arXiv:hep-ph/0306256v1 26 Jun 2003

Features of high energy pp and pA elastic scattering at small t

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Abstract

A method of determination of the real part of the elastic scattering amplitude is examined for high energy proton-proton and proton-nuclei elastic scattering at small momentum transfer. The method allows to decrease the number of model assumptions, to obtain the real part in the narrow region of momentum transfer and check up some different model approaches of hadron-nuclei scattering.

A large number of experimental and theoretical studies of the high energy elastic proton-proton and proton-antiproton scattering at small angles gives a rich information about this processes, which allows to narrow the circle of examined models and to solve a number of difficult problems. Especially this concerns the energy dependence of some of characteristics of these reactions and the contribution of the odderon.

Most of these questions are connected with the s and t dependence of the spin-non-flip phase of hadron-hadron scattering. The majority of the models define the real part of the scattering amplitude phenomenologically. Some

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models use local dispersion relations and the hypothesis of the geometrical scaling. As it is known, using some simplifying assumption, the information about the phase of the scattering amplitude can be obtained from the experimental data at small momentum transfers where the interference of the electromagnetic and hadron amplitudes takes place. On the whole, the obtained information confirms the local dispersion relations. The knowledge of the structure of the elastic proton-nuclei and nuclear-nuclear scattering is needed to distinguish different models describing high energy nuclei interactions. This is important for the QCD approach of the high energy nuclei interaction [1].

The standard procedure to extract the magnitude of the real part includes the fit of the experimental data taking the magnitude of total cross section, of the slope, of ρ, and, sometimes of the normalization coefficient as free parameters. This procedure requires a sufficiently wide interval of t and a large number of experimental points.

The spin-independent amplitude can be written as a sum of nuclear $\Phi^h(s,t)$ and electromagnetic $\Phi^e(s,t)$ amplitudes :

$$\Phi(s,t) = \Phi^h(s,t) + e^{i\alpha\varphi}\Phi^e(s,t) \ . \tag{1}$$

where $\Phi^e(s,t)$ are the leading-terms at high energies of the one-photon amplitudes as defined, for example, in [2] and the common phase φ is

$$\varphi = -[\gamma + \log\left(B(s,t)|t|/2\right) + \nu_1 + \nu_2] \tag{2}$$

where $B(s,t)$ is the slope of the nuclear amplitude, $\gamma = 0.577$, and ν_1 and ν_2 are small correcting terms define the behavior of the Coulomb-hadron phase at small momentum transfers (see [3]). At very small t and fixed s, these electromagnetic amplitudes are such that $\Phi^e_1(s,t) = \Phi^e_3(s,t) \sim \alpha/t$, $\Phi^e_2(s,t) = -\Phi^e_4(s,t) \sim \alpha \cdot \text{const.}$, $\Phi^e_5(s,t) \sim -\alpha/\sqrt{|t|}$., where α is the fine-structure constant. We assume, as usual, that at high energies and small angles the double-flip amplitudes are small with respect to the spin-nonflip one and that spin-nonflip amplitudes are approximately equal. Consequently, the observables are determined by two amplitudes: $F(s,t) = \Phi_1(s,t) + \Phi_3(s,t) = F_N + F_C exp(\ rmi\alpha\varphi)$.

In the standard fitting procedure

$$d\sigma/dt = \pi[(F_C(t))^2 + (\rho(s,t)^2+1)(ImF_N(s,t))^2) \\ + 2(\rho(s,t) + \alpha\varphi(t))F_C(t)ImF_N(s,t)]. \quad (3)$$

$F_C(t) = \mp 2\alpha G^2/|t|$ is the Coulomb amplitude; and $G^2(t)$ is the proton electromagnetic form factor squared. $Re\ F_N(s,t)$ and $Im\ F_N(s,t)$ are the real and imaginary parts of the nuclear amplitude and $\rho(s,t) = Re\ F_N(s,t)/Im\ F_N(s,t)$. The formula (3) is used for the fit of experimental data determining the Coulomb and hadron amplitudes and the Coulomb-hadron phase in order to obtain the value of $\rho(s,t)$.

