Communication Systems Laboratory

Trellis Codes With Low Ones Density For The OR Multiple Access Channel

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Outline

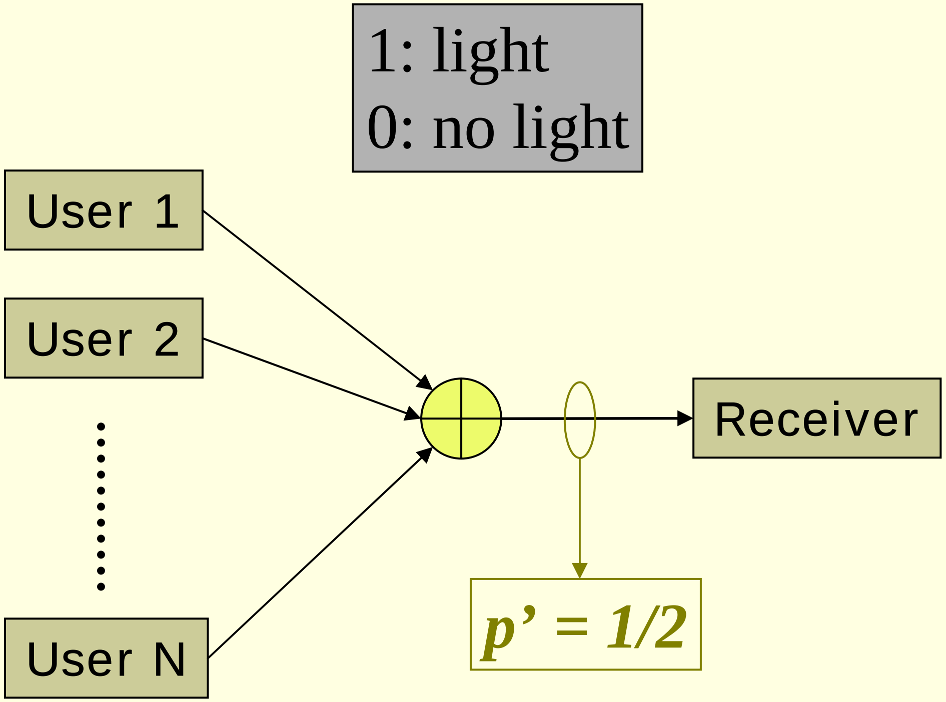
- Motivation : Uncoordinated Multiple Access to the Optical Channel : the OR Channel.
- IDMA-based architecture.
    - Treating other users as noise : The Z channel.
- The need for non-linear codes in this application.
- Non-linear Trellis Codes (NLTC).
  - Design.
  - Analytical bounds for the BER.
- Concatenation Block Code + NLTC.
- Simulations
- Conclusions
- Ongoing work

Motivation: Multiple Access to Optical Channels

- Uncoordinated Multiple Access to the Optical Channel.
- Optical Channels:
  - provide very high data rates, up to tens to hundreds of gigabits per second.
  - Typically deliver a very low Bit Error Rate $(BER < 10^{-9})$
- Wavelength Division (WDMA) or Time Division (TDMA) are the most common forms of Multiple Access today.
  - However, they require considerable coordination.
- Goal:
  - Provide uncoordinated access (for large number of users).
  - Maximize the rate at feasible complexity for optical speeds.
  - Satisfy $BER < 10^{-9}$.
- Strong complexity & latency constraint.

Model: The OR Multiple Access Channel (OR-MAC)

