Trajectory Tracking and Consensus of Networks of Euler–Lagrange Systems

Emmanuel Nuño∗, Romeo Ortega ∗∗∗, Luis Basañez ∗∗, David Hill ∗∗∗∗

* Department of Computer Science. University of Guadalajara.
Guadalajara, Mexico.

** Institute of Industrial and Control Engineering.
Technical University of Catalonia. Barcelona, Spain.

*** Laboratoire des Signaux et Systèmes. SUPÉLEC.
Gif-sur-Yvette, France.

**** School of Electrical and Information Engineering.
The University of Sydney. Sydney, Australia.

Emails: emmanuel.nuno@cucei.udg.mx, ortega@lss.supelec.fr, luis.basanez@upc.edu, david.hill@sydney.edu.au

Abstract: This paper addresses the problem of synchronizing networks of nonidentical, nonlinear dynamical systems described by Euler–Lagrange equations. It is assumed that the communication graph is simply connected and that the systems are fully actuated, with their states available for measurement. The communications can be subject to constant time-delays. The main result of the paper is a controller for each system in the network, capable of tracking a desired trajectory and, if such trajectory does not exist, capable of reaching a network consensus. Moreover, it is proved that, if there are no time-delays and the graph is balanced each system reaches a consensus arbitrarily near the average of the initial conditions of all the systems in the network. Simulations using a ten robot manipulator network with different time-delays are provided.

Keywords: Euler-Lagrange Systems, Synchronization, Consensus, Time-delays.

1. INTRODUCTION

Motivated by applications in physics, biology and engineering, the study of synchronization of collections of dynamic systems has become an important topic in control theory (Olfati-Saber et al., 2007; Scardovi and Sepulchre, 2009; Scardovi et al., 2009). Roughly speaking, the synchronization objective is to reach some kind of agreement between some variables of interest of the systems, also called agents here. Clearly, the two main ingredients in this problem are the individual dynamics of the agents and the interconnection pattern among them. The present paper is focused on the design of control laws to synchronize nonidentical, nonlinear dynamical systems described by Euler–Lagrange (EL) equations. The motivation to consider EL–systems stems from the fact that EL–models describe a very large class of physical systems of practical interest (Ortega et al., 1998).

A large literature is available on the problem of synchronization of networks composed by identical nodes, particularly when the nodes are linear time invariant systems and there are no communication delays (Olfati-Saber et al., 2007; Scardovi and Sepulchre, 2009). Some results for non–identical nonlinear nodes may be found in (Zhao et al., 2009) and references therein. In a remarkable paper, Pogromsky et al. (2002) introduce the new concept of semi–passivity to establish conditions for global synchronization of diffusively coupled (non-necessarily identical) nonlinear systems. Recently, the fundamental paper of Chopra and Spong (2006) points out that interconnecting the outputs of passive systems through strongly connected balanced graphs induces output synchronization. It also shows that synchronization is preserved in the presence of constant communication delays. The passivity property has also been used by Stan and Sepulchre (2007), Arcak (2007) and Pogromsky and Nijmeijer (2001) to analyze synchronization behavior. In an interesting paper, Scardovi et al. (2009), make use, instead of passivity, of the stronger property of incremental output strict passivity—called in the paper co-coercitivity—in order to prove synchronization to the average of the outputs if the passivity gains dominate the interconnection gains, in a small–gain theorem–like way. Such paper also consider the presence of groups of identical systems, called compartments, and, invoking the well–known diagonal dominance condition for large scale systems of Moylan and Hill (1978), impose an additional constraint on the passivity gains dominating the algebraic connectivities of the graphs interconnecting the same species.

The synchronization problem can also be formulated when a desired trajectory is explicitly defined. The first result along those lines for cooperative robot manipulators has been reported by Rodriguez-Angeles and Nijmeijer (2004), that propose a nonlinear observer–based controller with
all-to-all coupling, which achieves position synchronization. More recently, Chung and Slotine (2009) consider general EL-systems and present a generalization of the well-known Slotine and Li controller (Slotine and Li, 1988) for the case of multiple systems. The schemes of Rodriguez-Angeles and Nijmeijer (2004), Chung and Slotine (2009) and Chopra and Liu (2008), being designed to track a given reference, will obviously drive all positions to zero in the absence of a desired trajectory, i.e., they “synchronize to zero” the systems positions. Nuño et al. (2010) report and correct a similar undesired behavior of a recently proposed adaptive scheme for bilateral teleoperators.