$ReF_N(s,t)$ is obtained by fitting the differential cross sections either taking into account the value of σ_{tot} from another experiment, as made by the UA4/2 Collaboration, or taking σ_{tot} as a free parameter, as made in [4]. If one does not take the normalization coefficient as a free parameter in the fitting procedure, its definition requires the knowledge of the behavior of imaginary and real parts of the scattering amplitude in the range of small transfer momenta and the magnitude of $\sigma_{tot}(s)$ and $\rho(s,t)$.

In this talk, we briefly describe some new procedures of simplifying the determination of elastic scattering amplitude parameters.

From equation (3) one can obtain the equation for $ReF_N(s,t)$ for every experimental point i

$$ReF_N(s,t_i) = -ReF_C(s,t_i) \\ \pm[(1+\delta)/\pi d\sigma/dt(t=t_i) - (\alpha\phi F_C(t_i) + ImF_N(t_i))^2]^{1/2}. \quad (4)$$

where δ is the corrections coefficient which reflect the accuracity of the normalization parameter $n = 1+\delta$. As the imaginary part of scattering amplitude is defined by

$$ImF_N(s,t) = \sigma_{tot}/(0.389 \cdot 4\pi) exp(B/2t), \quad (5)$$

it is obvious from (4) that the determination of the real part depends on n, σ_{tot}, B. The magnitude of σ_{tot} determined from experimental data depends on the normalization parameter $n = 1+\delta$ which reflects the experimental error in determining $d\sigma/dt$ from dN/dt. The equation (4) shows the possibility to calculate the real part in every separate point of t_i if the imaginary part of scattering amplitude and n are determined and to check up the form of the obtained real part of the scattering amplitude or vice versa (see [5]). This form shows also a minimum value of n, as the expression situated under the square root cannot be less then zero.

Let us define the sum of the real part hadron and Coulomb amplitudes as Δ_R, so we can write:

$$\Delta_R(t_i) = [ReF_N(s,t_i) + ReF_C(s,t)]^2 =$$
$$[(1+\delta)/\pi \ d\sigma/dt(t=t_i) - (\alpha\phi F_C(t_i) + ImF_N(t_i))^2]. \quad (6)$$

This formula shows a significant property for the proton-proton cross section at a very high energy and proton-antiproton scattering at low energy, where the real part of the hadronic amplitude is sufficiently large and is opposite in sign relative to the Coulombic amplitude. We therefore get

$$\Delta_R(t_i) = (1+\delta)(ReF_N(s,t_i) + ReF_C(s,t))^2$$
$$-\delta(\alpha\phi F_C(t_i) + ImF_N(t_i))^2]. \quad (7)$$

Let us examine this expressions for the *pp*-scattering at energies above $\sqrt{s} = 540 \ GeV$ where the real part of the hadron spin-non-flip amplitude is positive and non-negligible. For this aim, let us make a gedanken experiment and calculate $d\sigma/dt$ with definite parameters ($\rho = 0.15$ and $\sigma_{tot} = 63$ taking them as experimental points. For the pp-scattering at high energies, the equation (4) has a remarkable property.

The real part of the Coulomb scattering amplitude of pp-scattering is negative and exceeds the size of $F_h(s,t)$ at $t \to 0$, but has a large slope. As the real part of the hadronic amplitude is positive at high energies, it results that Δ_R has a minimum situated in t independent of n and σ_{tot} as shown in Fig. 1.

The position of the minimum gives us the value t_R where $ReF_N = -ReF_C$. As we know the Coulomb amplitude, we estimate the real part of the pp-scattering amplitude at this point. Note that all other methods give us the real part only in a sufficiently wide interval of the transfer momenta. If we choose the right normalization coefficient and σ_{tot} our minimum will be equal zero. But if the normalization coefficient is not right one the minimum will be or above or lower then zero, but practically it is located at the same point t_R. So, the size of Δ_R shows us the valid determination of the normalization coefficient and σ_{tot}.