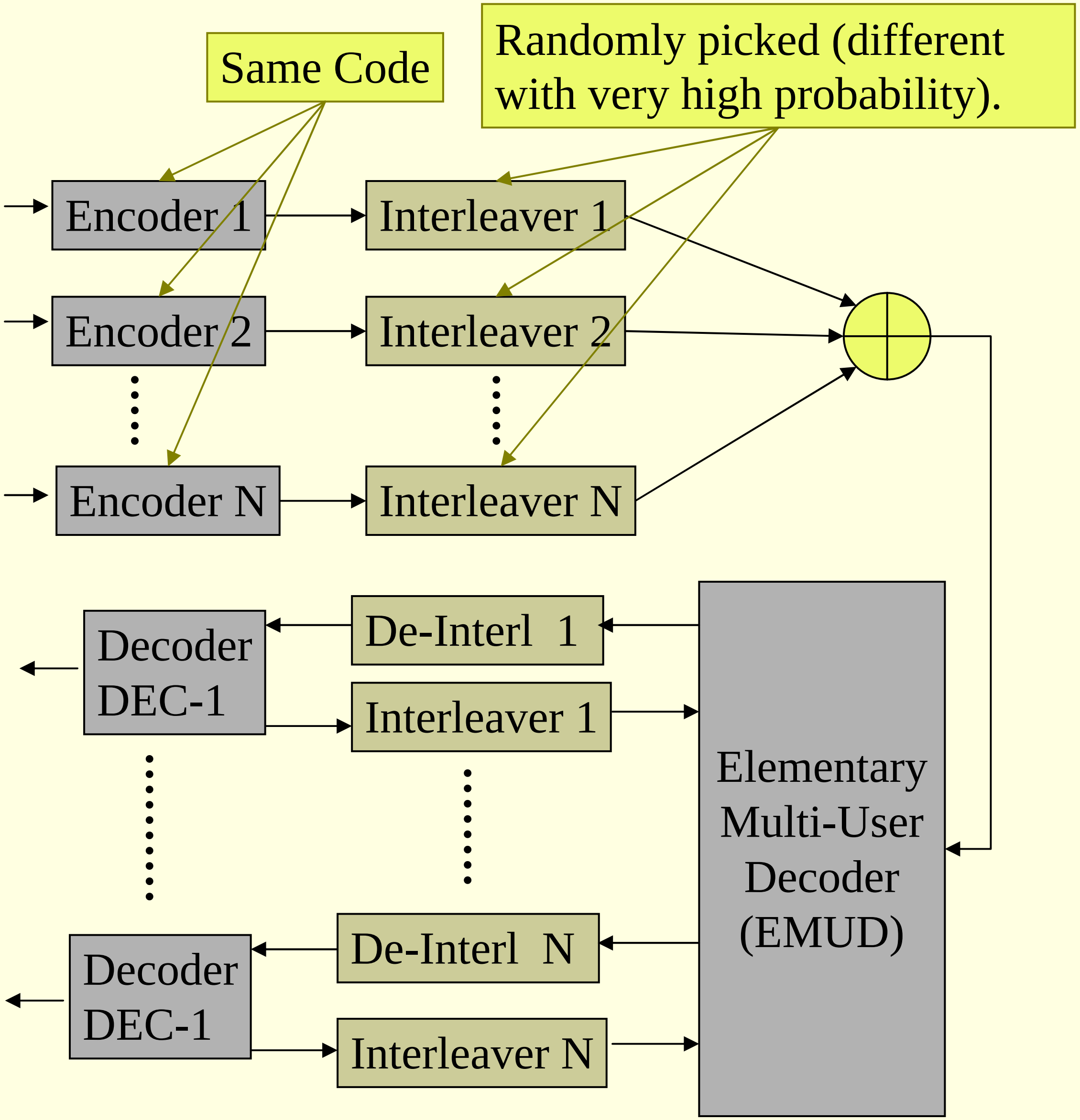
- Basic model for multiple-user optical channel with non-coherent combining.
- 0+X=X, 1+X=1
- N users, all transmitting with the same ones density p: P(X=1)=p, P(X=0)=1-p.



- Theoretically: Sum-rate = 1 (100% efficiency) can be achieved with a ones density in the transmission of

$$p(N) = 1 - (1/2)^{1/N} \approx \frac{\ln(2)}{N}$$

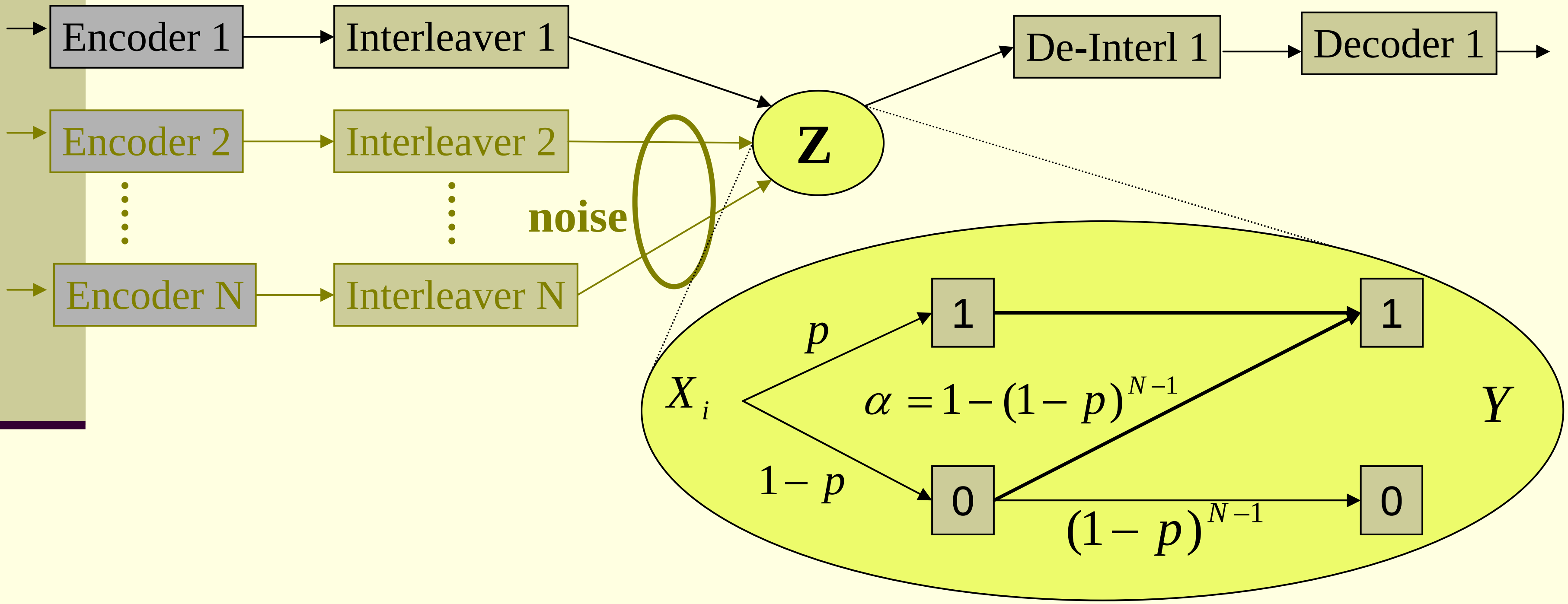
IDMA-Based Architecture



- [Ping *et al.*'03] for general MAC.
- With appropriately designed codes it works over the OR-MAC.
- Joint Iterative decoding.
- For a large number of users joint decoding may not be computationally feasible for optical speeds today.