This paper presents a unified scheme that can handle, with a slight modification, both versions of the synchronization problem—with or without reference signal. The main contribution is a controller that achieves global full state synchronization—that is, asymptotically drive to zero the difference between the states of all the agents for all initial conditions—avoiding the synchronization to zero behavior in the absence of a reference signal. When a desired trajectory is given, synchronization is achieved measuring only positions and velocities. Moreover, the paper also proves that the proposed controller can achieve a consensus on all the agents arbitrarily near the average of their initial conditions in the absence of time-delays and with a balanced graph. The extension to the case when the physical parameters are unknown has been recently reported in Nuño et al. (2011).

2. EULER-LAGRANGE SYSTEMS

The EL-equations of motion describe the dynamic behavior of a large class of physical systems. For a $n$–Degrees of Freedom system, they are given by $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau$, where $L : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is the Lagrangian function defined as: $L(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q} - U(q)$, with $q \in \mathbb{R}^n$, the generalized configuration coordinates, $M : \mathbb{R}^n \to \mathbb{R}^{n \times n}$, the generalized mass matrix, $U : \mathbb{R}^n \to \mathbb{R}$, the potential energy function and $\tau \in \mathbb{R}^n$, the vector of external forces acting on the system. See (Ortega et al., 1998; Ortega and Spong, 1989) for further details and applications of EL-systems. Although the generalized coordinates may represent physical quantities other than positions, e.g., charges or fluxes in electrical systems, extrapolating from the application to mechanical systems, in the sequel we will refer to $q$ as the systems position vector and $\dot{q}$ its corresponding velocity.

It is well-known that EL-systems satisfy the power-balance equation $\dot{H} = \tau^T \dot{q}$, where $H : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is the systems total co-energy: $H(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q} + U(q)$. Consequently, if $H$ is bounded from below, it define passive operators $\tau \to \dot{q}$. Besides this fundamental property, which has been extensively exploited for passivity–based control design, EL-systems enjoy another key property, reported by Ortega and Spong (1989), that is instrumental for the present developments. Namely, writing the EL-system as $M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + g(q) = \tau$, where $C(q, \dot{q})$ and $g(q)$ are the Coriolis and centrifugal forces, $C : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n \times n}$, and $g(q) := \frac{\partial U}{\partial q}$ are the forces due to the gravity potential field. If $C(q, \dot{q})$ is defined via the Kristoffel symbols of the first-kind, then $M(q) = C(q, \dot{q}) + C^T(q, \dot{q})$. See (Ortega et al., 1998) and also (van der Schaft, 1999) for an intrinsic, differential geometric, interpretation of this property. In this work, the following standard assumption regarding EL-systems is adopted:

3. NETWORK CONTROL PROBLEMS

This paper is focused on the control of networks of $N$ non–identical EL-systems, denoted as

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i,$$  \hspace{1cm} (1)

where $i \in N := \{1, ..., N\}$. The sub-indices of the agents functions usually range in the set $N$, this clarification is omitted for brevity. In particular, the aim of this paper is to solve the following problems:

**TSP** (Tracking Synchronization Problem). Given a desired trajectory for all agents $q_i \in \mathbb{R}^n$, assuming that $q_{id}, \dot{q}_{id}, \ddot{q}_{id} \in \mathcal{L}_\infty$, the following has to be satisfied

$$\lim_{t \to \infty} |\ddot{q}_i(t)| = 0, \hspace{1cm} (2)$$

$$\lim_{t \to \infty} |\dot{q}_i(t)| = 0, \hspace{1cm} (3)$$

where $\ddot{q}_i := q_i - q_{id}$ is the position tracking error.

