Almost Global Finite-Time Stable Observer for Rigid Body Attitude Dynamics

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Abstract—A state observer is proposed for rigid body attitude motion with a given attitude dynamics model. This observer is designed on the state space of rigid body attitude motion, which is the tangent bundle of the Lie group of rigid body rotations in three dimensions, $\text{SO}(3)$, and therefore avoids instability due to the unwinding phenomenon seen with unit quaternion-based attitude observers. In the absence of measurement noise and disturbance torques, the observer designed leads to almost global finite-time stable convergence of attitude motion state estimates to the actual states for a rigid body whose inertia is known. Almost global finite-time stability of this observer is shown using a Morse function as part of a Lyapunov analysis; this Morse function has been previously used for almost global asymptotic stabilization of rigid body attitude motion. Numerical simulation results confirm the analytically obtained stability properties of this attitude state observer. Numerical results also show that state estimate errors are bounded in the presence of bounded measurement noise and bounded disturbance torque.

I. INTRODUCTION

This paper presents a nonlinear observer for attitude and angular velocity states of a rigid body. The attitude dynamics of the rigid body is assumed to be given by Euler’s equation with a known external torque, known body inertia, and a bounded but unknown disturbance torque. The observer design presented here exhibits almost global finite-time stable convergence of state estimates to actual states in the absence of measurement noise and disturbance torque. This gives it fast convergence as well as robustness to bounded disturbance torques and bounded measurement noise. Since most unmanned and manned vehicles can be accurately modeled as rigid bodies, the attitude dynamics of such vehicles when operated in uncertain or poorly known environments, can be subject to unknown but bounded disturbance torques. Therefore, fast convergence of state estimates and robustness of the observer to such unknown disturbances is essential for feedback control of such vehicles. The attitude is described directly on the Lie group of rigid body orientations in this observer design, without using local coordinates or unit quaternions for attitude representation.

The attitude and angular velocity observer designed here assumes that attitude and angular velocity have been determined from available measurements. Attitude is determined from direction or angle measurements while angular velocity measurements are usually obtained directly through rate gyroes. The problem of attitude determination from a set of three or more vector measurements is commonly set up as an optimization problem called Wahba’s problem [1]. This problem has been solved by different methods in the prior literature, a sample of which can be obtained in [2], [3], [4], [5], [6], [7]. However, attitude determination algorithms based on vector measurements, which are obtained from sensors like star trackers, sun sensors and magnetometers, are not designed to filter out high frequency noise in these measurements. Moreover, several of these attitude determination schemes use coordinate (three parameter) representations or unit quaternion representations for attitude. Local coordinate representations, including commonly used quaternion-derived parameters like the Rodrigues parameters and the modified Rodrigues parameters (MRPs), cannot describe arbitrary or tumbling attitude motion, while the unit quaternion representation of attitude is known to be ambiguous. Each physical attitude corresponds to an element of the Lie group of rigid body rotations $\text{SO}(3)$, and can be represented by a pair of antipodal quaternions on the hypersphere $S^3$, which is often represented as an embedded submanifold of $\mathbb{R}^4$ in attitude estimation. For dynamic attitude estimation, this ambiguity in the representation could lead to instability of filtering and observer schemes due to unwinding, as is described for attitude feedback control using unit quaternions in [8], [9].

The attitude and angular velocity observer designed here can be used in conjunction with attitude determination schemes from vector measurements and angular velocity measurements, to filter out noise in these measurements. The observer described here uses the global representation of attitude on $\text{SO}(3)$. Attitude observers and filtering schemes on $\text{SO}(3)$ and $\text{SE}(3)$ have been reported in, e.g., [10], [7], [11], [12], [13], [14], [15], [16], [17], [18]. These observers do not suffer from kinematic singularities like observers using coordinate descriptions of attitude, and it does not suffer from the unstable unwinding phenomenon encountered by observers using unit quaternions. Further, the observer designed here has an associated Lyapunov function constructed on the tangent bundle of $\text{SO}(3)$, using which almost global finite time stability of state estimates in the absence of measurement noise and disturbance torques is proven. This Lyapunov function is very similar to the one used recently to construct an attitude stabilization scheme with almost global finite time stability [19]. This gives the observer strong robustness properties and a fast rate of convergence. This observer can also be used in conjunction with a discrete-time filtering scheme, like the unscented filtering scheme for rigid body motion developed in [20], to propagate state estimates between successive discrete-time measurement updates.
II. ATTITUDE DYNAMICS MODELS