Treating other users as noise: Z-Channel

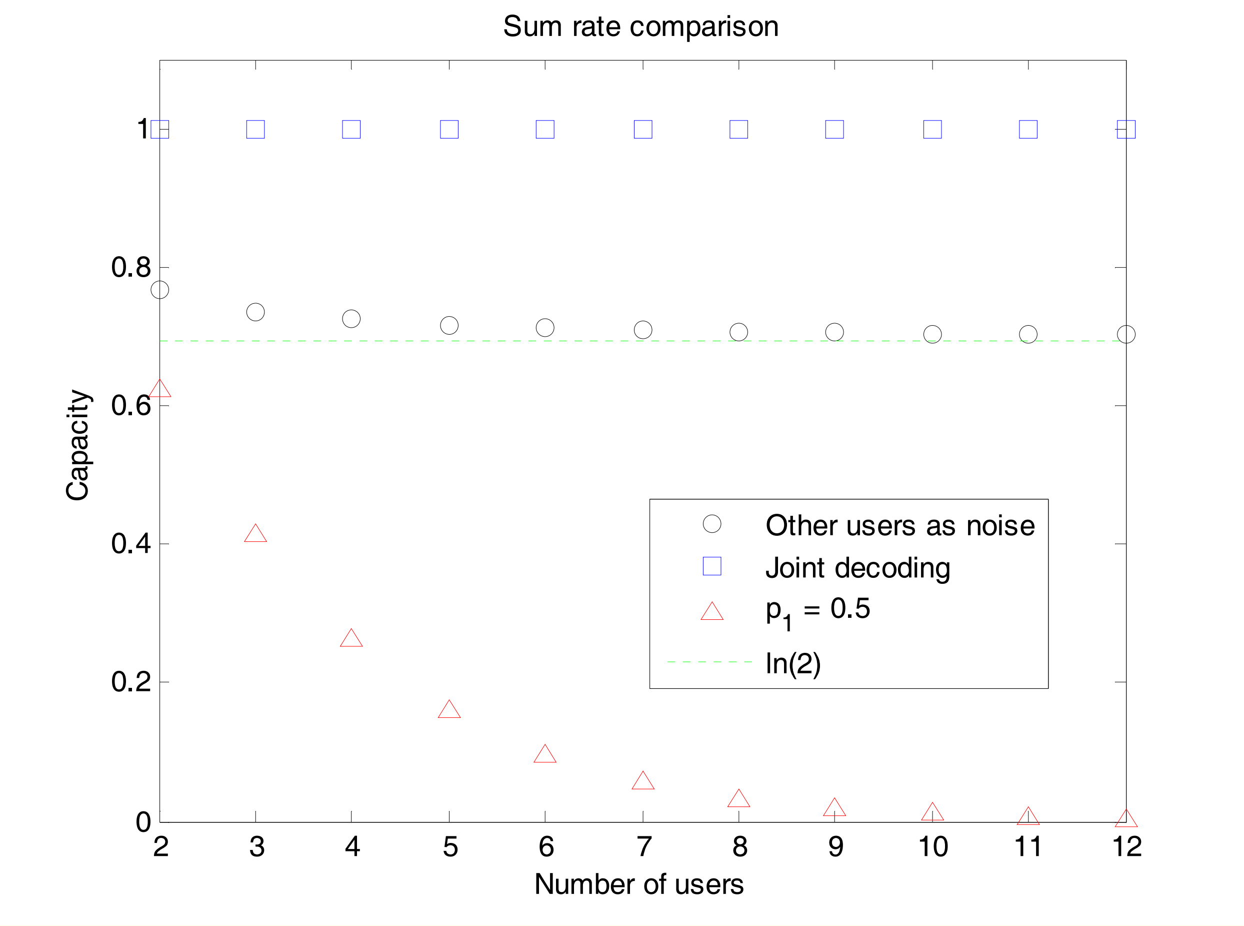
- A practical alternative is to treat all but a desired user as noise.
- When treating other users as noise in an OR-MAC, each user "sees" a Z-Channel.



- The sum-rate is lower bounded by ln(2) (around 70%), for any number of users.

