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Fuzzification of continuous-value spatial evidence for mineral prospectivity mapping

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Abstract

Complexities of geological processes portrayed as certain feature in a map (e.g., faults) are natural sources of uncertainties in decision-making for exploration of mineral deposits. Besides natural sources of uncertainties, knowledge-driven (e.g., fuzzy logic) mineral prospectivity mapping (MPM) is also plagued and incurs further uncertainty in subjective judgment of analyst when there is no reliable proven value of evidential scores corresponding to relative importance of geological features that can directly be measured. In this regard, analysts apply expert opinion to assess relative importance of spatial evidences as meaningful decision support. This paper aims for fuzzification of continuous spatial data used as proxy evidence to facilitate and to support fuzzy MPM to generate exploration target areas for further examination of undiscovered deposits. In addition, this paper proposes to adapt the concept of expected value to further improve fuzzy logic MPM because the analysis of uncertain variables can be presented in terms of their expected value. The proposed modified expected value approach to MPM is not only a multi-criteria approach but it also treats uncertainty of geological processes depicted by maps or spatial data in term of biased weighting more realistically in comparison with classified evidential maps because fuzzy membership scores are defined continuously whereby, for example, there is no need to categorize distances from evidential features to proximity classes using arbitrary intervals. The proposed continuous weighting approach and then integrating the weighted evidence layers by using modified expected value function, described in this paper can be used efficiently in either greenfields or brownfields.

Keywords: Weight assignment; Continuous field data; Fuzzy logic; Expected value; Uncertainty; Mineral prospectivity modeling.

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1. Introduction

Knowledge- and data-driven are two major types of approaches to create and integrate weighted evidential layers for mineral prospectivity mapping (MPM) to delineate target areas for further exploration of a certain deposit-type (Bonham-Carter, 1994; Carranza, 2008). The theory of fuzzy sets and fuzzy logic (Zadeh, 1965) has been applied in knowledge-driven assignment of evidential scores that need expert judgments reflecting realistic spatial as well as genetic associations between spatial evidence and mineral deposits of the type sought (e.g., D’Ercole et al., 2000; Knox-Robinson, 2000; Carranza and Hale, 2001; Porwal et al., 2003; Tangestani and Moore, 2003; Rogge et al., 2006; Nykänen et al., 2008; Lusty et al., 2012).

The assignment of fuzzy membership values to evidential features in the $[0, 1]$ range, also called fuzzification of spatial evidence, is the most important stage in fuzzy MPM (Carranza, 2008) because evidential scores should adequately represent the relative importance of geological features (or data) in the process of mineralization, however evidence is vaguely-known or completely unknown (e.g., Ye, 2011; Xu, 2007a,b). However, knowledge-driven evidential scores are assigned based on the analyst's expert judgment, which is inherently subjective, but an analyst usually cannot make an exact choice because of fuzziness (i.e., when there is vague evidence and no reliable proven value of evidential scores for a proposition). Thus, in cases of fuzziness, certain fuzzy membership values in the $[0,1]$ range can be preferred by an analyst as evidential scores of vague evidence. This practice has been used in fuzzy logic MPM to delineate target areas for further exploration (e.g., D’Ercole et al., 2000; Knox-Robinson, 2000; Carranza and Hale, 2001; Porwal et al., 2003; Tangestani and Moore, 2003; Rogge et al., 2006). However, because expert judgment is subjective, defining fuzzy membership values in the $[0,1]$ range as quantitative scores of evidential features is a source of uncertainty in MPM.

Natural resource management and exploration targeting are plagued with uncertainties of various kinds (Runge et al., 2011; McCuaig et al., 2010). Vagueness, ambiguity, similarity, possibility, probability, fuzziness, randomness, and imprecision are different types or sources of uncertainty (Celikyilmaz and Burhan Türksen, 2009). Dissimilarities of geological settings are also natural sources of uncertainties in decision-making for assignment of evidential scores in MPM (McCuaig et al., 2010; Lisitsin et al., 2013), even in areas with simple geology (Van Loon, 2002). Geological modeling of mineral systems is complex because there are significant uncertainties in knowledge as well as data about such systems (McCuaig et al., 2010) and they are rarely accurately and precisely represented in existing geological datasets (Lisitsin et al., 2013). For example, data may indicate the presence of evidential features of a deposit-type sought (e.g., geochemical anomaly of indicator elements, favorable host rocks) but mineral occurrence is not observed in the field, and vice versa. Besides natural sources of uncertainties, knowledge-driven (e.g., fuzzy logic) MPM is also plagued and incurs further uncertainty arising from subjective judgment of analyst to fuzzify evidential data.
Recently, Lisitsin et al. (2013) applied Monte Carlo simulation to model uncertainty of geological interpretations resulting from subjective expert opinion in fuzzy logic MPM. They assigned several evidential scores to a certain feature to obtain a distribution function of evidential scores as input probability distribution to support Monte Carlo simulation. This is an effective method for modeling uncertainty where there are some primary reliable historical data for supporting the analyst to obtain a probability distribution of uncertain variables (e.g., Fairbrother et al., 2007; Sari et al., 2009; Mun, 2006). However, the result of Monte Carlo simulation is affected by the probability distribution of the input uncertain variables (e.g., Mun, 2006). In knowledge-driven MPM, there is no reliable proven evidential score corresponding to relative importance of geological features that can be measured directly. Therefore, if evidential scores are assigned based on judgment of several analysts, the probability distributions of such scores carry the uncertainties of expert judgments as well.

Furthermore, in fuzzy logic MPM, distances to geological features are generally categorized into some proximity classes using arbitrary intervals of distance and then the same score is assigned for all distances in each proximity class (e.g., Carranza and Hale, 2001; Porwal et al., 2003; Rogge et al., 2006; Lisitsin et al., 2013). Therefore, this existing practice in fuzzy logic MPM is sensitive to the widths of classes of distances and the relative importance of every distance to geological features is not really evaluated as proxy evidence of mineral prospectivity. However, it has been the traditional practice in MPM to discretize continuous values into categorized data to facilitate understanding of the relation between predictor variables and the target variable (e.g., Bonham-Carter, 1994; Cheng and Agterberg, 1999; D’Ercole et al., 2000; Knox-Robinson, 2000; Carranza and Hale, 2001; Luo and Dimitrakopoulos 2003; Porwal et al., 2003, 2004, 2006; Rogge et al., 2006; Carranza, 2008; González-Álvarez et al., 2010; Markwitz et al., 2010; Lisitsin et al., 2013). Nevertheless, as has been shown in MPM by Nykänen et al. (2008) and as has been shown in other knowledge fields by many researchers (e.g., Clenshaw and Olver, 1984; Sakawa and Yauchi, 1999; Benitez-Read et al., 2005; Narmatha Banu and Devaraj, 2012; Ray, 2012; Guillén-Flores et al., 2013; Silva et al., 2014; Xie et al., 2014), it must be pointed out that discretization of continuous values is not needed in the fuzzification of evidence for a particular proposition.

