9]. In this procedure, the elements of the RGB cube are classified into clusters using a suitable metric. In our experiments, we used digital distance functions generated by neighbourhood sequences. In Figure 3 we show results for our clustering method.

A quantitative analysis of the proposed clustering method can be obtained by considering a suitable measure, like the specialisation of the uniformity measure of Levine and Nazif [12].

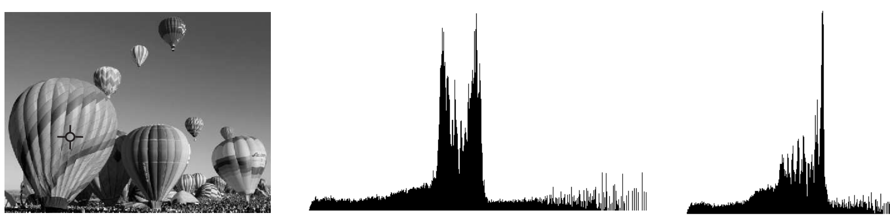
(a) (b) (c)

Fig. 3. Classifying colours into six clusters by neighbourhood sequences; (a) original image, (b) {12}, (c) {23}.

4 Tools for finding optimal distance measurement

Now we turn to present some tools and give guidelines to help with finding optimal distance functions and threshold values for the above procedures. Our approach is based on histogram analysis, and we propose two types of histograms which may be helpful.

Fuzziness histogram. This histogram type can be assigned to the "Fuzziness" procedure, and might be useful especially in region growing. We choose one or more seed colour(s) and for every non-negative integer K we calculate the number of image pixels that have the colour at the distance K from the seed(s). From this data we compose a histogram. Naturally, the shape of the histogram highly depends on the chosen distance function. For example, a "faster" distance function results a shorter histogram, but reasonable differences also may occur with respect to modality. These properties can be nicely observed in Figure 4.



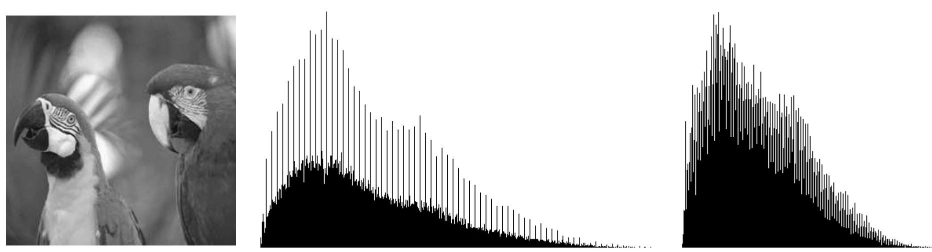
(a) (b) (c)

Fig. 4. Fuzziness histograms for the indicated seed colour, using different metrics; (a) original image, (b) L_1, (c) L_2.

The difference between two such histograms can be measured by suitable histogram distance measures [3] which reflect the different behaviour of different distance functions. The first mode of the histogram may play very important role in region growing, as we often do not take much care about colours far from the seed(s). The separation of this mode can be performed e.g. by density estimation

[17] to find the first local minimum. Distance functions which provide nicely separable first mode are expected to give good results in this case. However, note that a mode may include more than one colour classes (and thus more objects). By repeating the procedure only to the subimage defined by the investigated mode these colour classes can be further separated.

Global histogram. We propose another type of histogram which depends on only the distance function and the image. This case the K-th column of the histogram corresponds to the number of those pixel pairs in the image, whose colours have the distance K. Since this way no more additional parameters (e.g. seeds) are expected to be given, we refer to this descriptor as a global histogram. Global and fuzziness histograms can be processed similarly, but this time the first mode has no particular importance. These histograms nicely show if distance functions behave differently on a given image, and similarly to fuzziness histograms, histogram distances also can be calculated. Moreover, we gain a nice illustrative representation if the distance function is based on a neighbourhood sequence. For example, in Figure 5 the obtained global histograms nicely reflect the values in the period of the neighbourhood sequences. Finding local minima by density estimation, one can use global histograms to find colour classes (however, this time we should mind that the different modes of the histogram may contain same image pixels). Global and fuzziness histograms also seem to be well applicable in image retrieval as feature vectors extracted from the images. As these histograms correspond mainly to the colour distribution of the images, this approach is expected to be useful in the pre-filtering phase of image retrieval.



(a) (b) (c)

Fig. 5. Global histograms based on neighbourhood sequences; (a) original image, (b) {1111113}, (c) {123}.

5 Conclusions

The most image indexing and segmentation techniques are based on classical (e.g. Euclidean) metrics. In some cases other metrics may provide better results [7], but very few suggestions can be found in the literature how to choose them. The authors in [11] showed that distance functions based on neighbourhood sequences also might be worth taking into consideration in some digital image processing problems, since they are valid alternatives for integer domains. Especially, classic metrics also can be approximated by distance functions generated by neighbourhood sequences; see e.g. [4, 6, 10, 13]. In the present paper we propose some tools that may be helpful with choosing the appropriate distance functions, and nicely reflect the behaviour of neighbourhood sequences, as well. Our future aim is to make the choice of a suitable distance function as automatic as possible based on the tools presented here.

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