**CP** (Consensus Problem). When there is no reference signal, all agents’ positions must reach a consensus at a constant value $q_c \in \mathbb{R}^n$, moreover their velocities must converge to zero, that is,

$$\lim_{t \to \infty} q_i(t) = q_c \hspace{1cm} (4)$$

$$\lim_{t \to \infty} |\dot{q}_i(t)| = 0. \hspace{1cm} (5)$$

It is also especially interesting to find the conditions under which the proposed controllers solve the following particular consensus problem.

**AACP** (Approximate Average Consensus Problem). When there is no reference trajectory, all agents achieve a consensus arbitrarily close to the average of their initial positions. That is, (4) and (5) hold with

$$q_c = \frac{1}{N} \sum_{i=1}^{N} q_i(0) + \Delta, \hspace{1cm} (6)$$

where $\Delta \in \mathbb{R}^n$ can be made arbitrarily small.

4. NETWORK TOPOLOGY

Following the synchronization scenario, the EL-systems exchange information over a network described by a directed interconnection graph, which can be subject to unknown, constant transmission delays, and where each agent is a node of the graph. See, e.g., (Olfati-Saber et al., 2007; Scardovi and Sepulcre, 2009) for some basic preliminaries on graph theory as applied to consensus and synchronization problems. The interconnection graph can be modeled using standard tools, such as the Laplacian matrix $L \in \mathbb{R}^{N \times N}$, whose elements are defined as

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{N} a_{ik} & i = j \\ -a_{ij} & i \neq j \end{cases}, \hspace{1cm} (7)$$

where $a_{ij}$ are the adjacency matrix elements.
with $a_{ij} = 1$ if $j \in N_i$, $a_{ij} = 0$ otherwise, with $N_i$ the set of agents transmitting information to the $i$–th agent.

By construction, $L_1 N = 0$, where $1_N$ stands for a column vector of size $N$ filled with ones.

The following assumptions regarding the communication graph are made in the paper.

A2 The directed graph is simply connected, i.e., there exists a node such that all other nodes in the graph are connected to this node via a directed path.

A3 The information exchange between the $i$-th and the $j$-th EL–systems is subject to a constant, unknown time–delay, denoted by $T_{i,j} \geq 0$.

Assumption A2 ensures that rank$(L) = N-1$, that $L$ has a single zero–eigenvalue and that the rest of the spectrum of $L$ has positive real parts (Olfati-Saber and Murray, 2004).

5. PROPOSED CONTROL SCHEME

This section presents a solution to the TSP, CP and AACP formulated in the previous section. To simplify the notation, it is convenient to define a generalized position error, whose convergence to zero ensures the synchronization objectives. Namely,

$$e_i := \sum_{j \in N_i} [\hat{q}_i - \hat{q}_j(t - T_{i,j})] + \alpha \tilde{q}_i,$$  \hspace{1cm} (8)

where $\alpha = 1$, if a common trajectory is imposed to all agents and $\alpha = 0$, if there is no desired common trajectory. Let us also define the signals

$$e_i := \hat{q}_i + \Lambda e_i,$$  \hspace{1cm} (9)

where $\Lambda := \text{diag}\{\lambda_h\} > 0$ for $h \in [1 ... n]$. The proposed controller is given by

$$\tau_i = M_i [\hat{q}_d - \Lambda \tilde{q}_i] + C_i [\hat{q}_d - \Lambda \tilde{q}_i] + g_i(q_i) - K_i e_i,$$  \hspace{1cm} (10)

where $K_i := \text{diag}\{k_{ii}\} > 0$.

5.1 Cascaded Structure of the Closed–Loop

In this subsection we make the key observation that the closed-loop dynamics are described by the cascade of an exponentially stable system, depending only on the agents dynamics, and a stable linear time–invariant (LTI) delay–differential system, defined by the graph and the communication delays.

Replacing (10) in (1), and using (9), we obtain the well–known error equation

$$M_i e_i + C_i(q_i, \hat{q}_i) + K_i e_i = 0.$$  \hspace{1cm} (11)

It is important to note that since $\frac{W_i(0)}{m_{ii}}$ is independent of $\xi$, the rate of converge can be made arbitrarily large—increasing $k_i$—without inducing a peaking phenomenon (Sussman and Kokotovic, 1991). In Subsection 5.4 this property is invoked for the solution of the AACP.