Considering the caveats (i.e., subjective nature) of fuzzification of spatial evidence for MPM, recent works on fuzzification of geochemical anomalies (Yousefi et al., 2012, 2013, 2014), proximity to intrusive contacts (Yousefi et al., 2013), and fault density (Yousefi et al., 2014) strive to assign continuous fuzzy evidential scores. Following Nykänen et al. (2008), Yousefi et al. (2012, 2013, 2014) and Lisitsin et al., (2013), this paper aims for fuzzification of continuous-value spatial data used as proxy evidence in MPM. In addition, this paper proposes to adapt the concept of expected value to further improve fuzzy logic MPM because the problem of modeling evidential attributes that are incompletely known or completely unknown (Xu, 2007a,b) and the relative importance and integration of fuzzy evidential values can be and has been addressed by using expected values (e.g., Heilpern, 1992, 1997; Rubinstein, 1981; Wang and Chin, 2011; Ye, 2011; Liu, 2013; Gupta et al., 2013). The expected value approach is based on the idea that event level interaction and probabilities,
here evidential attributes and their corresponding evidential scores representing the probability of mineral deposit occurrence, can be averaged to produce unbiased estimates that properly account for potential future events in modeling (e.g., Mosher et al., 2010). The expected value method in conjunction with fuzzy models has been applied in ranking and decision-making problems (e.g., Heilpern, 1992; Guo and Tanaka, 2001; Wang and Zhang, 2009a,b; Wang and Chin, 2011; Ye, 2011; Gupta et al., 2013). Besides using fuzzy logic MPM with continuous weighted evidential maps, this paper proposes a modified expected value integrating approach whereby geo-exploration data inputs are first fuzzified using continuous fuzzy membership values, and then their expected values are used to support decision-making in MPM.

To demonstrate the procedure of fuzzification of continuous-value spatial evidence for MPM using a modified expected value approach that is suitable in greenfield areas, we chose a case study area in the Kerman province in southeast Iran where there are only nine known occurrences of porphyry-Cu deposits. This number of mineral deposit occurrences is inadequate for data-driven MPM (cf. Carranza, 2004). We used these few deposits only as a set of testing samples to evaluate efficiency of the methodology, developed in this paper, for mapping mineral prospectivity.

2. Methods and results

In this study, we used a pixel size of 100 m × 100 m in all of the maps stored in a GIS. This cell size was obtained by using the function of scale number recommended by Hengl (2006).

For fuzzification of continuous-value spatial evidence data, we first analyzed (i) geochemical multi-element data to derive a map of multi-element signature of porphyry-Cu mineralization, and (ii) extracted relevant features from the geological map to create a map of distances to intrusive contacts and a map of density of faults to depict, respectively, heat-source and structural controls on porphyry-Cu mineralization. The continuous values in the derived maps (i.e., factor scores representing multi-element geochemical signature, fault density (FD), and distance to intrusive contacts) do not lie within the [0,1] range, and thus are not appropriate for fuzzy MPM. In MPM, the main goal is to classify a region into highly prospective areas as targets for further exploration, areas with very low priority for prospecting, and some classes between them. Thus, MPM is a classification problem, and prospectivity models can be portrayed as classified maps. Transformation of data (e.g., binarization, multi-class representation, continuous-value fuzzification) provides a set of values with more discriminatory information and less redundancy for classification (Micheli-Tzanakou, 1999). Defining a suitable non-linear transformation into a new space could facilitate interpretation of a pattern (e.g., dispersion pattern of geochemical indicator elements) for a set of evidential values in MPM compared to their original space (Bishop, 2006; Yousefi et al., 2014). The transformation of continuous-value data using a logistic sigmoid (or S-shaped) function gains an optimal decision boundary for classification (Bishop, 2006). A logistic sigmoid transformation has played an important role in many classification algorithms and pattern recognition (Bishop, 2006), such as statistics, neural networks, machine learning, and expert systems (e.g., Micheli-Tzanakou, 1999; Berthold and Hand, 2002;
For example, Nykänen et al. (2008) used a logistic function to assign fuzzy membership scores to continuous-value spatial evidence of mineral prospectivity. Recently, Yousefi et al. (2014) compared linear and non-linear transformations of evidential data using logistic function and demonstrated that the latter is much better for weighting of evidential data in fuzzy MPM. A sigmoid function maps the whole data space into a finite interval, e.g. [0,1] range. There is a family of logistic functions (Theodoridis and Koutroumbas, 2006) that can be used to transform data into logistic space based on the minimum and maximum data values and slope variations between them. Accordingly, different but suitable logistic functions have been used for transforming evidential values into the [0,1] range to support fuzzy logic MPM (Carranza and Hale, 2002b; Carranza, 2008; Porwal et al., 2003, 2006; Theodoridis and Koutroumbas, 2006; Yousefi et al., 2012, 2013, 2014).

Likewise, in this paper, to transform unbounded values of FD and distance to intrusive contacts into the [0,1] range, we used a suitable logistic sigmoid function (Cox and Snell, 1989), viz.:

$$F_x = \frac{1}{1 + e^{-s(X - i)}}$$

where $F_x$ is a fuzzy membership (fuzzy score), $i$ and $s$ are inflexion point and slope, respectively, of the logistic function, and $X$ is a map value to be transformed in the [0,1] range.