The dynamics model considered here is the attitude dynamics of a rigid body rotating under the action of a disturbance torque. The rigid body’s attitude is represented by the rotation matrix \( R \in \text{SO}(3) \) from a body-fixed coordinate frame to an inertial reference frame. The angular velocity of the rigid body is represented in the body-fixed frame and denoted by the vector \( \Omega \in \mathbb{R}^3 \). The attitude kinematics is given by

\[
\dot{R} = R \Omega^\times,
\]

(1)

where \((\cdot)^\times : \mathbb{R}^3 \to \mathfrak{so}(3)\) is the skew-symmetric cross product operator (matrix), and \(\mathfrak{so}(3)\) is the Lie algebra of \(\text{SO}(3)\), identified with the linear space of \(3 \times 3\) skew-symmetric matrices. The attitude dynamics of the rigid body is given by

\[
J\dot{\Omega} = J\Omega \times \Omega + \tau_e,
\]

(2)

where \(\tau_e \in \mathbb{R}^3\) is an external disturbance torque applied to the rigid body about its center of mass and \(J\) is the body’s inertia matrix represented in the body-fixed coordinate frame.

III. FINITE-TIME OBSERVER

Let \((\tilde{R}, \tilde{\Omega}) \in \text{SO}(3) \times \mathbb{R}^3\) denote the state estimates provided by the observer, based on the actual states \((R, \Omega)\), in continuous time. Define \(\tilde{\Omega} := \tilde{R}^T R\) and the attitude and the angular velocity state estimate errors as

\[
\begin{align*}
\hat{R} & := \tilde{R}^T R, \\
\hat{\Omega} & := \Omega - \tilde{\Omega}.
\end{align*}
\]

(3)

(4)

The proposed observer for the attitude and angular velocity takes the form

\[
\begin{align*}
\dot{\hat{R}} & = \hat{R} \hat{\Omega}^\times, \\
J\dot{\hat{\Omega}} & = J\hat{\Omega} \times \hat{\Omega} + \tau_e - \tau_\Omega,
\end{align*}
\]

(5)

(6)

where \(\tau_\Omega\) is a feedback term to be designed such that the angular velocity estimate \(\hat{\Omega}\) can be obtained from the intermediate variable \(\tilde{\Omega}\) that satisfies equation (6). From the attitude kinematics and dynamics equations (1)-(2) and the observer equations (5)-(6), the estimation error states \((\hat{R}, \hat{\Omega})\) are found to satisfy

\[
\begin{align*}
\dot{\hat{R}} & = \hat{R} \hat{\Omega}^\times, \\
J\dot{\hat{\Omega}} & = \tau_\Omega.
\end{align*}
\]

(7)

(8)

A. Some Preliminary Results

A couple of lemmas, that are used to prove the main result, are given below.

Lemma 3.1: Let \(a\) and \(b\) be non-negative real numbers and let \(p \in (1, 2)\). Then

\[
a^{(1/p)} + b^{(1/p)} \geq (a + b)^{(1/p)}.
\]

(9)

Moreover the above inequality is a strict inequality if both \(a\) and \(b\) are non-zero.

Below is another lemma required to prove the main result.

Lemma 3.2: Let \(A = \text{diag}(a_1, a_2, a_3)\) where \(a_1 > a_2 > a_3 \geq 1\) and let \((R, \Omega)\) denote the attitude (rotation matrix) and angular velocity vector of a rotating rigid body. Define

\[
s(R) = \sum_{i=1}^3 a_i (R^T e_i) \times e_i
\]

such that

\[
\frac{d}{dt} \text{Tr}(A - AR) = \Omega^T s(R),
\]

(10)

where \(e_i\) for \(i = 1, 2, 3\) are the column vectors of the \(3 \times 3\) identity matrix, \(I\). Let \(S \subset \text{SO}(3)\) be a closed subset containing the identity in its interior, defined by

\[
S = \{ R \in \text{SO}(3) : R_{ii} \geq 0 \text{ and } R_{ij} R_{ji} \leq 0 \forall i, j \in \{1, 2, 3\}, i \neq j \}.
\]

(11)

Then for \(R \in S\), we have

\[
s(R)^T s(R) \geq \text{Tr}(A - AR).
\]

(12)

The proof of 3.1 and 3.2 can be found in [19].