This method works only in the case of the positive real part of the nucleon amplitude, and it is especially good in the case of large ρ. So, it is interesting for the experiment $PP2PP$ at RHIC and the future TOTEM experiment at LHC.

Though in the range of ISR we have small $\rho(s, t \approx 0)$ and few experimental points, let us try to examine one experiment, for example, at $\sqrt{s} = 52.8\ GeV$. This analysis is shown in Fig. 2. One can see that in this case the minimum is sufficiently large, and $-t_{min} = (3.3 \pm 0.1)10^{-2}\ GeV^2$. The corresponding real part equals $0.442 \pm 0.014\ GeV$.

Our analysis gives $\rho = 0.063 \pm 0.003$, while the paper [6] gives $\rho = 0.077 \pm 0.08$.

For RHIC energies we can simulate the "experimental" data taking the calculated theoretical curve with certain parameters σ_{tot}, B, ρ and the magnitude of errors which are expected in the future experiment. Then we calculate the deviation from the theoretical curve in units of errors using a Gaussian random procedure in order to calculate the probability of the deviation by a number of errors. As a result, we obtain the differential cross sections modeling the future experimental data, for example, with the posible size of $\rho = 0.135$ and $\rho = 0.175$. Then we can determing the value of Δ_R from these "gedanken" experimental data , which are shown in Fig.3 (a,b) correspondingly. The difference between these two modeling data representations is obvious. The pure theoretical representation of Δ_R with the same values of ρ and with $\rho = 0$ are shown also.

There is another interesting feature: the magnitude and position of second maximum. It is easy to connect the size of the maximum with the magnitude of the real part of the scattering amplitude. Let us separate the main t-dependence in the equation in the representation of Δ_R and then put the differential equal zero:

$$\frac{d}{dt}[\Delta_R] = \frac{d}{dt}[h_1^2 \frac{1}{t^2} e^{Dt} + 2h_1 h_2 \frac{1}{t} e^{Dt/2} e^{Bt/2} + h_2^2 e^{Bt}] = 0, \tag{8}$$

where h_1 and h_2 are some electromagnetic and hadronic constants; D is the effective slope of the exponential form of the dipole form-factor.

At the point of maximum $-t = t_m$, we get

$$\begin{aligned} &2h_1^2 e^{-Dt_m} + h_1^2 D t_m e^{-Dt_m} - 2h_1 h_2 \frac{D+B}{2} t_m e^{-Dt_m/2} e^{-bt_m/2} \\ &-2h_1 h_2 t_m^2 e^{-Dt_m} e^{-Bt_m/2} + h_2^2 t_m^3 \frac{B}{2} e^{-Bt_m} = 0. \end{aligned} \tag{9}$$

Note that we can not neglect any term in this equation, the term with t_m^3 included as in the interesting region of momentum transfer all these terms are of the same order. We therefore obtain the equation:

$$\frac{ReF_h^2}{ReF_c^2} \frac{Bt_m}{2} + \frac{ReF_h}{ReF_c}(1 + \frac{D+B}{2} t_m) + \frac{D}{2} t_m + 1 = 0. \tag{10}$$

Equation (10) leads to the simple relation

$$B/2 = (1 + \frac{D}{2})\ \frac{ReF_C}{t_m ReF_h}. \tag{11}$$

Remembering the definition of Δ_R, we obtain

$$B/2 = (1 + \frac{D}{2})\ \frac{1}{t_m(\Delta_R^{1/2}/ReF_C - 1)}. \tag{12}$$

So, we can determine the slope of the real part of the hadron elastic scattering amplitude without any fitting procedure on a large interval of momentum transfer.