To transform factor scores values representing multi-element geochemical signature of the deposit-type sought, we applied the following logistic sigmoid function (Yousefi et al., 2012; 2014):

$$GMPI = \frac{e^{FS}}{1 + e^{FS}}$$

where $FS$ is the factor score of each sample per indicator component obtained in a factor analysis. The $GMPI$ is, therefore, a weight of each stream sediment geochemical sample in the [0,1] range for each indicator component. In this way, the evidential scores of stream sediment samples are calculated continuously based on the $FS$s of samples per indicator component obtained in the factor analysis (Yousefi et al., 2012, 2014). The main difference between the logistic functions in Eqs. (1) and (2) is that in the latter there is no need to define any parameter using judgment of analyst, but it is suitable mainly for transformation of input values with a limited range like factor scores from factor analysis of geochemical data in which values greater than 1 often represent anomalies of exploration interest (e.g., Yousefi et al., 2012, 2014). The advantage of Eq. (1) over Eq. (2) is that the former can transform integers and floating values of different ranges into [0,1] by using suitable values of $i$ and $s$. Suitable values of $i$ and $s$ are sought by trial-and-error such that the application of Eq. (1) results in highest and lowest fuzzy scores (for example 0.99 and 0.1), respectively for the maximum and minimum values of spatial evidence.

After the assignment of continuous fuzzy evidential scores using logistic function, fuzzy evidence maps can be combined to derive a mineral prospectivity map for delineation of target areas for further exploration of porphyry-Cu deposits. In this paper, besides combining fuzzy evidence maps using a fuzzy logic operator, uncertainties of geological interpretations (different geo-exploration data sets) were considered in the final prospectivity model by applying a modified expected value function for comparative analysis. To achieve this comparative, maps of fuzzy evidence layers were integrated.
using their weighted averages to gain a final expected value prospectivity model (e.g., Yaffee, 2000; Bragg, 2006). Therefore, in this paper, the two prospectivity models – a fuzzy prospectivity model and an expected value prospectivity model – were generated, which were then compared and contrasted in regard to supporting decision-making for the selection of target areas for further exploration of the deposit-type sought.

2.1. Generation of fuzzy heat-source control evidence map

The study area is situated in the southern part of the Urumieh-Dokhtar Volcanic Belt of Iran, which is an Andean magmatic arc (Alavi, 1980; Berberian et al., 1982) with huge potential for porphyry-Cu deposits (Tangestani and Moore, 2002a; Hezarkhani, 2006a, b; Atapour and Aftabi, 2007; Boomeri et al., 2009). MPM can be used in every scale of prospecting from regional- to large-scale based on the data available and the conceptual model of the mineral deposit type sought (e.g., Pan and Porterfield, 1995; Carranza, 2008; Abedi and Norouzi, 2012) even for determining exploration drilling sites (Abedi and Norouzi, 2012) in large scale mineral prospecting. In this paper the study area measures ~2,500 km² and is covered by the 1:100,000 scale quadrangle map of Sabzevaran prepared by the Geological Survey of Iran (GSI) (Grabeljsek, 1956). The simplified lithostratigraphic map of the study area is shown in Fig. 1.

Porphyry-Cu deposits consist of disseminations copper minerals in host rocks or as open-space fillings in veins and breccias that are distributed in relatively large volumes, forming high tonnage but low to moderate ore grades. Most porphyry-Cu deposits form in subduction-related magmatic arcs parallel to convergent plate margins, both in continental and oceanic settings (e.g., Sillitoe, 1972; Sillitoe and Hedenquist, 2003; Sillitoe, 2010). Porphyry-Cu deposits are mostly centered in high-level intrusive complexes. Wide varieties of intrusive rocks with dioritic to granitic compositions are spatially associated with, or host, porphyry-Cu deposits (e.g., Sillitoe, 2010). Quartz monzonite, diorite, granodiorite, quartz diorite, and monzonite are the most commonly reported rock types (Lundmark et al., 2005; Hezarkhani, 2006a, b, 2009; Boomeri, et al., 2009; Peytcheva et al., 2009).

In the study area, porphyry-Cu deposits are genetically as well as spatially related with intrusive rocks, and particularly close to intrusive contacts, in the Urumieh-Dokhtar volcanic belt (Campos et al., 2002; Lundmark et al., 2005; Hezarkhani, 2006a, b, 2009; Atapour and Aftabi, 2007; Qu et al., 2007; Boomeri et al., 2009; Peytcheva et al., 2009). Hence, areas proximal to intrusive contacts have stronger likelihood of porphyry-Cu mineralization than distal areas. To represent that empiricism and to create a fuzzy map of proximity to intrusive contacts (Fig. 2), we created a map of distance from intrusive contacts, in which distances are represented as floating point values. After creating the distance map, to represent favorability for porphyry-Cu occurrence we used the inverse of distance to contacts of intrusive bodies, representing \( X \) in Eq. (1). In the distance map, the contact, feature itself, has a value of 0 but 1/0 does not exist in the map of inverse of distances although it should have the highest fuzzy score. However, contacts to intrusive bodies were assigned the highest value obtained by using Eq. (1). Assigning of continuous fuzzy membership values to continuous floating point
values has been used in the literature in other fields of study (Clenshaw and Olver, 1984; Sakawa and Yauchi, 1999; Benitez-Read et al., 2005; Narmatha Banu and Devaraj, 2012; Ray, 2012; Guillén-Flores et al., 2013; Silva et al., 2014; Xie et al., 2014). Therefore, the smallest distances from the intrusive contacts are assigned with the highest fuzzy scores whereas the largest distances from the intrusive contacts are assigned with the lowest fuzzy scores. The approach of continuous weighting avoids classification or discretization of evidential values using arbitrary intervals.

2.2. Generation of fuzzy geochemical signature evidence map

Analysis of significant anomalies in geochemical landscapes based on stream sediment geochemical data is important for creating and integrating layers of geochemical evidence in MPM for the deposit-type sought (Carranza, 2010c; Yousefi et al., 2012). In this regard, we used multi-element (Cu, Ag, Zn, Pb, As, and Sb) concentration data from 821 samples of ~80 mesh (<177 μm) fraction of stream sediments, collected, analyzed, and prepared by the GSI. To determine a multi-element anomalous signature of the deposit-type sought, we performed staged factor analysis, SFA, (Yousefi et al., 2012, 2014). Prior to performing SFA, we applied isometric logratio or ilr transformation (Egozcue et al., 2003; Filzmoser et al., 2009) of the multi-element geochemical data to address the closure problem inherent in compositional data (Filzmoser et al., 2009; Carranza, 2011).