B. Observer Design

The main result and its proof are given here.

Theorem 3.1: Consider the attitude error dynamics of a rigid body as given by (8). Further, let \(k_1 > 0\) and define

\[
\begin{align*}
\dot{z}(\tilde{R}) & = s(\tilde{R}) \\frac{s(\tilde{R})}{(s(\tilde{R}) T s(\tilde{R}))^{(1-1/p)}}, \\
\Psi(\tilde{R}, \tilde{\Omega}) & = \tilde{\Omega} + k_1 z(\tilde{R}), \\
w(\tilde{R}, \tilde{\Omega}) & = \frac{d}{dt} s(\tilde{R}) = \sum_{i=1}^3 a_i e_i \times (\tilde{\Omega} \times \tilde{R}^T e_i).
\end{align*}
\]

(13)

(14)

(15)

Let \(k_p, k_v > 0\) be positive observer gains such that \(k_1 k_p \geq k_v\), let \(L\) be a positive definite \(3 \times 3\) observer gain matrix such that \(L - J\) is positive semi-definite, where \(J\) is the moment of inertia matrix of the rigid body, and let \(p \in (1, 2)\) be a ratio of two positive odd integers. The observer feedback law

\[
\tau_\Omega = -\frac{k_v}{(\Psi(\tilde{R}, \tilde{\Omega})^T L \Psi(\tilde{R}, \tilde{\Omega}))^{(1-1/p)}} L \Psi(\tilde{R}, \tilde{\Omega}) - k_p s(\tilde{R})
\]

\[
- \frac{k_1}{(s(\tilde{R})^T s(\tilde{R}))^{(1-1/p)}} J H(s(\tilde{R})) w(\tilde{R}, \tilde{\Omega}) - \tau_e,
\]

(16)

where \(H : \mathbb{R}^3 \to \text{Sym}(3)\), the space of symmetric \(3 \times 3\) matrices, is defined by

\[
H(x) = I - \frac{2(1 - 1/p)}{x^T x} x x^T,
\]

(17)

stabilizes the attitude error dynamics of the rigid body to \((\hat{R}, \hat{\Omega}) = (I, 0)\) almost globally over the state space in finite time.

Proof: Define the Lyapunov function

\[
V(\tilde{R}, \tilde{\Omega}) = \frac{1}{2} \Psi(\tilde{R}, \tilde{\Omega})^T J \Psi(\tilde{R}, \tilde{\Omega}) + k_p \text{Tr}(A - A\tilde{R}),
\]

(18)
where $\Psi(\tilde{R},\tilde{\Omega})$ is as defined in (14). The time derivative of this Lyapunov function is
\[
\dot{V} = \Psi(\tilde{R},\tilde{\Omega})^T \left( \frac{k_1}{(s(\tilde{R})^Ts(\tilde{R}))^{1-1/p}} JH(s(\tilde{R})) w(\tilde{R}, \tilde{\Omega}) \right) + J\dot{\tilde{\Omega}} + k_p \dot{\tilde{\Omega}}^T s(\tilde{R}).
\]
On substituting the observer law (16) in the above expression, we get
\[
\dot{V} = -k_v (\Psi(\tilde{R}, \tilde{\Omega})^T L \Psi(\tilde{R}, \tilde{\Omega}))^{1/p} - k_1 k_p (s(\tilde{R})^Ts(\tilde{R}))^{1/p},
\]
which is negative semi-definite in the states.