Non-linear codes are required

- Optimal ones density: $p(N) = 1 - (1/2)^{1/N} \approx \frac{\ln(2)}{N}$

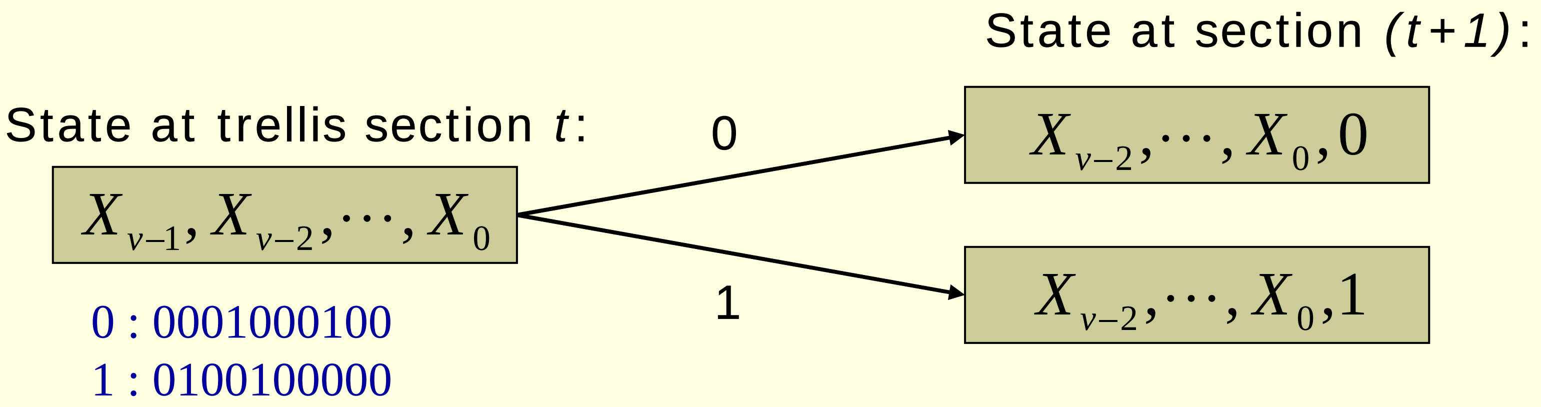


Optimal ones densities:

Users	Joint	Others noise
2	0.293	0.286
6	0.109	0.108
12	0.056	0.056

Non-linear Trellis Codes

- Desired ones density *p* is given (by number of users *N).*
- (n,1) feed-forward encoder: 1 input, n output bits per trellis section
- $S = 2^{v}$ states.



State at trellis section *t*:

$X_{v-1}, X_{v-2}, \cdots, X_0$

$0 : 0001000100$

$1 : 0100100000$

State at section *(t+1)*:

$X_{v-2}, \cdots, X_0, 0$

$X_{v-2}, \cdots, X_0, 1$

- Outputs are given by a look-up table.
- Design: Create the look-up table, assign output values to the 2S branches of the trellis
- Goal: Maximize the minimum distance of the code maintaining the desired ones density *p.*

Metric for Z-Channel

- The metric for the Viterbi decoding algorithm for the Z-Channel is the number of 0-1 transitions.
- Since the Z-Channel is asymmetric, the Hamming distance is not a proper definition of distance between codewords.
- Directional distance between two codewords c_1 and c_2 (denoted $d_D(c_1, c_2)$) is the number of positions at which c_1 has a 0 and c_2 has a 1.
- ‘Greedy’ definition of pairwise distance:

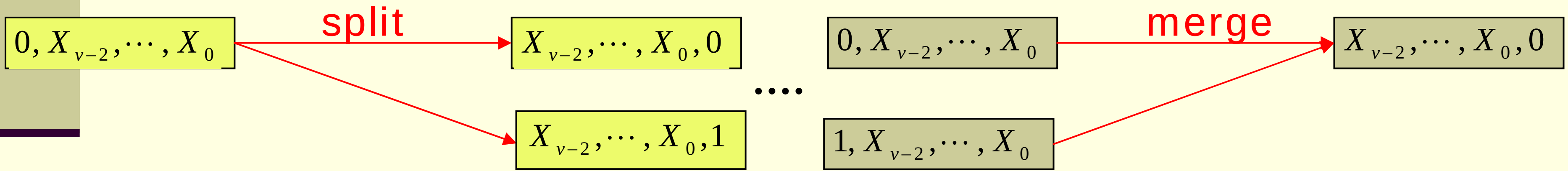
$$d\left(c_i, c_j\right) = d\left(c_j, c_i\right) = \min\left[d_D\left(c_i, c_j\right), d_D\left(c_j, c_i\right)\right]$$

Design technique

1. Choose *n*, the number of output bits per trellis section, to satisfy a certain target sum-rate *N/n*.
2. Assign the Hamming weight of the output of each branch, to satisfy the optimal ones density *p*. $$\begin{cases} w = \text{floor}(p \cdot n) \\ w+1 \end{cases}$$
3. For each branch, choose the positions of each of the *w (w+1)* ones.

Extension to Ungerboeck's rule

- Ungerboeck:
  - Every incorrect codeword, in its trellis representation, departs from the correct path (split), and returns to the correct path (merge) at least once.
  - Maximize the distance between a split.
  - Maximize the distance between a merge.



split

$0, X_{v-2}, \cdots, X_0$

$X_{v-2}, \cdots, X_0, 0$

$X_{v-2}, \cdots, X_0, 1$

....

merge

$0, X_{v-2}, \cdots, X_0$

$1, X_{v-2}, \cdots, X_0$

$X_{v-2}, \cdots, X_0, 0$

0 : 0100100000

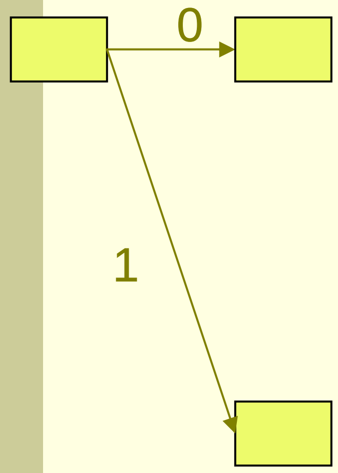
1 : 0010000010

Example: w = 2, n = 10

Maximum possible distance between two branches : 2

Extending Ungerboeck’s rule

- One can extend Ungerboeck’s rule into the trellis.