For the SFA, we used principal components analysis with varimax rotation (Kaiser, 1958) for extracting the common factors, and considered only factors with eigenvalues of >1 for interpretation. In addition, we used 0.6 as threshold loading of an element on a factor to interpret a multi-element geochemical signature of the deposit-type sought (e.g., Borovec, 1996; Chandrajith et al., 2001; Helvoort et al., 2005; Treiblmair and Filzmoser, 2010; Yousefi et al., 2012, 2014). In the result of SFA, two multi-element associations (i.e., factors) were recognized, F1 (Zn-Sb-Ag-As) and F2 (Pb-Cu). Derived sample factor scores (FSs) depicting significant multi-element signatures of the deposit-type sought (here F1 and F2) usually lie outside the [0,1] range. Therefore, we used a logistic sigmoid function (Eq. 2) to calculate a geochemical mineralization prospectivity index, GMPI, to create fuzzy geochemical evidence maps (Yousefi et al., 2012, 2014). The two GMPI maps, depicting Zn-Sb-Ag-As and Pb-Cu factors, were integrated to a single map of geochemical multi-element signature as a fuzzy spatial evidence of porphyry-Cu prospectivity (GMPI_{porphyry-cu}; Fig. 3) by using equations like Eq. (5) in Yousefi et al. (2012). As Yousefi et al. (2012, 2014) demonstrated, transforming FS values in the results of SFA, calculation of GMPI and finally combining different indicators of the deposit-type sought using proper equations gains better discrimination between geochemical multi element populations.

2.3. Generation of fuzzy structural control evidence map

Areas at or near intrusive contacts are strongly fractured and, thus, enable passage of hydrothermal fluids that exchange heat and mass with intruded rocks (e.g., Richards et al., 2001; Carranza and Hale, 2002a; Guillou-Frottier and Burov, 2003; Sillitoe, 2010). Therefore, high FD
represents strong favorability for porphyry-Cu mineralization (Guillou-Frottier and Burov, 2003; Storti et al., 2003; Qu et al., 2007; Pirajno, 2010; Chen et al., 2011). To represent that empiricism and to create a fuzzy map of FD (Fig. 4), we created a map of FD (total length of faults per pixel in the study area) representing \( X \) in Eq. (1), so that the highest values of FD have the highest fuzzy scores whereas the lowest values of FD have the lowest fuzzy scores.

2.4. Integration of fuzzy evidence maps

For integrating the generated continuous fuzzy evidential maps, we used two different functions, fuzzy operator and modified expected value approach for comparison purpose.

2.4.1 Fuzzy prospectivity model

In fuzzy MPM, fuzzy evidence maps (here Figs. 2, 3, and 4) are combined to obtain a map of fuzzy prospectivity values for delineating target areas for further exploration of the mineral deposit-type sought (Bonham-Carter, 1994; Carranza, 2008). Fuzzy evidence maps are integrated using suitable fuzzy operators (An et al., 1991). In this regard, any of the existing fuzzy operators can be used considering the mineralization type sought and the purpose of the integration. We used the fuzzy gamma operator to integrate the weighted evidential maps (Fig. 5a) because it involves both fuzzy algebraic sum and fuzzy algebraic product operators in a scheme. The output of the fuzzy algebraic product is less than or equal to the lowest fuzzy score at every location in the input fuzzy evidence maps. Thus, the fuzzy algebraic product has a ‘decreasive’ effect, meaning that the presence of very low but non-zero fuzzy scores tend to deflate or under-estimate the overall support for the proposition, and so it is appropriate in combining complementary sets of evidence. The output of the fuzzy algebraic sum is greater than or equal to the highest fuzzy score at every location in the input fuzzy evidence maps. Thus, the fuzzy algebraic sum has an ‘increasive’ effect, meaning that the presence of very high fuzzy scores (but not equal to 1) tend to inflate or overestimate the overall support for the proposition, and so it is appropriate in combining supplementary sets of evidence. Considering that target areas for prospecting porphyry-Cu deposits must exhibit the presence of supplementary evidential features representing interactions of conditions favorable for mineral deposit formation, such areas should have high prospectivity values. Consequently, to model target prospective areas, using the fuzzy algebraic sum to model the ‘increasive’ effect of supplementary sets of evidence is more (but not totally) suitable than using the fuzzy algebraic product to model the ‘decreasive’ effect of supplementary sets of evidence. To achieve this in a single operation, we used the fuzzy gamma operator with a high value of gamma (=0.9).

2.4.2 Expected value prospectivity model

In decision theory, and in particular in choice-making under uncertainty (e.g., here selecting target areas for further exploration), expected value is used for making an optimal choice in the context of incomplete information. In probability theory, the expected value refers, intuitively, to the value of a
random variable (e.g., here mineralization) one would "expect" to find. The expected value is a weighted average of all possible values, and each possible value that the random variable can assume is multiplied by its assigned weight. The resulting products are added together to obtain the expected value (e.g., Yaffee, 2000; Bragg, 2006). The evidential scores used in computing this average are probabilities in the case of a discrete random variable, or the values of a probability density function in the case of a continuous random variable (e.g., Yaffee, 2000; Bragg, 2006). Suppose a random variable $V$ can take values $v_1$ to $v_n$ with their corresponding probability from $p_1$ to $p_n$. Since all probabilities $p_i$ add up to one ($p_1 + p_2 + ... + p_n = 1$), the expected value of $V$ can be viewed as the weighted average, with $p_i$'s being evidential scores (cf. Wikipedia, 2014):

$$E_V = \frac{v_1p_1 + v_2p_2 + ... + v_np_n}{p_1 + p_2 + ... + p_n}$$

(3)

Therefore, the expected value for variable $V$ is defined as:

$$E_v = \sum_{i=1}^{n} v_i p_i$$

(4)

The mathematical functions of expected value can be found in literatures (e.g., Rubinstein, 1981; Mun, 2006). The simplistic approach for predicting uncertain variables is to study all possible expected values of the variables (e.g., Bragg, 2006). In MPM, the absence or presence of an evidence in a certain pixel of the study area is portrayed by the presence or absence of other indicator features, and several evidential features can be present simultaneously at the same location (pixel). Therefore, unlike the classical expected value, Eq. (3) and Eq. (4), the sum of the all evidential scores is not equal to 1.