From the expression (19), we see that the set where $\dot{V} = 0$ is given by
\[
\dot{V}^{-1}(0) = \{(\tilde{R}, \tilde{\Omega}) : s(\tilde{R}) = 0 \text{ and } \Psi(\tilde{R}, \tilde{\Omega}) = 0 \}
= \{(\tilde{R}, \tilde{\Omega}) : s(\tilde{R}) = 0 \text{ and } \tilde{\Omega} = 0 \}. \tag{20}
\]
From prior research on almost global (asymptotic) attitude stabilization and tracking [21], [22], [9], [23], [24], [25], we know that the subset of $SO(3)$ where $s(\tilde{R}) = 0$, is the set of critical points for $\text{Tr}(A - A\tilde{R})$, which is given by:
\[
\mathcal{C} \hat{=} \{ (I, \text{diag}(1,-1,-1), \text{diag}(-1,1,-1), \text{diag}(-1,-1,1)) \subset SO(3). \tag{21}
\]
Therefore, the subset of the state space where $\dot{V} = 0$ is
\[
\dot{V}^{-1}(0) = \{(\tilde{R}, \tilde{\Omega}) : \tilde{R} \in \mathcal{C}, \tilde{\Omega} = 0 \} \subset SO(3) \times \mathbb{R}^3. \tag{22}
\]
This subset is also the set of equilibria of the system given by equations (7)-(8) and its largest invariant set. As shown in [21], [22], [9], $\text{Tr}(A - A\tilde{R}) : SO(3) \to \mathbb{R}$ is a Morse function and the identity element $I \in SO(3)$ is the minimizer of this function. Hence, the equilibrium $(I, 0)$ of the feedback system is locally attractive. Moreover, since $\text{Tr}(A - A\tilde{R})$ is a Morse function, the other three elements of its set of critical points $\mathcal{C}$ are nondegenerate critical points [26] that also give rise to equilibria of the feedback system (7)-(8) and (16). We label these equilibria as follows:
\[
\mathcal{E}_1 = \{ \tilde{R} = \tilde{R}_1 = \text{diag}(1,-1,-1), \tilde{\Omega} = 0 \}, \tag{23a}
\mathcal{E}_2 = \{ \tilde{R} = \tilde{R}_2 = \text{diag}(-1,1,-1), \tilde{\Omega} = 0 \}, \tag{23b}
\mathcal{E}_3 = \{ \tilde{R} = \tilde{R}_3 = \text{diag}(-1,-1,1), \tilde{\Omega} = 0 \}. \tag{23c}
\]
In the almost globally asymptotically stable attitude control schemes in [21], [22], [9], these equilibria in the set $\dot{V}^{-1}(0)$ are hyperbolic equilibria of the feedback attitude dynamics. For the observer obtained here, the feedback system is Hölder continuous with exponent less than one, and not Lipschitz continuous. However, the dynamics of the estimation errors given by (8) and (16) can be transformed to that of the following Lipschitz continuous system:
\[
J\hat{\tilde{\Omega}} = W(\tilde{R}, \tilde{\Omega}) \tau_{\tilde{\Omega}}, \quad \text{where} \quad W(\tilde{R}, \tilde{\Omega}) = (\Psi(\tilde{R}, \tilde{\Omega})^T L \Psi(\tilde{R}, \tilde{\Omega}))(s(\tilde{R})^Ts(\tilde{R}))^{2+2/p}. \tag{24}
\]
This Lipschitz continuous system can be shown to be almost globally asymptotically stable at $(\tilde{R}, \tilde{\Omega}) = (I, 0)$ with the same domain of attraction as the equivalent schemes in [21], [22], [9]. Note that $W(\tilde{R}, \tilde{\Omega})$ is positive except at the equilibrium states of this observer. Therefore, the phase diagrams of the finite-time stable Hölder continuous system (7), (8) and (16) and the asymptotically stable Lipschitz continuous system (7), (24) and (16) are similar. In particular, the domains of attraction of the equilibria of the two systems are identical. Therefore, the stable equilibrium $(\tilde{R}, \tilde{\Omega}) = (I, 0)$, has a domain of attraction that is almost global.

Trajectories starting in the almost global domain of attraction of $(I, 0)$ start in the subset $S \times \mathbb{R}^3 \subset SO(3) \times \mathbb{R}^3$ or enter this subset in finite time, where $S$ is the neighborhood of $I \in SO(3)$ defined by (11) in Lemma 3.2. Substituting the inequality (12) from Lemma 3.2 in the expression (19), we obtain for $(\tilde{R}, \tilde{\Omega}) \in S \times \mathbb{R}^3$,
\[
\dot{V} \leq -k_v \left( (\text{Tr}(A - A\tilde{R}))^{1/p} + (\Psi(\tilde{R}, \tilde{\Omega})^T L \Psi(\tilde{R}, \tilde{\Omega}))^{1/p} \right), \tag{26}
\]
since $k_1 k_p \geq k_v$ and $L - J$ is positive semi-definite. This gives a guaranteed rate of decrease of the Lyapunov function along the system given by equations (7)-(8) of the rigid body. Finally, using the inequality (9) in Lemma 3.1, we obtain
\[
\dot{V} \leq -k_v \left( (\text{Tr}(A - A\tilde{R})) + (\Psi(\tilde{R}, \tilde{\Omega})^T \Psi(\tilde{R}, \tilde{\Omega})) \right)^{1/p}.
\]
\[
\dot{V} \leq -k_v V^{1/p}. \tag{27}
\]
Therefore, since $p > 1$, all initial states of the system given by equations (7)-(8) which start in the open and dense subset $SO(3) \times \mathbb{R}^3 / \mathcal{M}$ and for which the value of the Lyapunov function $V$ is finite, converge to the equilibrium $(I, 0)$ in finite time [27]. Moreover, as $p < 2$ in the observer feedback law given by (16), this observer feedback remains bounded and in fact converges to the zero vector as $s(\tilde{R})$ and $\tilde{\Omega}$ both converge to the zero vector, due to the fact that
\[
\lim_{\|x\| \to 0} \frac{x}{\|x\|^{2-2/p}} = 0
\]
for $x \in \mathbb{R}^3$. If $V_S$ is the value of $V$ when the state first enters $S \times \mathbb{R}^3$, then the remaining time duration in which $V$ converges to zero is upper bounded by
\[
\Delta t = \frac{pV_S^{(1-1/p)}}{k_v(p - 1)}.
\]
Therefore, the stable equilibrium $(I, 0)$ of the system given by eqns. (7), (8), and (16) is almost globally finite time stable.