Maximize

Extending Ungerboeck’s rule

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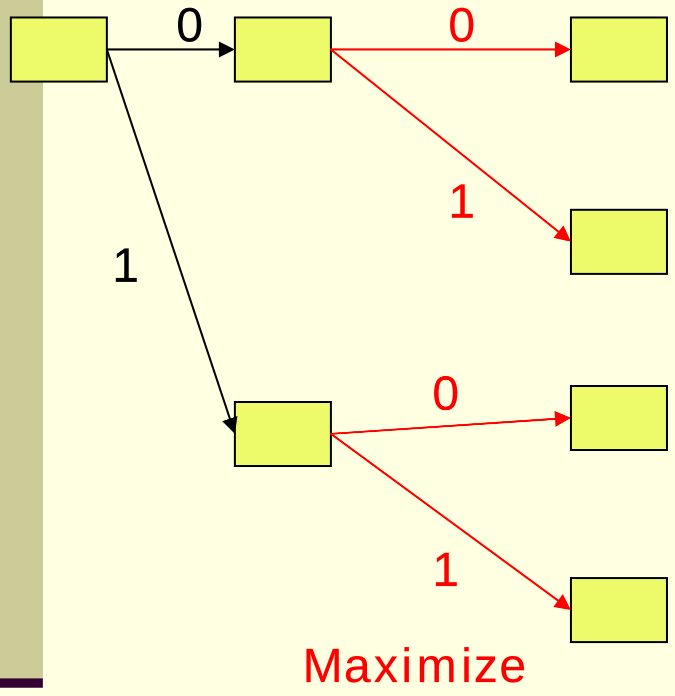

Extending Ungerboeck's rule

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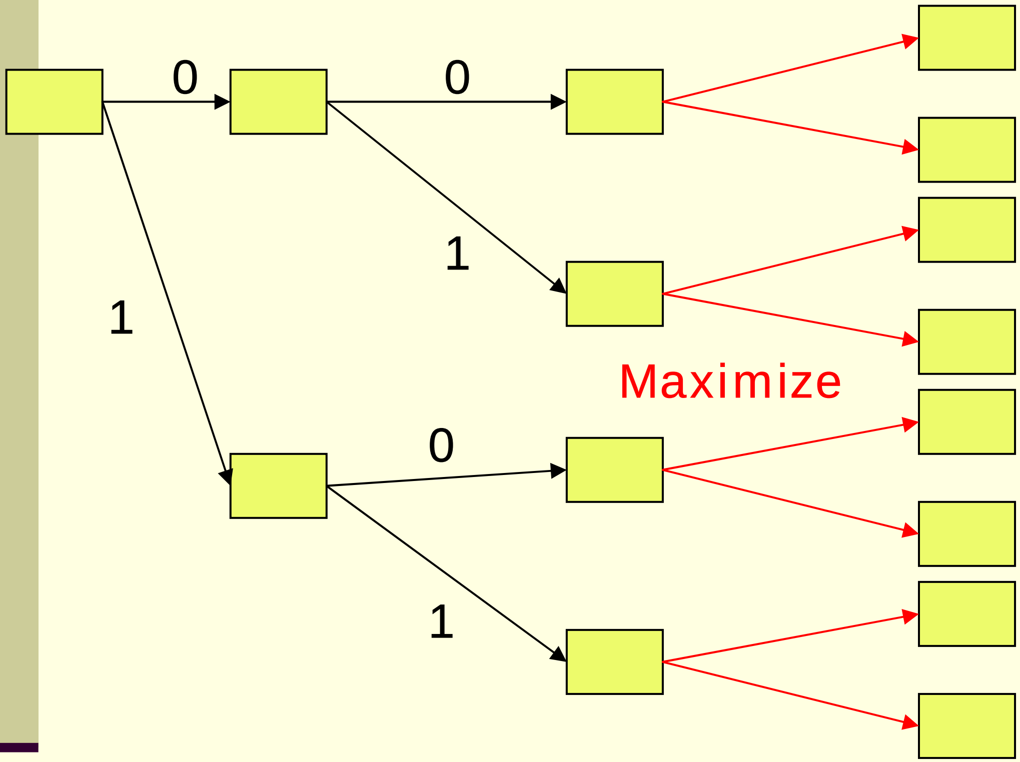


Note that by maximizing the distance between the 8 branches, coming from a split 2 trellis section before, we are maximizing all groups of 4 branches coming from a split in the previous trellis section, and all splits.

- The same idea can be applied for the merge, moving backwards in the trellis.
- If we move *h* trellis sections forward from a split (including the split), and *g* sections backwards from a merge (including the merge), then:

Of course, there is a limit for *h* and *g*, given by the Hamming weights of the outputs, *h*, and the number of states (branches).

$$d_{\min} \geq (w-1)(h+g)+v+1$$

Bit Error Bound for the Z-Channel

- We use the transfer function bound technique on [Viterbi '71] for linear codes, and extended by [Biglieri '90] for non-linear codes, modifying the pairwise error probability measure.
- Given two codewords $X^n, \hat{X}^n$