Note that the point t_R is important for the determination of the real part of spin-flip amplitude also [7]. At high energies and small angles the analyzing power can written in form

$$-A_N \frac{d\sigma}{dt}/2 = ImF^h_{nf}(ReF^c_{sf} + ReF^h_{sf}) + ImF^c_{nf}(ReF^c_{sf} + ReF^h_{sf}) \tag{13}$$
$$-ImF^c_{sf}(ReF^c_{nf} + ReF^h_{nf}) - ImF^h_{sf}(ReF^c_{nf} + ReF^h_{nf}).$$

We obtain for proton-proton scattering at high energies at the point t_R where $ReF_h^{nf} = -ReF_c^{nf}$,

$$ReF^h_{sf}(s,t) = \frac{-1}{2(ImF^h_{nf}(s,t) + ImF^c_{nf}(t))} A_N(s,t)\ \frac{d\sigma}{dt}\ - ReF^c_{sf}(t). \tag{14}$$

At this point some terms in the definition of analyzing power will be canceled. Such a representation can be used for the determination of the real part of the hadron spin-flip amplitude at high energy and small angles.

It is interesting to apply this new method to the proton-nuclear scattering at high energies. The size of the hadron amplitude grows only slightly less then proportional to A. If $\sigma_{tot}(pp) = 38$ mb in the region of hundred GeV, the $\sigma_{tot}(p^{12}C) = 335$ mb. The most important difference with pp-scattering is that the slope is very high, near 70 mb/GeV^2. The electromagnetic amplitude grows like Z and its slope also grows. It is interesting that the simple calculations which take the hadron amplitude at small momentum transfer in the usual exponential form with large slope leads the practically the same results as for the proton-proton scattering.

Let us take the Coulombic amplitude of $p^{12}C$ scattering in form

$$F_{em} = \frac{2\alpha_{em}\ Z}{t}\ F_{em}^{^{12}C} F_{em1}^{p} \tag{15}$$

where F_{em1}^{p} and F_{em2}^{p} are the electromagnetic form factors of the proton, and $F_{em}^{^{12}C}$ that of ^{12}C. We use

$$F_{em1}^{p} = \frac{4m_p^2 - t(\kappa_p + 1)}{(4m_p^2 - t)(1 - t/0.71)^2}, \tag{16}$$

$$F_{em2}^{p} = \frac{4m_p^2 \kappa_p}{(4m_p^2 - t)(1 - t/0.71)^2}, \tag{17}$$

where m_p is the mass of the proton and κ_p its anomalous magnetic moment. We obtain $F_{em}^{^{12}C}$ from the electromagnetic density of the nucleus

$$D(r) = D_0\ \left[1 + \tilde{\alpha}\left(\frac{r}{a}\right)^2\right] e^{-\left(\frac{r}{a}\right)^2}. \tag{18}$$

$\tilde{\alpha} = 1.07$ and $a = 1.7$ fm give the best description of the data [8] in the small-$|t|$ region, and produce a zero of $F_{em}^{^{12}C}$ at $|t| = 0.130$ GeV^2. We also calculated $F_{em}^{^{12}C}$ by integration of the nuclear form factor given by a sum of Gaussians [9] and obtained practically the same result with the zero now at $|t| = 0.133$ GeV^2.

Let us take the hadronic amplitude with standard exponential form with $\sigma_{tot} = 335$ mb. and with the slope $B = 62$ mb/GeV^2 [10]. The calculations are shown on Fig.4 for two variants with $\rho = 0.1$ and for $\rho = 0.075$. We can see that the minimum in t is approximately in the same region as the minimum in proton-proton scattering. There is also a significant difference of the size of the maximum for this two values of rho which is connected with the large slope in proton-nuclear scattering.

All previous results were obtained under the assumption that hadron scattering amplitude has the same exponential behavior as the *pp*-scattering amplitude at high energies. However the Glauber model is often used for the description of hadron-nuclei reactions. This model gives different behaviors of hadron scattering amplitudes at small momentum transfer. The slope of the hadronic amplitude increases sufficiently faster when $|t|$ increases while in pp-reaction the slope is constant or slightly decreases when $|t|$ increases. At low energies up to some GeVs a good description is obtained for different nuclei reactions in the framework of the Glauber model. Note that the proton-proton scattering at low energy also has the same behavior at low transfer momentum as the one given by the Glauber model for nuclear reactions. In Fig.5 the slopes of the real and imaginary part of the hadronic amplitude of $p^{12}C$-elastic scattering are shown.