In MPM, the parameters of Equations (3 and 4) for a unit cell of a study area can be modified as $v_1 = i_1$ as an indicator value (e.g., FD) with a fuzzy score $f_1$, $v_2 = i_2$ as another indicator value (e.g., proximity to intrusive contacts) with a fuzzy score $f_2$, $v_3 = i_3$ as the next indicator value (e.g., multi-element geochemical signature) with a fuzzy score $f_3$, and so on, up to indicator with value of $v_n = i_n$ with a fuzzy score $f_n$. Because in MPM the study area is gridded into individual pixels as discrete variables, a modified expected value function for a unit cell of the area can be written as:

$$Ep = \frac{\sum_{i=1}^{n} F_i I_i}{\sum_{i=1}^{n} F_i}$$

(5)

where $Ep$ is the expected prospectivity value of the unit cell, $I_i$ is the value of the unit cell in the $i^{th}$ indicator evidence map, and $F_i$ is its corresponding fuzzy score that has been assigned continuously using a logistic function. Because in the expected value functions, Eq. (3-5), the values in individual evidential maps are added, they should have the same unit. In this study the unit for values of FD is “kilometer”, the unit of distances from intrusive contacts is “kilometer” as well. So for using the same unit in another evidential map, geochemical evidential map, we used the distances from anomalies in “kilometer” like other layers. For this, anomalies in the map of multi element geochemical signature, $GMPICu$-porphyry values in Fig. 3, were recognized and extracted using C-A fractal model (Cheng et al.,
Then a map of distances from anomalies in “kilometer” was created. In the next stage, a continuous weighted map of proximity to anomalies was generated, like the map of distances from intrusive. So we used values of different geo-exploration data that all have the same unit. For the present case study, the $E_p$ for a certain pixel in terms of prospectivity for porphyry-Cu deposit is defined as:

$$E_{E_{Cu-porphyry}} = \frac{F_{FD}I_{FD} + F_{IC}I_{IC} + F_{GS}I_{GS}}{F_{FD} + F_{IC} + F_{GS}}$$

where $I_{FD}$ is the value of FD, $I_{IC}$ is the value of proximity to intrusive contacts (represented as inverse of distance to contacts of intrusive bodies) and $I_{GS}$ is the value of proximity to multi element geochemical signature (represented as inverse of distance to anomalies) and $F_{FD}$, $F_{IC}$, and $F_{GS}$ are the corresponding fuzzy scores calculated using Eq. (1). After calculating the values of $E_{E_{Cu-porphyry}}$ per pixel in the study area, which is the integrated expected prospectivity value, they were mapped to generate a model of expected value prospectivity of the deposit type sought (Fig. 6a).

Mathematically, the expected value treats uncertainty and probability (e.g., Mosher et al., 2010; Runge et al., 2011; Gupta et al., 2013) because each event value (here, evidential value like FD) is multiplied by its probability of occurrence (here, fuzzy score) and then divided by sum of the probabilities (here, fuzzy scores) as a weighted average multicriteria decision-making function (cf. Bonham-Carter, 1994; Carranza, 2008; Feizizadeh et al., 2014). Using a weighted average is an approach that has been widely applied to combine information from several criteria into a single evaluation model to modulate uncertainty (Chen et al., 2010; Feizizadeh et al., 2014), such as index overlay for MPM (e.g., Bonham-Carter, 1994; Carranza, 2008) or geo-hazard susceptibility mapping (e.g., Feizizadeh et al., 2014), and other application fields as expected value approach for example in well-logging (Mosher et al., 2010), financial supports (e.g., Gupta et al., 2013), and biological sciences (e.g., Runge et al., 2011). Furthermore, weighted average integrating approaches like expected value have been used to synthesize continuous fuzzy values (e.g., Xue et al., 2008; Chen et al., 2014) in fuzzy systems (e.g., Heilpern, 1992).

2.5. Evaluation of the model

2.5.1 Using known mineral occurrences

Prospectivity values in the maps of fuzzy prospectivity model (Fig. 5a) and expected value prospectivity model (Fig. 6a) are first categorized into some classes. To define threshold prospectivity values for classification of the prospectivity models, we used the concept of fractals (Mandelbrot, 1977, 1983; Mandelbrot et al., 1984). Several fractal methods have been developed and successfully applied to classify values in maps to be used as spatial evidence of mineral prospectivity, such as geochemical anomalies (e.g., Cheng, 1995, 1999, 2007; Cheng and Agterberg, 2009; Cheng et al., 1994, 1996, 2010; Carranza, 2008, 2010b, 2010c; Deng et al., 2009, 2010, 2011; Wang et al., 2011b;
Zuo, 2011a, 2011b, 2011c; Zuo and Cheng, 2008; Zuo and Xia, 2009; Zuo et al., 2009b), structures like faults (e.g., Zhao et al., 2011), and geological features (e.g., Ford and Blenkinsop, 2008; Wang et al., 2011a; Zuo et al., 2009a). Here, we used the concentration–area model (C–A) (Figs. 5b and 6b), proposed by Cheng et al. (1994), to determine threshold prospectivity values for classification of the prospectivity models. Based on Figs. 5b and 6b, we obtained two classes of prospectivity resulting in a binary prospectivity model (Fig. 5c) from the fuzzy prospectivity model, and three classes of prospectivity resulting in a ternary prospectivity model (Fig. 6c) from the expected value prospectivity model. For evaluating the relative importance of different classes of evidential features, Good (1950), Bonham-Carter et al. (1989), Jaccard (1908), Yule (1912) and Bayes (1764) divided scores of classes of evidential features by their corresponding occupied area with respect to total area, because in smaller areas of spatial evidence it will be empirically “easier” to find undiscovered deposits than in larger areas of spatial evidence. Furthermore, Yousefi et al. (2012, 2013) used a graph in which classes of prospectivity are plotted versus percentage of known mineral occurrences delineated by (i.e., prediction rate of) respective prospectivity classes as another way to evaluate different prospectivity models. In this paper, we used both the percentage of known mineral occurrences delineated by the corresponding prospectivity classes and the areas occupied by the corresponding classes of prospectivity (with respect to the total study area) in integrating with each other to make one plot. Therefore, by overlaying the locations of known mineral deposits on the classified prospectivity map, the models are evaluated based on the association of known mineral deposits with different classes of prospectivity (e.g., Carranza et al., 2005; Porwal et al., 2003, 2004, 2006; Yousefi et al., 2012, 2013, 2014) considering their occupied area with respect to the total area of the region being studied to obtain Figs. 5d and 6d. These plots are called prediction-area (P-A) plots, and have been drawn based on Figs. 5c and 6c, respectively, for the fuzzy prospectivity model and the expected value prospectivity model.