To complete the description of the system, rewrite (9), for one DOF, as: $e_h = \hat{q}_h + \lambda_h \tilde{q}_h$, where $h \in [1 ... n]$ and $\hat{q}_h, \tilde{q}_h$ and $e_h \in R^N$ are defined piling up their $N$ components $\hat{q}_h, \tilde{q}_h$ and $e_h$, respectively. Note that, by using this notation all $n$ DOFs are decoupled.

Now, with $d := \sum_1^n \text{card } N_i$, and defining $\tau_1 := T_{1,1}, \tau_2 := T_{1,2}, \ldots, \tau_d := T_{N,N}$, the position error in (8) can be expressed, for all agents, as

$$e_h = A_0 \tilde{q}_h - \sum_{k=1}^{d} A_k \hat{q}_h(t - \tau_k) + \alpha \tilde{q}_h$$ \hspace{1cm} (13)

where the matrix $A_0 := \text{diag}\{e_i\} \in R^{N \times N}$ and the matrices $A_k \in R^{N \times N}$ have all elements equal to zero, except one off-diagonal element equal to one of the $a_{ij}$, placed (in an obvious way) such that the systems Laplacian satisfies

$$L = A_0 - \sum_{k=1}^{d} A_k.$$  \hspace{1cm} (14)

Using the previous expressions for $e_h$ and $\hat{q}_h$ we get

$$\dot{\hat{q}}_h = - \lambda_h (A_0 + \alpha 1_N) \hat{q}_h + \lambda_h \sum_{k=1}^{d} A_k \hat{q}_h(t - \tau_k) + e_h$$ \hspace{1cm} (15)

In view of (12), which is independent of $q_0$, the stability analysis boils down to the study of the LTI delay–differential system (15) driven by an exponentially decaying term.

5.2 Tracking Synchronization

The proposition below gives the solution to the TSP.

**Proposition 1.** Consider a network of $N$ agents of the form (1) verifying Assumption A1. Assume they are interconnected through a communication graph satisfying Assumptions A2 and A3. Under these conditions, the controller (8), (9), (10), with $\alpha = 1$, solves the TSP.

**Proof.** The proof is established showing that the transfer matrix from $e_h$ to $\hat{q}_h$, of the LTI system (15), is asymptotically stable. In this case, since the input is exponentially decaying, we have that, $\forall h \in [1 ... n], |\hat{q}_h|, |\tilde{q}_h| \rightarrow 0$.

The fact that $e_i \in L_2$ ensures that its Laplace transform exists and, from (15), we have that $\hat{q}_h(s) = \Phi(s)e_h(s)$, where

$$\Phi(s) := \left[ s1_N + \lambda_h (A_0 + 1_N) - \lambda_h \sum_{k=1}^{d} e^{-s\tau_k} A_k \right]^{-1}.$$ \hspace{1cm}

Invoking Gershgorin Theorem we conclude that all poles of $\Phi(s)$ lie on disks centered at $\Re{s} = -\lambda_h(k_{ii} + 1)$ with radius $\lambda_h \sum_{k=1}^{d} e^{-s\tau_k} A_k$. Noting that, for the possible unstable poles with $\Re{s} > 0$, $|e^{-s\tau_k}| > 1$ and, from (7),...
that $\sum_{k=1}^{d} A_{ik} = \ell_{ii}$ we conclude that $\Phi(s)$ has no poles in the open half–plane $\Re s > 0$. Moreover, none of the Gershgorin disks intersect the origin of the complex plane, thus $s = 0$ is not a pole of $\Phi(s)$. This implies that (15) is an asymptotically stable LTI system, as required. Thus, the TSP problem is solved.

5.3 Consensus

In the consensus problems we set $\alpha = 0$ and $q_{d} = \dot{q}_{d} = 0$, in this scenario, (15) becomes

$$\dot{q}_{h} = -\lambda_{h} A_{0} q_{h} + \lambda_{h} \sum_{k=1}^{d} A_{k} q_{h}(t - \tau_{k}) + e_{h}. \tag{16}$$

Comparing the latter with (15) we underscore the absence of the term $\alpha I_{N}$, which is instrumental to prove asymptotic stability of $\Phi(s)$. An alternative argument is then needed to complete the proof.