The calculations were obtained by using the formulas of [11] and [12]. It is clear that the real part has a fast decrease and changes sign at $-t = 0.06$ GeV^2. The calculation of the value of Δ_R with $\rho = 0.1$ shows a wide minimum in the region of $0.025 \leq |t| \leq 0.045$ GeV (see Fig. 6). But if we made such a calculation for $\rho = -0.1$ the ordinary minimum will be obtained but situated sufficiently far in t. All the results arise from the behavior of the slope of the hadronic amplitude in the Glauber model. In the conclusion we obtain very different behaviors of Δ_R for the two models. So, research of such values Δ_R can help us to choose between different approaches.

The precise experimental measurements of dN/dt and A_N at RHIC, as well as, if possible, at the Tevatron, will therefore give us unavailable information on the hadron elastic scattering at small t. New phenomena at high energies could be therefore detected without going through the usual arbitrary assumptions (such as the exponential form) concerning the hadron elastic scattering amplitudes. It is interesting to apply this method to the proton-nuclei scattering at high energies, especially at RHIC energies. This method offers a unique possibility search for the behavior of the real part of the hadronic amplitude in nuclear reactions.

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Figure captions

FIG.1. The model calculations of Δ_R for the pp-scattering at RHIC energy $\sqrt{s} = 540\ GeV$ and with $\sigma_{tot} = 63\ mb$ and different n.

FIG.2. The calculation of Δ_R for the pp-scattering using the experimental data of $d\sigma/dt$ at $\sqrt{s} = 52.8\ GeV$ [6]. The lines are the polynomial fit of the points calculated with experimental data and with different n.

FIG.3. The calculation of Δ_R for the pp-scattering at RHIC with a) $\rho_1 = 0.135$ and b) $\rho_2 = 0.175$ The solid, short-dashed, and dotted lines are the theoretical curves for $\rho_2 = 0.175$, $\rho_1 = 0.135$, and $\rho_0 = 0$ respectively.

FIG.4. The calculation of Δ_R for the $p^{12}C$ -scattering with $\rho = 0.1$ and $\rho = 0.075$ (the solid and dashed lines, correspondingly) using of the exponential behavior of the hadronic amplitude.

FIG.5. The slopes B of the real (hard line) and imaginary (dashed line) parts of the hadronic amplitude calculated in the Glauber model for $p^{12}C$ scattering.

FIG.6 The calculation of Δ_R for $p^{12}C$ -scattering with $\rho = 0.1$ and $\rho = -0.1$ (the solid and dashed lines, correspondingly) in the Glauber model.