In Figs. 5d and 6d, the intersection point of the two curves, the curve of prediction rate of known mineral occurrences corresponding to prospectivity classes, and the curve of percentage of occupied areas corresponding to the prospectivity classes, is a criterion to evaluate and compare the two prospectivity models, the fuzzy prospectivity model (Fig. 5a) and the expected value prospectivity model (Fig. 6a). This is because if an intersection point appears in a higher place in the P-A plot, it depicts a smaller area containing larger number of mineral deposits. So, it is “easier” to find undiscovered deposits type sought in such a smaller area. The intersection point of the P-A plot for the expected value prospectivity model (Fig. 6d) plots higher than the intersection point of the P-A plot for the fuzzy prospectivity model (Fig. 6d). Based on the intersection points in Figs. 5d and 6d, 79% of mineral occurrences are delineated in 21% of the study area according to the fuzzy prospectivity model whereas 91% of mineral occurrences are delineated in 9% of the study area according to the expected value prospectivity model. This comparison illustrates that target areas with high values in the expected value prospectivity model should be given more priority for further exploration, rather than target areas with high values in the fuzzy prospectivity model.
2.5.2 Using field observations

As MPM aims to delimit target areas for further exploration of the deposit-type sought, fieldwork can and should be conducted in the delimited target areas to evaluate/validate MPM results. In this paper, considering the prospectivity maps (Figs. 5c and 6c), six areas were selected for further evaluation of the prospectivity models, and consequently to evaluate the efficiency of the proposed approaches based on initial follow-up field observations. These six areas (labeled by A-F in Figs. 5c and 6c) were selected far from any known porphyry-Cu occurrences based on their prospectivity values and their accessibility for field observation. Initial fieldwork shows there are no outcrops or mineralization in two of these six chosen areas (labeled by B and D), but at the other four areas (labeled by A, C, E and F) various indicators of porphyry-Cu mineralization are present and explained as follows. There are outcropping intrusive rocks consisting mainly granodiorite, granite, diorite, gabbro, monzosyenite, and monzonite. Other outcropping rocks are volcanic rocks comprised of andesite, latite, tuff, basaltic rocks, rhyodacite, agglomerates, and pyroclastics. The granitoids and volcanic rocks are commonly affected by silicic, potassic, pyrite, iron-oxide, argillic, chlorite, and sericite alterations (Fig. 7). In the zones of contact between the intrusive and pyroclastics rocks, indicators of porphyry-Cu mineralization include stockwork (Fig. 8), breccia and porphyritic textures, silicified veins commonly associated with Cu and Au mineralization, dikes, economic minerals including mainly chalcopyrite and secondary malachite (Figs. 9 and 10). From the mineralized outcrops, some rocks samples were collected to determine element concentrations (Table 1). Based on initial field observations of geological indicators (Figs. 7-10) and the results of analyses of rock samples (Table 1), it is worthwhile to follow-up further exploration in the four selected areas.

3. Discussion

Although the continuous-value fuzzy evidence maps created in this paper have been applied for fuzzy and expected value MPM, they can be used in other knowledge-driven methods of MPM, namely: (a) analytical hierarchy process (AHP) modeling (e.g., De Araújo and Macedo, 2002; Moreira et al., 2003; Carranza, 2008); (b) modeling with evidential belief functions (e.g., Moon, 1990; Carranza, 2008; Tangestani and Moore, 2002b; Rogge et al., 2006), in which the fuzzy evidence maps can be applied as belief maps; and (c) wildcat MPM (Carranza, 2008; Carranza and Hale, 2002b), in which instead of discrete maps of proximity to features the continuous maps of fuzzy scores of proximity to features can be used. Each of the continuous-value fuzzy evidence maps created in this paper can also be used in multi-index overlay modeling (e.g., Bonham-Carter, 1994; Harris et al, 2001; Chico-Olmo et al, 2002; De Araújo and Macedo, 2002; Billa et al., 2004). In all of these methods, continuous evidential scores of distances in each proximity value of evidential features can be derived directly using a suitable logistic function even without using the locations of known mineral deposit occurrences as in traditional data-driven MPM approaches and even without
discretization of evidential values into some arbitrary classes and assigning weights to classes based on an analyst's expert opinion as in traditional knowledge-driven MPM approaches.

The method of assignment of continuous fuzzy scores to classes of geochemical anomalies, described in this paper, can be used efficiently for assigning fuzzy scores to continuous values of a numerical data set (e.g., values of a geophysical data set, like total aeromagnetic intensity values). If in an area, based on conceptual model of prospectivity for the mineral deposit-type sought, density of a spatial feature is an indicator of mineral prospectivity, the method for generating fuzzy FD in this paper can be applied. In situations where proximity to spatial features is an indicator of mineral prospectivity, the method to assign fuzzy proximity scores (e.g., proximity to faults and proximity to alterations) can be applied. The conceptual model of prospectivity for the mineral deposit-type sought serves as a guide to use the method of assignment continuous fuzzy score to values of spatial evidence using logistic function like Eq. (1).

Fuzzy T-norm (e.g., minimum or fuzzy AND) and S-norm (e.g., maximum or fuzzy OR) operators (e.g., Wang and Fang, 2007; Trillas et al., 2008) and fuzzy operators, like fuzzy gamma, product, and sum do not treat uncertainty functionally, but there are some approaches that treat uncertainty mathematically. In this regard, Lisitsin et al. (2013) used Monte Carlo simulation in conjunction with fuzzy logic to map prospectivity by using classified evidential layers. Here, we used an adapted expected value function to integrate continuous weighted evidential layers to overcome the problems mentioned in the introduction and to further improve fuzzy logic MPM in terms of treating uncertainty. Expected value in conjunction with fuzzy logic has been used in many field of study in ranking and MCDM problems (e.g., Heilpern, 1992, 1997; Guo and Tanaka, 2001; Wang and Zhang, 2008, 2009,a,b; Xu, 2011; Wang and Chin, 2011; Ye, 2011; Gupta et al., 2013). We used expected value function because uncertain variables of fuzzy sets, where there is vague knowledge about evidential scores (Xu, 2007a,b), can be integrated using expected value functions (e.g., Heilpern, 1992, 1997; Rubinstein, 1981; Wang and Chin, 2011; Ye, 2011; Liu, 2013; Gupta et al., 2013). Thus, the expected value function can be used to integrate uncertain variables with different probabilities of evidence present (here, fuzzy scores) in MCDM problems. In this regard, the final model is affected by the variables with higher probabilities of evidence present (or higher fuzzy scores) more than the variables with lower probabilities of evidence present (or lower fuzzy scores).