$$P_e\left(X^n \to \hat{X}^n\right) + P_e\left(\hat{X}^n \to X^n\right) =$$

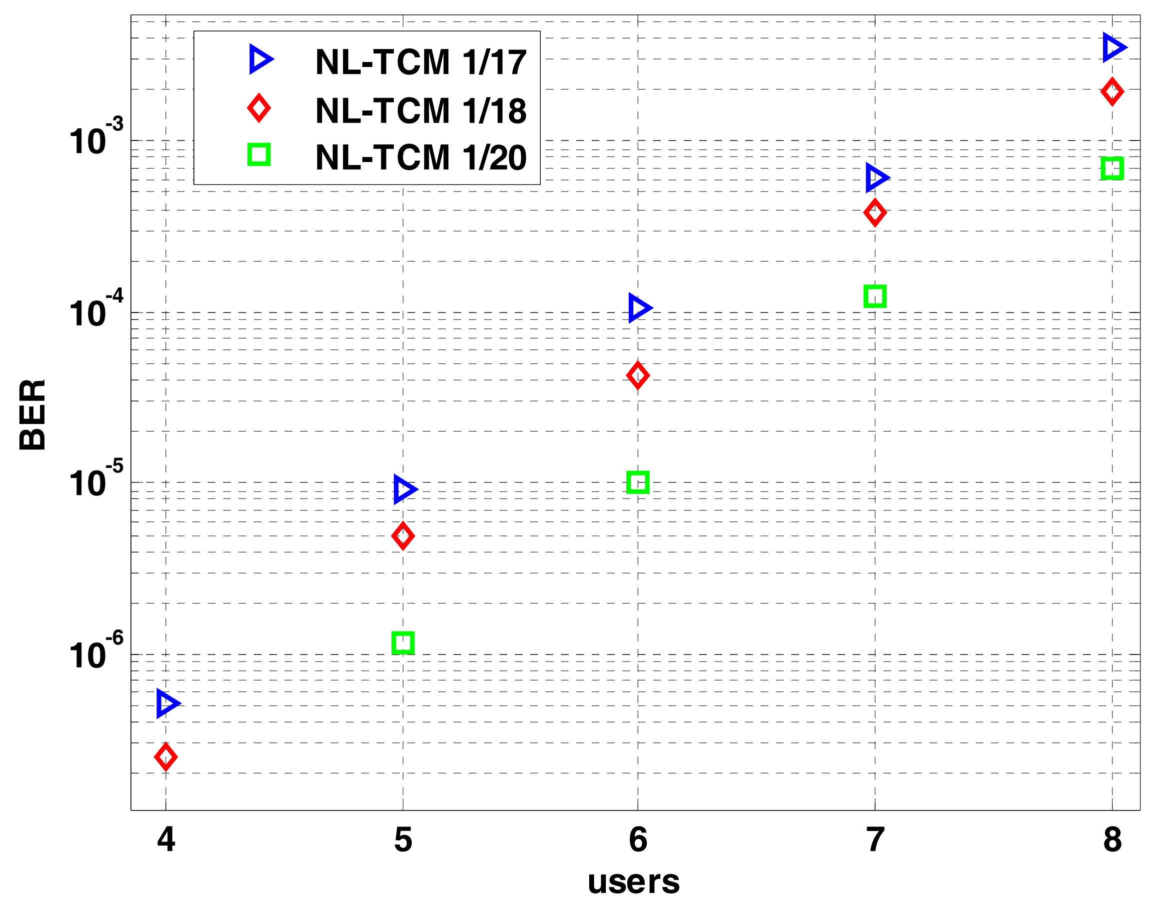
$$\alpha^{\max\left(d_D(X^n,\hat{X}^n),d_D(\hat{X}^n,X^n)\right)} \le \frac{\alpha^{d_D(X^n,\hat{X}^n)}}{2} + \frac{\alpha^{d_D(\hat{X}^n,X^n)}}{2}$$

- Replace $P_e\left(X^n \to \hat{X}^n\right)$ with $\dfrac{\alpha^{d_D(X^n,\hat{X}^n)}}{2}$ and the

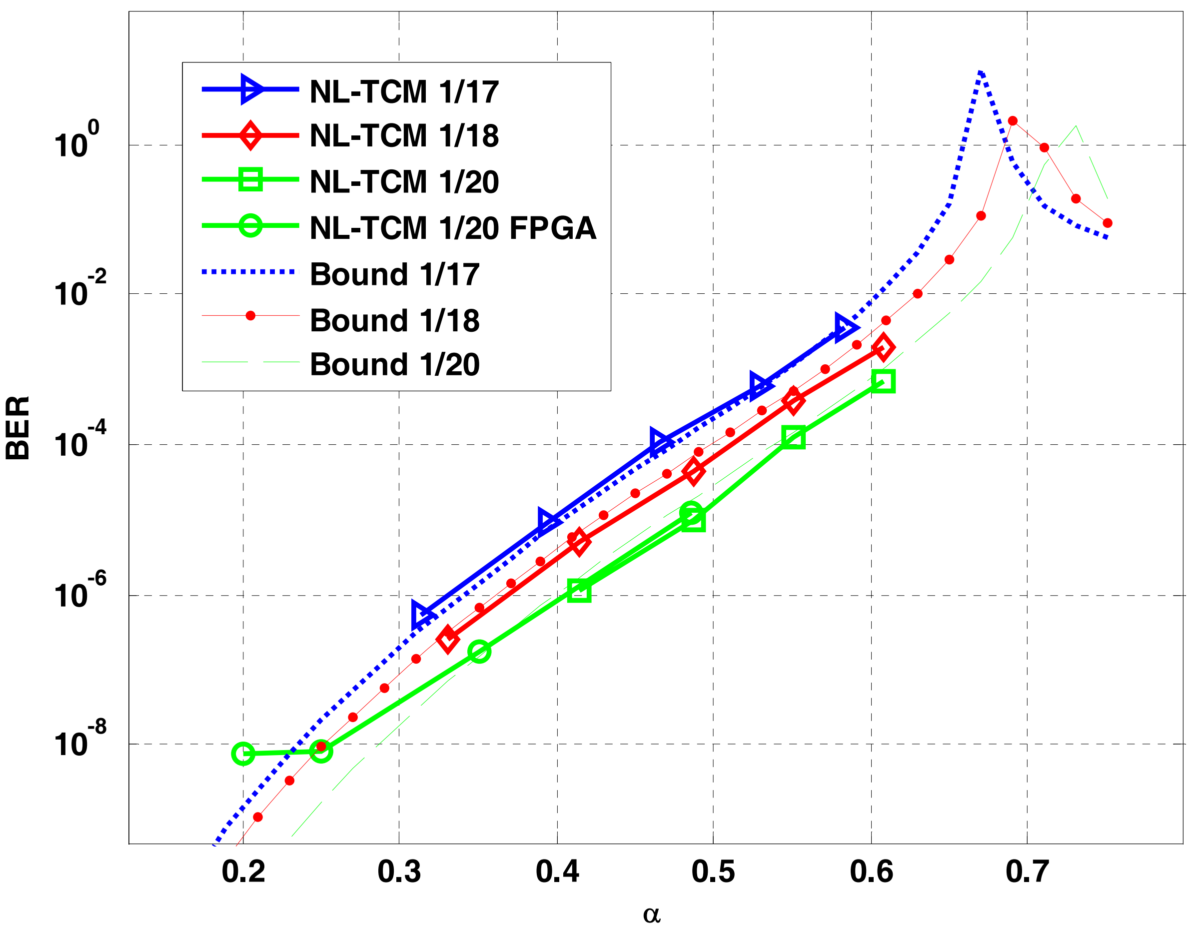
  transfer function bound technique can be readily applied to the NLTC to yield an upper bound to its BER over the Z-Channel.