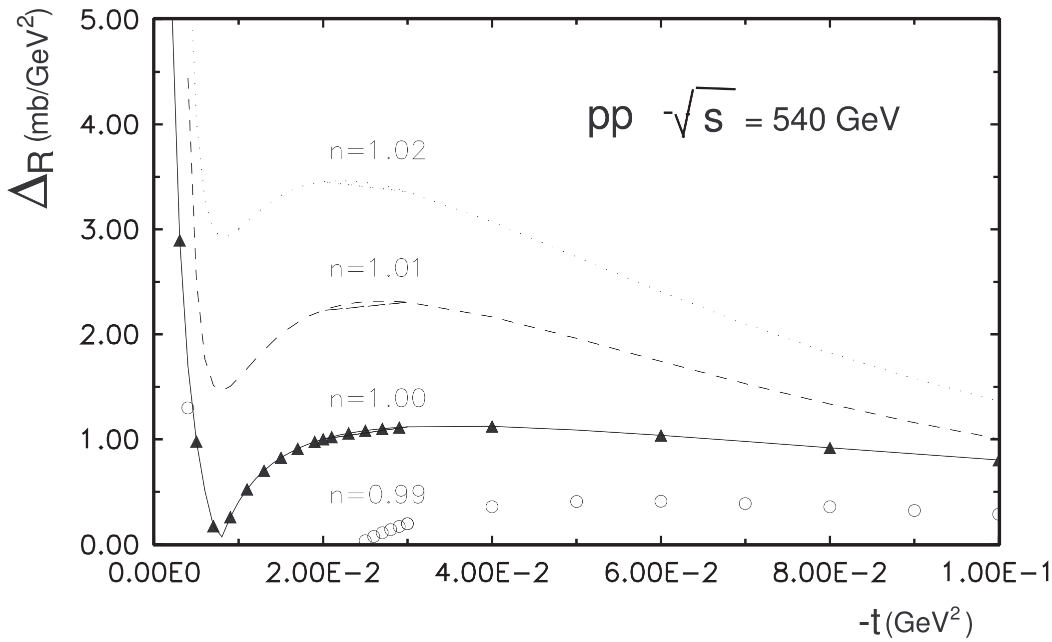

pp $\sqrt{s}$ = 540 GeV
n=1.02
n=1.01
n=1.00
n=0.99
Δ_R (mb/GeV^2)
5.00
4.00
3.00
2.00
1.00
0.00
0.00E0
2.00E−2
4.00E−2
6.00E−2
8.00E−2
1.00E−1
-t (GeV^2)

Fig.1

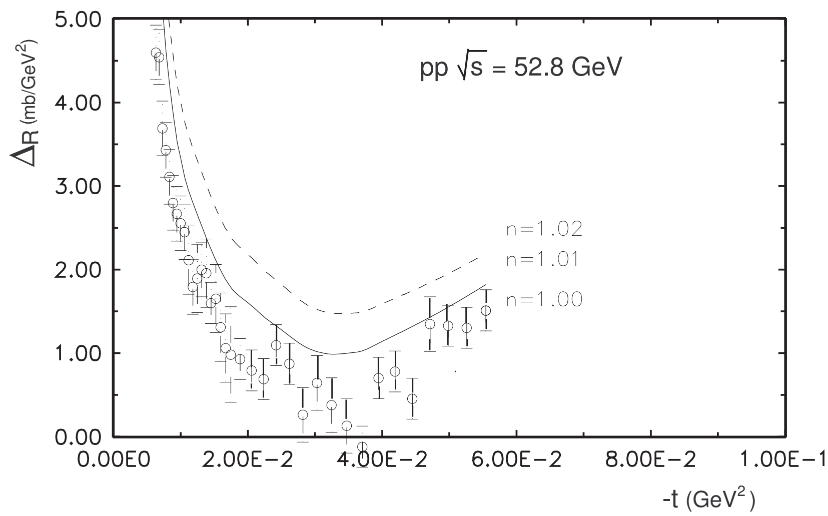
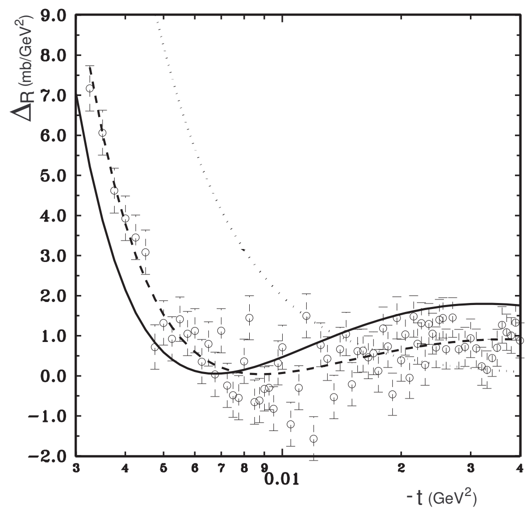
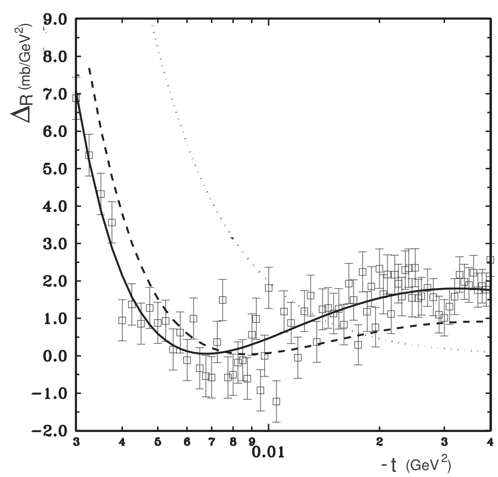