Proposition 2. Consider a network of $N$ agents of the form (1) verifying Assumption A1. Assume they are inter-connected through a communication graph satisfying Assumptions A2 and A3 and that there is no desired reference trajectory. Under these conditions, for any time-delays, the controller (8), (9), (10), with $\alpha = 0$, solves the CP.

Proof. From (16), we have that $q_{h}(s) = \Psi(s) e_{h}(s)$, where

$$\Psi(s) := \left[s I_{N} + \lambda_{h} (A_{0} - \sum_{k=1}^{d} e^{-st_{k}} A_{k})\right]^{-1}. \tag{17}$$

Notice that, using the Gershgorin Theorem and the fact that, for the possible unstable poles with $\Re s > 0$, $|e^{-st_{k}}| > 1$, we conclude that $\Psi(s)$ has no poles on the in the open half–plane $\Re s > 0$. However, the Gershgorin disks intersect (tangentially) the origin of the complex plane and $s = 0$ may be a root—possibly repeated. The fact that this is indeed one root follows immediately $\Psi^{-1}(0) = \lambda_{h} L$ and the connected assumption. Hence $\Psi(s)$ is not asymptotically stable. Our first objective is then to prove that $|q_{h}(s)| \to 0$. For, compute

$$\lim_{s \to 0} s^{2} q_{h}(s) = \lim_{s \to 0} s^{2} \Psi(s) e_{h}(s) = \lim_{s \to 0} s \Psi(s) \lim_{s \to 0} s e_{h}(s)$$

where the third expression holds because both limits exist. Indeed,

$$\lim_{s \to 0} s \Psi(s) = \lim_{s \to 0} \left[I_{N} + \frac{\lambda_{h}}{s} (A_{0} - \sum_{k=1}^{d} e^{-st_{k}} A_{k})\right]^{-1}$$

$$= \lim_{s \to 0} \left[I_{N} + \frac{\lambda_{h}}{s} \left[\text{diag} \left\{ \frac{s}{\lambda_{h}} \right\} + (A_{0} - \sum_{k=1}^{d} e^{-st_{k}} A_{k})\right]\right]^{-1}$$

$$= \lim_{s \to 0} \left[I_{N} + \frac{\lambda_{h}}{s} \left[\text{diag} \left\{ \frac{s}{\lambda_{h}} \right\} + L + s \sum_{k=1}^{d} \tau_{k} A_{k}\right]\right]^{-1}$$

for which the third equation we used the fact that $e^{-st_{k}}$ and $1 - st_{k}$ are infinitesimal equivalent expressions when $s \to 0$. The facts that $\text{rank}(L) = N-1$ and $A_{k}$ has only off-diagonal elements ensure that $\text{rank} \left( \sum_{k=1}^{d} \tau_{k} A_{k} \right) = N$, thus the inverse always exists and it can be proved that $\lim_{s \to 0} s \Psi(s) = H$, where $H \in \mathbb{R}^{N \times N}$ is a constant matrix.

On the other hand, the fact that $|e_{h}| \to 0$ and the final value theorem, prove that $\lim_{s \to 0} s e_{h}(s) = 0$. Replacing these two limits, and invoking again the final value theorem, we show that $\lim_{t \to \infty} q_{h}(t) = \lim_{s \to 0} s^{2} q_{h}(s) = 0$, thus (5) holds.

We proceed now to prove (4). First, note that with $|\dot{q}_{h}| \to 0$ and $|e_{h}| \to 0$ we have, via (9), that $|e_{h}| \to 0$. Now, (13) can be written as: $e_{h} = L q_{h} + \sum_{k=1}^{d} A_{k} \int_{t-\tau_{k}}^{t} \dot{q}_{h}(\sigma) d\sigma$, and since both $\dot{q}_{h}$ and $e_{h}$ converge to zero, $|L q_{h}| \to 0$. Since $\text{span}\{I_{N}\}$ is in the kernel of $L$, it can be concluded that $q_{h} \to q_{c_{a}} I_{N}$, for some $q_{c_{a}} \in \mathbb{R}$. Note that this last is the same as $q \to (I_{N} \otimes q_{c})$. This completes the proof. \(\diamondsuit\)

5.4 Approximate Average Consensus

In this subsection we prove that the controller (10) solves the AACP provided there are no time-delays in the communications, i.e., $T_{ij} = 0$, and that the interconnection graph is balanced. As shown in (Olfati-Saber and Murray, 2004), the latter ensures that $I_{N} L = 0$.