Results : 6-user OR-MAC

- 64-State non-linear trellis code.



Results : 6-user OR-MAC



Large number of users

- Main result:
  - For any number of users, we achieve the same sum-rate with similar performance.

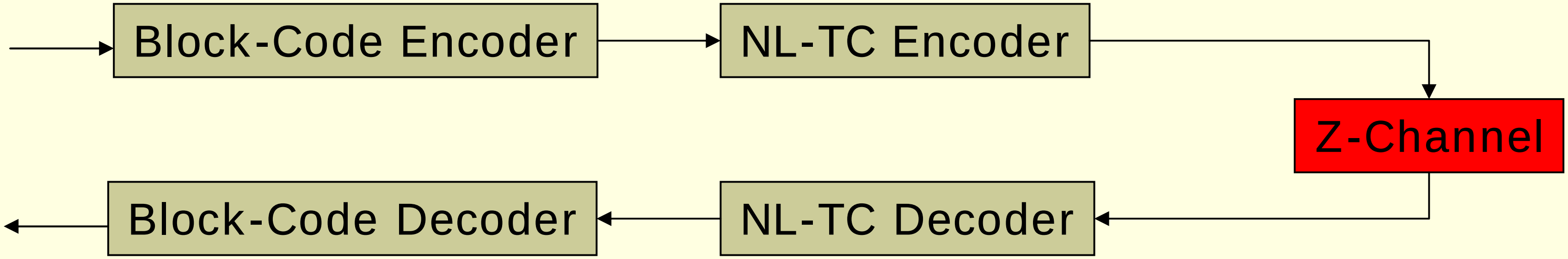
N	n	SR	α	BER
6	20	0.3	0.439	1.0214×10^{-5}
100	344	0.291	0.4777	1.1046×10^{-5}
300	1000	0.3	0.4901	1.2157×10^{-5}
900	3000	0.3	0.4906	1.2403×10^{-5}
1500	5000	0.3	0.4907	1.2508×10^{-5}

Large number of users

- **For any number of users we achieve the same sum-rate with similar performance.**
- Intuitive explanation:
  - As the number of users increases:
  - The optimal ones density decreases.
  - The individual rate decreases: *n* increases.
  - The output Hamming weight *w* stays the same.
  - The cross-over probability α increases.
  - We can extend further into the trellis Ungerboeck’s idea, increasing the minimum distance.
  - There is a point in which all the outputs have maximum distance between each other, and the minimum distance code can no longer be increased. However, α doesn’t increase much either.

Concatenation with Outer Block Code

- **A concatenation of an NLTC with a high rate block code provides a very low BER, at low cost in terms of rate.**



Block-Code Encoder

NL-TC Encoder

Z-Channel

NL-TC Decoder

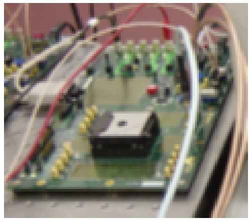
Block-Code Decoder

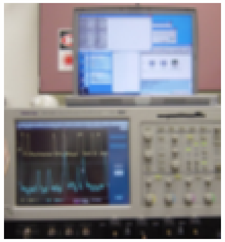
- **Results:**
  - **A concatenation of the rate-1/20 NL-TCM code with (255 bytes,247 bytes) Reed-Solomon code has been tested for the 6-user OR-MAC scenario.**
  - **This RS-code corrects up to 8 erred bits.**