pp $\sqrt{s}$ = 52.8 GeV
n=1.02
n=1.01
n=1.00
Δ_R (mb/GeV^2)
5.00
4.00
3.00
2.00
1.00
0.00
0.00E0
2.00E−2
4.00E−2
6.00E−2
8.00E−2
1.00E−1
-t (GeV^2)

Fig. 2



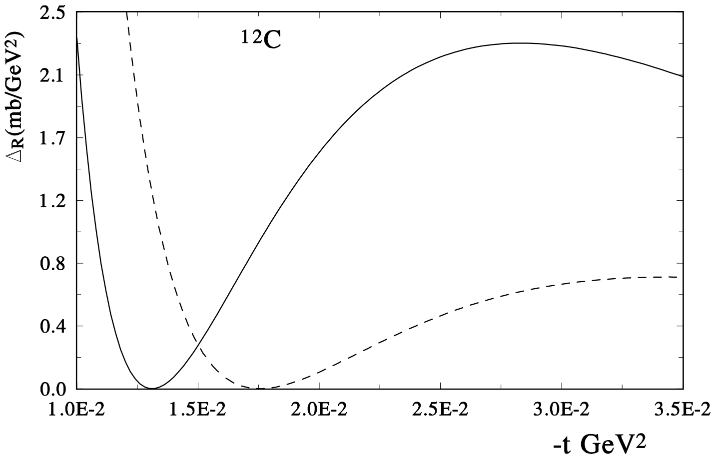
Δ_R (mb/GeV^2)
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
−1.0
−2.0
3
4
5
6
7
8
9
0.01
2
3
4
- t (GeV^2)

Fig.3 a



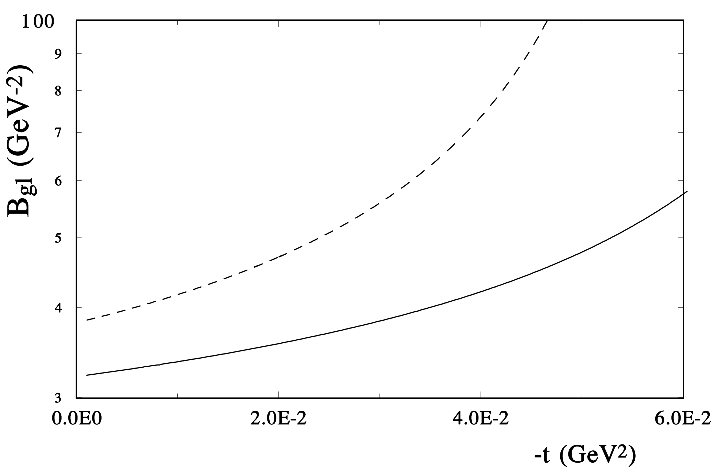
Δ_R (mb/GeV^2)
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
−1.0
−2.0
3
4
5
6
7
8
9
0.01
2
3
4
- t (GeV^2)

Fig.3 b



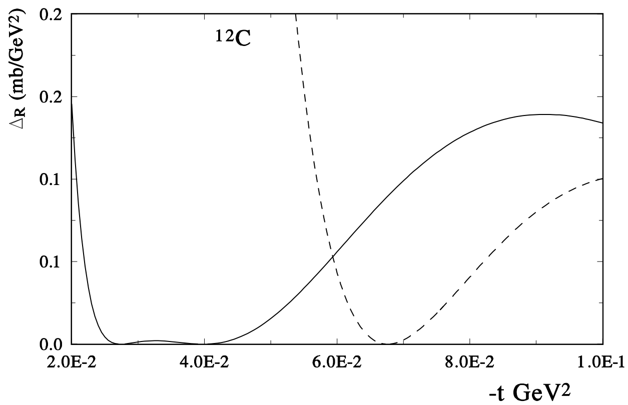
2.5
2.1
1.7
1.2
0.8
0.4
0.0
Δ_R(mb/GeV^2)
^{12}C
1.0E-2
1.5E-2
2.0E-2
2.5E-2
3.0E-2
3.5E-2
-t GeV^2