By using the aggregation function of expected value prospectivity model, e.g., Eq. (5), and several other functions for integration weighted evidential maps in MPM, like fuzzy operators (e.g., Bonham-Carter, 1994; Carranza, 2008), it is explicitly assumed that all evidential layers have the same influence regardless of the relevance of each map. To respect the relative importance of different maps, the index overlay weighted average function (Bonham-Carter, 1994) and the methods in which class weights and map weights are multiplied (e.g., Porwal et al., 2003, 2004, 2006; Lisitsin et al., 2013) can be used. For this, the continuous weighted evidential maps, developed in this paper, instead of classified or discretized spatial values can be multiplied with map weight in the modeling for example in the index overlay MPM (Bonham-Carter, 1994).
The important advantages of using modified expected value and continuous fuzzy evidence maps to MPM introduced in this paper are as follows. As mentioned in the Introduction, discretization of continuous values in a map to be used as spatial evidence is not needed in fuzzification because fuzzy evidence layers can be generated by using continuous fuzzy membership values as have been shown in Nykänen et al., 2008, here and in other fields of study (e.g., Clenshaw and Olver, 1984; Sakawa and Yauchi, 1999; Benítez-Read et al., 2005; Narmatha Banu and Devaraj, 2012; Ray, 2012; Guillén-Flores et al., 2013; Silva et al., 2014; Xie et al., 2014). However, it has been a traditional practice in MPM to discretize continuous-value spatial evidence into categorized maps (e.g. Bonham-Carter, 1994; Cheng and Agterberg, 1999; D’Ercole et al., 2000; Knox-Robinson, 2000; Carranza and Hale, 2001; Luo and Dimitrakopoulos 2003; Porwal et al., 2003, 2004, 2006; Rogge et al., 2006; Carranza, 2008; González-Álvarez et al., 2010; Markwitz et al., 2010; Lisitsin et al., 2013). Therefore, using a continuous weighting method in this paper is the first advantage of proposed method in which there is no need to categorize distances to geological features into some proximity classes with arbitrary intervals for MPM, and the continuous fuzzification method facilitates judgment by an analyst in regard to estimating evidential scores of spatial features and overcoming the problem of estimating classes intervals of distance to spatial features.

Secondly, uncertainties of [mapped] geological features used as fuzzy evidence layers can be alleviated by integrating fuzzy evidence layers using their weighted averages, such that there is no need to estimate a probability distribution of evidential scores of classes of evidential features. For example, because continuous fuzzy scores are assigned to continuous-value evidential maps, there is no need to estimate the probability distribution of evidential scores as input to Monte Carlo simulation for modeling uncertainty that needs historical data of evidential scores, and there is no need to assign several arbitrary evidential scores to a class of evidential feature.

Thirdly, decision-making for selecting better target areas by using the modified expected value approach to MPM is easier than by using the fuzzy logic approach to MPM. This is because the former approach uses a function based on weighted average, expected value, which mathematically treats uncertainty and probability together (e.g., Mosher et al., 2010; Runge et al., 2011; Gupta et al., 2013) but the latter does not. Therefore, as shown in Fig. 6b, values of prospectivity using the modified expected value MPM approach are classified better than those using the fuzzy logic MPM approach as shown in Fig. 5d. Furthermore, the prediction rate of mineral occurrences with respect to the occupied areas using the modified expected value approach to MPM is higher than that of the fuzzy logic approach to MPM (refer to the Figs. 5d and 6d).

There are some data-driven MPM approaches in which evidential scores of spatial features are assigned by user-functions without using training data (Luo, 1990; Bonham-Carter, 1994; Cheng and Agterberg, 1999; Carranza, 2008, 2010a; Luo and Dimitrakopoulos 2003; Porwal et al., 2003, 2004, 2006). Thus, the fourth advantage of the proposed MPM approach over existing data-driven methods of defining evidential scores or fuzzy memberships of evidential values (e.g., Luo, 1990; Bonham-Carter, 1994; Cheng and Agterberg, 1999; Carranza, 2008; Luo and Dimitrakopoulos 2003; Porwal et
al., 2003, 2004, 2006) is that a suitable logistic function can be used to generate continuous weighted evidential maps that can be used efficiently in different areas for different types of mineralization being sought; however, suitable values of $i$ and $s$ must be defined by the analyst for each data set of evidential values. The logistic functions used in this paper to assign continuous evidential scores to spatial values is a mathematical-based function that have been used in different fields of study (e.g., Micheli-Tzanakou 1999; Berthold and Hand 2002; Alpaydm, 2004; Bishop, 2006; Theodoridis and Koutroumbas, 2006; Fink, 2007) for transforming different ranges of values of different variables into the same space, and it is not an empirical or generic function (e.g., Luo, 1990; Bonham-Carter, 1994; Cheng and Agterberg, 1999; Carranza, 2008; Luo and Dimitrakopoulos 2003) that is defined by analysts. Therefore, the logistic-based MPM approach is much simpler than existing data-driven methods of defining fuzzy membership values in MPM without using training data in which different functions should be defined for different evidential maps in different areas (e.g., Luo, 1990; Bonham-Carter, 1994; Cheng and Agterberg, 1999; Carranza, 2008; Luo and Dimitrakopoulos 2003). In this regard, defining continuous fuzzy membership values using logistic function facilitates the quantification problem where there are no measurements for determining values, and there is relevant expert knowledge but it is not directly applicable (Kurowicka and Cooke, 2006). Thus, it can also be called a data-driven MPM approach without using training data (Luo, 1990; Bonham-Carter, 1994; Cheng and Agterberg, 1999; Carranza and Hale, 2002b; Carranza, 2008; Luo and Dimitrakopoulos 2003) or knowledge-guided data-driven MPM approach (Chung and Fabbri, 1993; Carranza et al., 2008; Carranza, 2010a). However, as have been shown by Nykänen et al. (2008), Yousefi et al. (2012, 2013, 2014), and further demonstrated in this paper, by using a logistic function to assign weights to numerical data there is no need to discretize continuous values into categorical data with arbitrary intervals as has been practiced traditionally in GIS-based MPM in the last two to three decades.