\section*{IV. Numerical Simulation Results}

This section presents numerical simulation results for the almost global finite time attitude observer outlined in the previous section. For these numerical simulations, a numerical integration scheme based on the Lie group variational integrator given in the following discussion, is used. Attitude and angular velocity are assumed to be obtained from sensor measurements.
A. Numerical Scheme and Discrete Equations of Motion

The continuous equation of motion (1) ensures that \( R \) evolves on the special orthogonal group \( SO(3) \). However, general-purpose numerical integration methods, such as unstructured Runge-Kutta schemes, do not preserve first integrals nor the geometry of the state space of rigid body attitude motion. Variational integrators and Lie group methods provide a systematic method of constructing structure-preserving and state space geometry-preserving numerical integrators [28]. Using the results given in [29], a Lie group variational integrator for the attitude dynamics of a rigid body is given by

\[
\begin{align*}
\Omega_{k+1} &= J^{-1}\left(F_k^T J \Omega_k + h(\tau_e)_k\right), \\
(J\Omega_k)^x &= \frac{1}{h}(F_k J - J F_k^T), \\
R_{k+1}^b &= \exp(-h(\omega_{0_e} e_2)^x) R_k^b F_k,
\end{align*}
\]

where the subscript \( k \) denotes variables at the \( k \)th time step, and \( h \in \mathbb{R} \) is the integration step size. \( J \in \mathbb{R}^{3 \times 3} \) is a nonstandard moment of inertia matrix defined by \( J = \frac{1}{2} \text{Tr}(J) I - J \). The constant \( \omega_0 \in \mathbb{R} \) is the orbital angular velocity, and \( R^b \) is the rotation matrix from the body fixed frame to the Local Vertical Local Horizontal (LVLH) coordinate frame respectively. An iterative process such as Newton-Raphson is used to solve for \( F_k \) from equation (29).

The external moment in discrete time, \( (\tau_e)_k \), is obtained from the primary external torques acting on a satellite in a circular low Earth orbit (LEO), as follows:

\[
(\tau_e)_k = (\tau_g)_k + (R_k^b)^T (\tau_d)_k,
\]

where \( (\tau_g)_k \) is the gravity gradient moment given in [29] as \( (\tau_g)_k = 3\omega_0^2(R_k^b)^T e_3 \times J(R_k^b)^T e_3 \). The torque \( \tau_d \) is a disturbance torque that is constant in the LVLH frame, and accounts for torques produced by atmospheric drag and smaller effects due to solar radiation pressure, geomagnetism, etc.

B. Numerical Simulations

The observer designed here is applied to two numerical simulations of a satellite in a circular low Earth orbit (LEO), with an orbit altitude of 200 km. In the first simulation, perfect measurements with no disturbance torque acting on the spacecraft is considered. The second simulation takes measurement noise and an external disturbance torque into consideration. For both simulations the following parameters are used. The moment of inertia of the spacecraft is given by \( J = \text{diag}(12, 3, 4) \) kg m\(^2\), the step size \( h = 0.01 \) s, the diagonal gain matrix \( A = \text{diag}(1.2, 1.1, 1.0) \), and the positive definite observer gain matrix \( L = [13 1 1; 1 4 1; 1 1 5] \).

