Modeling Inter-vehicle Communication in Multi-lane Highways: A Stochastic Geometry Approach

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APPENDIX A

PROOF OF LEMMA 1

Following the PPP definitions, the vehicles are uniformly distributed over the traffic lanes. Hence, from the length of the traffic chord, the pdf of the horizontal distance between the origin and the selected transmitter (c.f. Fig. 1) is given by $f_X(x) = \frac{1}{\sqrt{r^2 - (md)^2}}, 0 \leq x \leq \sqrt{r^2 - (md)^2}$. Since the distance $r_o$ is related to $X$ via Pythagoras theorem, Lemma 1 can be directly obtained.

APPENDIX B

DISCUSSION OF CONTENTION DOMAIN LENGTH

In the 1-D PPP with each point having a protection radius of $R_s$, the MHCPP-II considers $2R_s$ in the calculation of $E[|N|]$ to account for the contention domain at both sides of the transmitter. Note that, by definition, the MHCPP-II underestimates the intensity $\Lambda$ due to the role of unselected points (see [5] for details). However, due to the natural order of points in 1-D lines, the underestimation problem of the MHCPP-II can be directly related to the contention domain calculations (i.e., $E[|N|]$). We argue that considering the contention domain on both sides of the transmitters, along with retaining one transmitter per contention domain, results in underestimating $\Lambda$ for the following two reasons; i) starting from a retained transmitter and moving in each direction along the line, the first transmitter after a void distance of $R_s$ will contend only with the nodes in the next $R_s$ distance; ii) the MHCPP-II saturates at the intensity of $\frac{1}{2R_s}$ which can be considered as a loose packing density. This is because if $R_s$ is the void
region on each side of the transmitters, the intensity of concurrent transmitters should saturate at \( \frac{1}{R_s} \). Therefore, throughout the paper, our proposed contention domain length is \( R_s \) as opposed to \( 2R_s \) used by the MHCPP-II. We show that by using the simple approximation of the contention domain length, we can effectively mitigate the underestimation problem in the 1-D case.

**APPENDIX C**

**PROOF OF THEOREM 1**

Since the set of interfering vehicles is approximated by a PPP with intensity \( \Lambda \), the LT of the aggregate interference can be expressed as:

\[
L_I(s) = \mathbb{E}_{\Phi_i}(e^{-sl}) = \mathbb{E}_{\Phi_i} \left[ \exp \left( -s \sum_{i=1}^{N} \sum_{\psi_i \in \Phi \setminus \psi_0} P h_{ij} \|v_{ij}\|^{-\eta} \right) \right]
\]

\[
= \prod_{i=1}^{N} \sum_{\psi_i \in \Phi_i} \left[ \prod_{\psi_j \in \Phi_i} L_{\Lambda}(sP \|v_{ij}\|^{-\eta}) \right]
\]

\[
= \prod_{i=1}^{N} \sum_{\psi_i \in \Phi_i} \left[ \prod_{\psi_j \in \Phi_i} \frac{\mu}{\mu + sP \|v_{ij}\|^{-\eta}} \right]
\]

\[
= \prod_{i=1}^{N} \exp \left( - \int_{\psi_i \in \Gamma} \tilde{\Lambda}(\gamma_i, i) \left( 1 - \frac{\mu}{\mu + P \gamma_i^{-\eta}} \right) d\gamma_i \right)
\]

\[
= \prod_{i=1}^{N} \exp \left( - \int_{\psi_i \in \Gamma} \tilde{\Lambda}(\gamma_i, i) P \gamma_i^{-\eta} \left( \frac{\mu}{\mu + P \gamma_i^{-\eta}} \right) d\gamma_i \right)
\]

(1)

where (a) follows from the PGFL of a PPP. \( \psi_i \) represents the interference region on the \( i^{th} \) traffic lane. Thus the limits of integration are from \( \gamma_i \) to \( \infty \) and \( -\infty \) to \( \gamma_i \) as shown in Fig. 2. Substituting (1) in (2) of the paper proves the theorem.

**REFERENCES**