The method for defining continuous fuzzy membership values is generally appropriate for continuous-value spatial data. For discrete spatial features, Yousefi et al., (2013) used a logistic function and developed a method of weighted drainage catchment basin (WDCB) to assign evidential scores to catchments for generating a discrete weighted geochemical evidential map. However, further work is needed to develop a method for assigning evidential scores to discrete classes of other geo-evidential features like geological units (e.g., different kinds of rocks) with different relative importance for prospecting a certain type of mineral deposit. Using evidential values with the same unit is the limitation of the expected value prospectvity model. Thus, if geo-evidential values with different units are integrated with expected value function their units must be converted to the same, as we carried out for geochemical anomalies in this paper. Further work also is needed to facilitate selection of parameters in logistic function, $i$ and $s$ values. For example, one may develop a program to examine suitable values of $i$ and $s$ values considering input continuous-value spatial evidence.
4. Concluding remarks

1. By selecting a suitable logistic function, continuous fuzzy scores can be assigned to every value in a map of spatial data. This avoids the intermediate step of classifying continuous spatial data based on arbitrarily chosen threshold values. Assignment of continuous fuzzy scores to continuous-value spatial data, as described in this paper, is advantageous over the traditional knowledge-driven fuzzy modeling approach whereby fuzzy evidential scores are assigned to classified (or discretized) spatial data.
2. The proposed modified expected value approach to MPM is not only a multi-criteria approach but it also treats uncertainty of spatial evidence in terms of vaguely-defined evidential scores more realistically because continuous fuzzy membership values are defined whereby there is no need to categorize continuous-value spatial data into arbitrary classes. Modeling with the modified expected value approach to MPM provides more reliable information, compared to fuzzy logic approach to MPM, for exploration decision-making to select target areas.
3. Target areas generated in this paper exhibit strong positive spatial association with known porphyry-Cu occurrences. In selected target areas, fieldwork shows a variety of surface features suggesting the presence of porphyry-Cu mineralization in the subsurface.
4. The fuzzy prospectivity method and modified expected value approach, described in this paper, can be used efficiently in either greenfield or brownfield areas. In either of these types of areas, locations of known mineral occurrences can be used as testing points to evaluate the mineral prospectivity map.

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Table caption:

Table 1. Element concentrations in rock samples taken from mineralized outcrops in selected target areas based on initial fieldwork (Au in ppb, other elements in ppm).

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<th>Sample No.</th>
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<th>Ag</th>
<th>Cu</th>
<th>Mo</th>
<th>Ba</th>
<th>Pb</th>
<th>Sb</th>
<th>Zn</th>
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Figure captions:

**Fig. 1.** Simplified geological map of the study area and position of Cu known mineral occurrences.

**Fig. 2.** Fuzzified map of distances to intrusive contacts (white curvilinear features).

**Fig. 3.** Fuzzified map of $GMI_{Cu-porphyry}$ values, representing multi-element signature of porphyry-Cu mineralization.

**Fig. 4.** Fuzzified map of fault density.

**Fig. 5.** (a) Map of integrated fuzzy prospectivity scores generated by combining maps of fuzzy proximity to intrusive contacts (Fig. 2), multi-element geochemical signature ($GMI_{Cu-porphyry}$, Fig. 3), and fault density (Fig. 4) using the fuzzy “gamma=0.9” operator. White curvilinear features represent intrusive contacts. (b) Concentration–area model (C–A), log–log plots for fuzzy prospectivity values in Fig. 5a. (c) Classified fuzzy prospectivity map, white curvilinear features represent intrusive contacts. (d) Prediction-area (P–A) plot for the classified fuzzy prospectivity model in Fig. 5e. Because there are only two classes in the fuzzy prospectivity model (Figs. 5b and 5c), the curves in Fig. 5d are linear.

**Fig. 6.** (a) Map of expected value prospectivity scores generated by combining maps of fuzzy proximity to intrusive contacts (Fig. 2), fuzzy proximity to anomalies in multi-element geochemical signature extracted from $GMI_{Cu-porphyry}$ map in Fig. 3, and fault density (Fig. 4) using the expected value function, Eq. (6). White curvilinear features represent intrusive contacts. (b) Concentration–area model (C–A), log–log plots for expected prospectivity values in Fig. 6a. (c) Classified expected prospectivity map, white curvilinear features represent intrusive contacts. (d) Prediction-area (P–A) plot for the classified expected prospectivity model in Fig. 6c.

**Fig. 7.** Pyrite and argillic alterations (left) and sericitic, argillic, and silicic alterations (right)

**Fig. 8.** Stockwork texture (left) and sericite altered rock (right).

**Fig. 9.** Monzosyenite (left) and distribution of chalcopyrite, pyrite, and magnetite in polished section (right).

**Fig. 10.** Distribution of chalcopyrite in polished sections of two different samples.
HIGHLIGHTS: SCHEME

- Fuzzy evidential scores are assigned by using logistic function
- A modified expected value approach is proposed as well
- The modified expected value approach treats uncertainty of mapped geological features
Figure 1

- Quaternary
- Neogene volcanic rocks
- Oligo-miocene sedimentary rocks
- Oligo-miocene intrusive rocks
- Lower-middle Eocene sedimentary rocks
- Eocene volcanic rocks
- Cretaceous volcanic rocks
- Post-cretaceous intrusives
- Jurassic volcanic rocks
- Triassic sedimentary rocks
- Paleozoic metamorphic-sedimentary rock

Cu occurrence
Figure 2

Fuzzy score of proximity to intrusive contacts

0.99

0.15
Fuzzy score of geochemical signature (Distribution of GMPI)

0.97

0.004
Fuzzy score of fault density

0.99

0.14

Figure 4
Figure 5
Figure 6

Expected value prospectivity score

9.33
0.045

Cu occurrence

Class
High prospectivity
Moderate prospectivity
Low prospectivity

Cu occurrence

Field observation

B

Log (Area) vs. Log expected value prospectivity

Percentage of known Cu occurrences (%) vs. Expected value prospectivity score

Intersection point

Area
Prediction rate