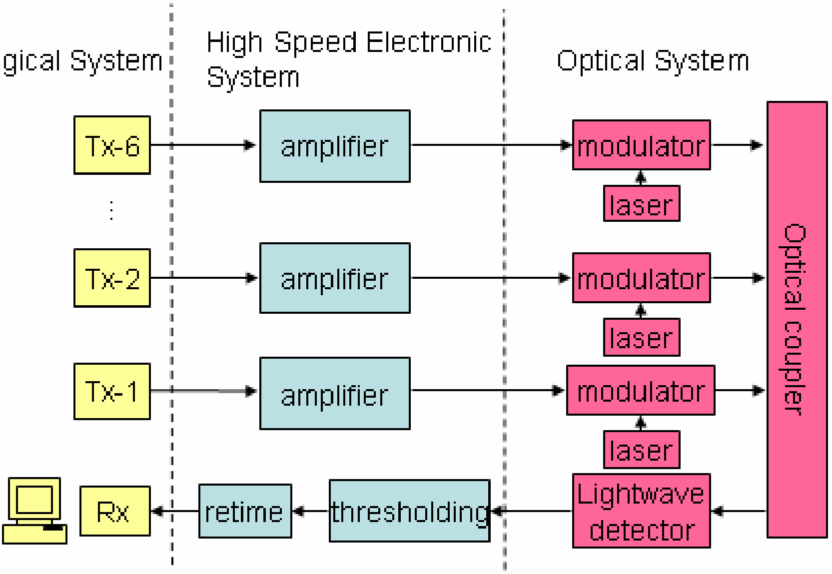
Rate	Sum-rate	p	α	BER
0.0484	0.29	0.125	0.4652	2.48×10^{-10}

- Although we don't have simulations for the 100-user case, it may be inferred that a similar BER would be achieved.

System Implementation

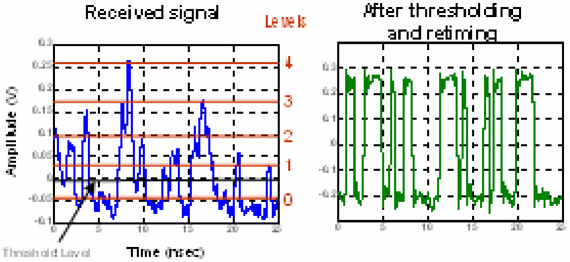






Optical Receiver

- Lightwave detector converts optical signal to multilevel electrical signal
- Threshold level of flip flop converts to two level signal output
- Variable delay line matches clock phase to that of desired transmitter.



Logic Implementation

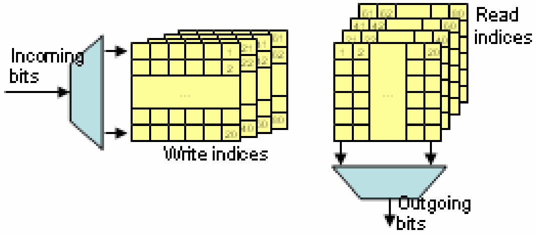
Channel coding implemented on VirtexII-Pro FPGAs

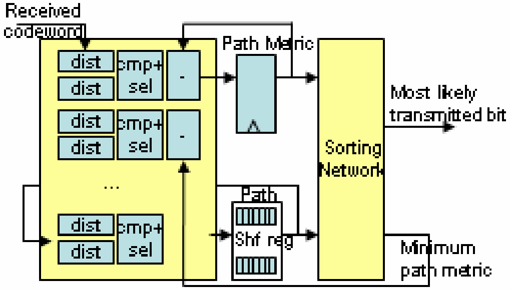
Interleaver:

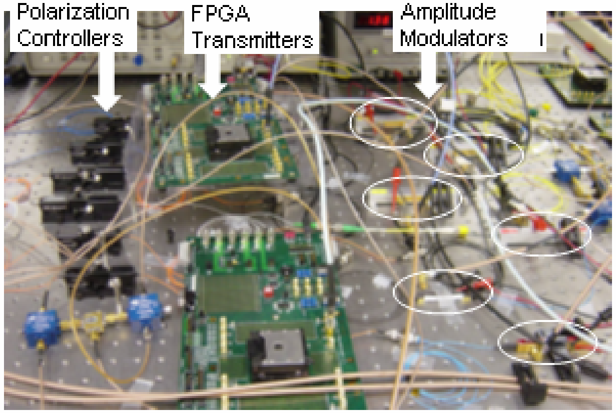
- 1600 bit interleaver
- Use novel random write-by-row, read-by, column scheme

Trellis code:

- 64 state parallel Viterbi decoding
- Heavily pipelined







- ***Winner of 1st Prize on Student Design Contest organized jointly by the 2006 ACM-DAC and IEEE International Solid State Circuits.***

Conclusions

- We have presented an IDMA-based architecture, where every user treats the others as noise, to provide uncoordinated multiple access to the OR-Channel.
- The goal is to provide access to a large number of users with feasible complexity.
- Non-linear trellis codes
  - Very low complexity and latency, not capacity achieving.
  - Efficiency of 30% with very low BER $(BER < 10^{-9})$ when concatenated with Reed-Solomon Code.
- Tight bit error bounds for NLTC over the Z-Channel have been presented.
- Real implementation for 6-user Optical MAC.

Ongoing work

- Non-linear turbo codes: parallel concatenation of NLTCs.
  - To be presented in Globecom’06.
  - We achieve similar BER at sum-rates of ~60%.
- More general models:
  - Allow 1-0 transitions: Binary Asymmetric Channel.
  - Soon to be submitted to *Trans. on Comm.*

Thank you!