Let the scalar control gains be \( k_1 = 0.00025, k_p = 40, k_v = 0.01 \), and \( p = 21/19 \). We chose the initial value for \( R_0^b \) to be \( \expm(\nu) \) where \( \nu = [-1, 0.5, 2]^T \) and the initial estimated attitude is \( \hat{R}_0 = I_{3 \times 3} \). The initial angular velocity is \( \Omega_0 = [\pi/120, \pi/60, -\pi/90]^T \) rad/s and the initial estimated angular velocity is \( \hat{\Omega}_0 = [0, 0, 0]^T \) rad/s.

1) First simulation: In all plots of vector quantities, the \( x-, \) \( y-, \) and \( z- \) components are given by the blue, green, and red lines, respectively. Fig. 1 shows the principal angle of rotation over a time period of 50 s for the natural motion of the tumbling satellite. Fig. 2 shows the principal angle of rotation over a time period of 50 s for the natural motion of the tumbling satellite.

Fig. 1. Principal angle of rotation of satellite

Fig. 2. Angular velocity components of satellite

Fig. 3. Principal angle estimation error

Fig. 4. Angular velocity estimation error
converges after about 10 s to the zero vector. These numerical results agree with the theoretical result of Theorem 3.1, which says that in the absence of external disturbances and measurement noise, one obtains almost global finite-time stable convergence of $(\tilde{R}, \tilde{\Omega})$ to $(I, 0)$.

2) Second simulation: In this simulation, the performance of the given observer is simulated in the presence of an external disturbance torque and measurement noise. The noise in the attitude measurement is given by $\delta R \in \mathbb{R}^3$, which is a vector of zero mean Gaussian processes with standard deviation $\sigma_R = 1^\circ$. The angular velocity measurement is corrupted by a vector of zero mean Gaussian processes with standard deviation $\sigma_\Omega = 0.1^\circ/\text{s}$ and is denoted by $\delta_\Omega$.

The measurements are described by

$$R_n = \exp((\delta R)\times)R, \quad \Omega_n = \Omega + \delta_\Omega.$$  

External torques acting on the satellite are given by the expression (31), with the gravity-gradient torque $\tau_g$ known, and the constant disturbance torque given by $\tau_d = [3, -3, 2]^T \times 10^{-3}$ Nm. Fig. 5 displays the measured principal of rotation, which is corrupted by measurement noise and includes the effect of the disturbance torque. In Fig. 6 the noisy angular velocity measurement that includes the influence of the disturbance torque is displayed. Fig. 7 gives the time plot of the principal angle of the attitude estimation error in the presence of measurement noise and the disturbance torque. The attitude estimation error converges to a small neighborhood of the identity after about 10 s and the principal angle corresponding to this error stays bounded within a band of $\pm 0.06$ rad.

Fig. 8 displays the components of the vector of angular velocity estimation errors in the present of measurement noise and the disturbance torque. The components of the angular velocity estimation error converge to a small neighborhood of zero after around 10 s, and stay bounded within a $\pm 0.05$ rad/s band.

In Fig. 9 the time plots of the undisturbed attitude estimation error (dark color) from the first simulation (without noise and external disturbance) and the attitude estimation error (light color) from the second simulation (with noise and external disturbance) are plotted together for comparison.

Fig. 10 displays the difference between the time evolutions of the angular velocity estimation error from both simulations. The darker color plots are for the system without noise and external disturbance and the lighter color plots are for the system with noise and external disturbance.
The unmodeled disturbance torque components acting on the spacecraft are displayed in Fig. 11.

V. CONCLUSION

An almost global finite-time stable state observer for rigid body attitude motion is presented. When the rigid body attitude dynamics model is known and there is no measurement noise, the observer guarantees almost global finite-time stable convergence of attitude motion state estimates to the actual states. In the presence of bounded measurement noise and bounded unknown disturbance torque, the state estimates converge to a neighborhood of the desired final attitude state.

These convergence properties are verified through numerical simulation of a realistic scenario for a rigid satellite in a low earth orbit with gravity gradient torque and an unmodeled disturbance torque acting on the rigid body, along with measurements that are corrupted by zero mean Gaussian noise processes. In future work, this scheme will be extended to a finite-time stable state observer for the coupled attitude and translational motion of a rigid body, and then combined with a discrete-time filtering scheme to propagate state estimates between successive discrete-time state measurements.

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