Before establishing the solution to the AACP problem, we present the following result, which is a minor extension of Theorem 5 in (Olfati-Saber and Murray, 2004).

Proposition 3. Suppose that $L \in \mathbb{R}^{N \times N}$ is the Laplacian matrix of a balanced and simply connected graph. Then, for any positive definite, diagonal matrix $\Lambda \in \mathbb{R}^{n \times n}$,

$$\lim_{t \to \infty} e^{-(L \otimes \Lambda)t} = \frac{1}{N} I_{N} I_{N}^{T} \otimes I_{A}. \tag{17}$$

Proof. First note that using the properties of the Kronecker product and matrix exponentials, and expressing $L$ with its associated Jordan normal form $D \in \mathbb{R}^{N \times N}$, i.e., $L = V D V^{-1}$, we have that $e^{-(L \otimes \Lambda)t} = (V \otimes I_{A}) e^{-(D \otimes \Lambda)t} (V^{-1} \otimes I_{A})$, where $V = [V_{1}, V_{2}, \ldots, V_{N}]$, with $v_{i}$ the eigenvectors of $L$. From the fact that all eigenvalues of $L$ have positive real parts except for one at zero, and that $\lambda_{A} > 0$, it follows that $\lim_{t \to \infty} e^{-(D \otimes \Lambda)t} = (Q \otimes I_{A})$, where $Q \in \mathbb{R}^{N \times N}$ has only one non-zero element at $Q_{11} = 1$ (all the other elements vanish as $t \to \infty$). Finally,

$$\lim_{t \to \infty} e^{-(L \otimes \Lambda)t} = (V Q^{-1} \otimes I_{A}) = \frac{1}{N} (I_{N} I_{N}^{T} \otimes I_{A}) \tag{18}$$

where, to get the third expression, we used the fact that the first row of $V^{-1}$ is $1/N$. \(\diamondsuit\)

Proposition 4. Consider a network of $N$ agents of the form (1) verifying Assumption A1. Assume they are interconnected through a communication graph verifying Assumption A2, which is moreover balanced, and that the time-delays are zero. Under these conditions, controller (8), (9), (10) solves the AACP. Namely, it ensures (4), (5), (6), where $|\Delta|$ can be made arbitrarily small increasing $k_{i}$.

Proof. When $T_{i,j} = 0$, (16) can be expressed, for all $h$, as the first-order non-homogeneous linear differential equation $\ddot{q} = - (L \otimes \Delta)q + e$, whose solution is
\[ q(t) = e^{-\left((L \otimes \Lambda) t\right)} q(0) + \int_0^t e^{\left((L \otimes \Lambda)(\sigma - t)\right)} e(\sigma) d\sigma. \]

From (17) in Proposition 3 we get
\[ \lim_{t \to \infty} e^{-\left((L \otimes \Lambda) t\right)} q(0) = 1_N \otimes \ldots \]

Hence, \[ \lim_{t \to \infty} q(t) = 1_N \otimes \frac{1}{N} \sum_{i=1}^{N} q_i(0) + \Delta, \]

where \[ \Delta := \lim_{t \to \infty} \int_0^t e^{\left((L \otimes \Lambda)(\sigma - t)\right)} e(\sigma) d\sigma. \]
The proof is completed noting that, in view of (12) and (18), \( |\Delta| \) can be made arbitrarily small increasing \( k_i \).

6. NUMERICAL EXAMPLE

Fig. 1. Balanced and connected network used in the simulations. Each node consists of a 2-DOF nonlinear manipulator with revolute joints. The network is composed by three different groups of manipulators, with all members in each group equal.

Table 1. Physical parameters and initial positions for each manipulator in the network.