Fig.4



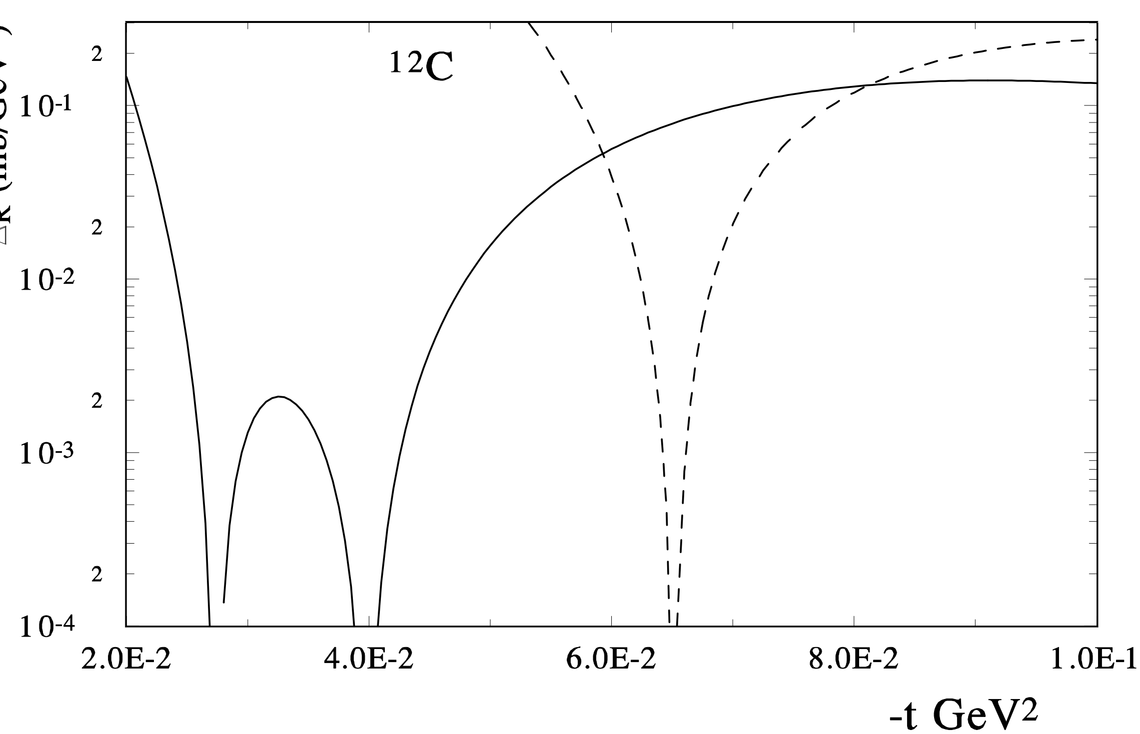
100
9
8
7
6
5
4
3
B_{gl} (GeV^{-2})
0.0E0
2.0E-2
4.0E-2
6.0E-2
-t (GeV^2)

Fig.5



0.2
0.2
0.1
0.1
0.0
Δ_R (mb/GeV^2)
^{12}C
2.0E-2
4.0E-2
6.0E-2
8.0E-2
1.0E-1
-t GeV^2

Fig.6



12C
Δ_R (mb/GeV2)
2
10-1
2
10-2
2
10-3
2
10-4
2.0E-2
4.0E-2
6.0E-2
8.0E-2
1.0E-1
-t GeV2