<table>
<thead>
<tr>
<th>Index</th>
<th>( m_1, m_2 ) (kg)</th>
<th>( l_1, l_2 ) (m)</th>
<th>( q^T(0) ) (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4, 2</td>
<td>0.4, 0.4</td>
<td>[1 -1]</td>
</tr>
<tr>
<td>2</td>
<td>3, 2.5</td>
<td>0.6, 0.5</td>
<td>[-1 2]</td>
</tr>
<tr>
<td>3</td>
<td>3.5, 2.5</td>
<td>0.3, 0.35</td>
<td>[1 -1]</td>
</tr>
</tbody>
</table>

The simulations are performed with a network of ten 2-DOF nonlinear manipulators with revolute joints (Fig. 1). Each manipulator nonlinear dynamics follows the EL-equations (1), and the inertia matrices, Coriolis and centrifugal matrices, and gravity vectors are given, respectively, by

\[ M_i = \begin{bmatrix} \alpha_1 + 2\beta_i c_{i2} & \delta_i + \beta_i c_{i2} \\ \delta_i & \delta_i \end{bmatrix}, C_i = \begin{bmatrix} -2\beta_i s_{i2} \hat{q}_{i2} - \beta_i s_{i1} \dot{q}_{i1} \\ \beta_i s_{i1} \hat{q}_{i1} \end{bmatrix}, \]

and \( g_i = g \begin{bmatrix} l_{i2} & \frac{1}{l_{i2}} \delta_i c_{i12} + \frac{1}{l_{i1}} (\alpha_i - \delta_i) c_{i1} \\ \frac{1}{l_{i2}} \delta_i c_{i112} + \frac{1}{l_{i1}} (\alpha_i - \delta_i) c_{i1} \end{bmatrix} \).

In these expressions, \( c_{i1}, s_{i1} \) are short notation for \( \cos(q_{i1}) \) and \( \sin(q_{i1}) \); \( q_{i1} \) is the articular position of link \( k \) of manipulator \( i \), with \( k \in \{1, 2\} \); \( c_{i12} \) stands for \( \cos(q_{i1} + q_{i2}) \); \( g \) is the acceleration of gravity constant, \( \alpha_i = l_{i2} m_{i2} + l_{i1}^2 (m_{i1} + m_{i2}) \), \( \beta_i = l_{i1} l_{i2} m_{i12} \) and \( \delta_i = l_{i2}^2 m_{i2} \), where \( l_{i1} \) and \( m_{i1} \) are the respective lengths and masses of each link. The physical parameters together with the initial positions for each manipulator are shown in Table 6. The initial velocities are all zero and, for all the simulations, \( \Delta = I_2 \).

The first simulations show the tracking capabilities of the proposed controller. Fig. 2 shows the tracking results with the desired trajectory \( \mathbf{q}_d = [0.5 \sin(0.1\pi t), 0.8 \sin(0.2\pi t)]^T \), for \( T_{i,j} = 1s \) with the control gain \( K = 20I_2 \), and Fig. 3 depicts the results when \( T_{i,j} = 2s \). In both figures it can be seen that the network of EL-systems asymptotically tracks the desired trajectory despite time-delays. The second set of simulations show the consensus results. Fig. 4 depicts the results when the control gain is not sufficiently large, \( i.e., K = 20I_2 \), thus, the network reaches a consensus, however not close to the average of the initial conditions.

When increasing \( K \) from \( 20I_2 \) to \( 50I_2 \), it can be seen in Fig. 5, that the consensus value of the network approaches the average of the initial conditions.

7. CONCLUSIONS

This paper presents a controller for a network of nonlinear and non-identical Euler-Lagrange systems. The controller is capable of providing asymptotic tracking of a reference trajectory, even in the presence of (arbitrarily large) time-delays in the communications. It is also shown that such controller can drive the network, in the absence of a
Fig. 4. The network reaches a consensus with $K = 20I_2$.

Fig. 5. The network approaches the average consensus, that is $q_c = [0.25, 0.3]^{\top}$, for $K = 50I_2$.

reference trajectory, towards a general consensus, and if there are not time-delays, the network approaches the average consensus, i.e., all the EL-systems states converge arbitrarily near the average of their initial conditions. The paper shows some simulations that support the reported theoretical results.

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REFERENCES


