Application of the Optimal Homotopy Asymptotic Method for Fins with Variable temperature Surface Heat Flux

Z.A. Zaidi and A. Shahzad

Department of Mathematics, COMSATS Institute of Information Technology, University Road, Post code 22060, Abbottabad, NWFP, Pakistan

Abstract: In this paper, Optimal Homotopy Asymptotic Method (OHAM) is employed to obtain the temperature distribution, efficiency and heat transfer of rectangular fin with temperature dependent heat transfer coefficient i.e. (power-law function of temperature). OHAM provides the optimal analytical solution in the form of an infinite series. Obtained results of OHAM and numerical results for different values of heat transfer mode (n = -1, 0.1, 1, 2, 3, 4, 5) are compared graphically. Optimal values of constants for efficiencies and heat transfer are tabulated for different values of conductive convective parameter M = 0.01, 0.4, 0.8, 1.2, 1.6, 2. It is observed that the results of OHAM are simple, effective and easy.

Key words: Optimal homotopy asymptotic method, heat transfer, fins

INTRODUCTION

The aim of this article is to present an approximate solution of the nonlinear differential equation for steady-state one dimensional temperature distribution in rectangular fin by a well-known Optimal Homotopy Asymptotic Method (OHAM). Nonlinearity in the differential equation is due to the assumption of the variable local heat transfer coefficient. The local heat transfer coefficient is assumed to be a power-law function of temperature. Consequently, the exact analytic solution of the differential is not possible. In recent decades, numerical methods become more and more important due to their good means of analyzing the equations, but in parallel it is also believed that the approximate solutions are also useful and practical.

Perturbation method is one of the well-known method that produces solutions for small parameters. Aziz [1] presented the approximate solution for convective fin with internal heat generation and temperature dependent thermal conductivity by perturbation method. He observed the accuracy of the method by comparing the results with numeric results and maximum error was found to be 2%. Due to the presence of small parameter, perturbation method cannot explain the behaviour of physical problem for large values parameters. Singular differential equation of order two considered by Mohyud-Din et al. [2] using He's polynomial. Homotopy Perturbation Method introduced by He [3-5] to avoid the presence of small parameter. Noor et al. [6, 7] and Mohyud-Din et al. [8] investigated the solution of higher-order linear boundary value equations by using variational iteration method and homotopy perturbation method by taking He's polynomial. As an application of engineering and mathematical physics, Mohyud-Din et al. [9] uses He's polynomial for the solution of seventh-order KdV equation for the traveling waves. Also Mohyud-Din et al. [10] obtained analytical soliton solution for kaup-kupershmidt equation without direct transformation. To solve the nonlinear partial differential equations Asif et al. [11] proposed the method by combining the homotopy analysis method and Laplace decomposition method.

Recently, Herisanu et al. [12] introduced Optimal Homotopy Asymptotic Method (OHAM) while studying the nonlinear dynamic behaviour of an electric machine exhibiting nonlinear vibration. This method is flexible than Homotopy Analysis Method (HAM) due to its built in convergence criteria. Marinca et al. [13-15] investigated the solution of nonlinear heat transfer problems and fluid flow problems with OHAM. They compared OHAM results with numeric results and found good agreement of OHAM solutions. Idrees et al. [16, 17] solved fourth and sixth order boundary value problem with OHAM. Iqbal et al. [18] investigated the effectiveness of the OHAM in solving the heat transfer flow of a third grade fluid between two heated parallel plates. They represented a comparative analysis between OHAM and numerical solution by finite element method and found that the method of OHAM is simple, precise, effective and easy to use.
Ansari et al [19] applied OHAM to the problem of flow through a porous channel. On comparison the results of OHAM with numerical solutions they found that this method is effective and easy. It has been also observed that N. Herisanu, V. Marinca and Javed et al [20-22] included functions of a physical parameter to define the auxiliary function \( H(p) \) in order to increase the efficiency and accuracy of the procedure. Javed et al. [23] considered boundary value problem of twelfth-order and compared the results obtained from OHAM and numerical method.

Fins are used in engineering to enhance heat dissipation from hot primary surface. There are different shapes and profiles of fins with different boundary conditions documented Kraus et al and Sunden et al. [24, 25]. Fins are used in air conditioning, air cooled craft engine, refrigeration, cooling of computer processor etc. The study on different fins with different assumptions is present in literature. For example, Minkler et al. [26] studied rectangular and triangular fins with uniform internal heat generation but constant thermal conductivity and heat transfer coefficient. By considering the annular fin of trapezoidal shape Razelos et al. [27] studied the heat transfer rate and effect of thermal conductivity. The assumption of their study was that with the distance from the base of the fin, the heat transfer coefficient increases as power law function. A close-form solution was presented by Unal [28] for the rectangular profile of fin for the one-dimensional temperature distribution. It was assumed that the heat transfer coefficient was a power function of the difference between the temperature of the fin and the ambient fluid. Exact solution was obtained for the straight fin with rectangular profile by Sen et al. [29]. He considered coefficient of heat transfer as temperature dependent of power law type function. Hamad et al. [30] investigated numerically the heat transfer to the power law fluid over the vertical stretching sheet. Laor et al [31] investigated the numerical results for the annular, triangular, rectangular and parabolic profiled fins and spines. They analyzed the efficiency and fin dimensions at optimum level by variational heat transfer coefficient of Power law function. Chang [32] use Adomian decomposition method to obtained series solution for thermal properties of fin by considering temperature as function of Power law.

**NOMENCLATURE**

- \( A_c \) Cross sectional area of the spone, \( \text{m}^2 \)
- \( a \) Dimensional constant
- \( B \) Boundary operator
- \( C_i \) Unknown constant of auxiliary function: \( i = 1, 2 \)
- \( g(x) \) Known function involved in OHAM
- \( h(T) \) Heat transfer coefficient, \( \text{W/m}^2\text{.K} \)
- \( H(p) \) Auxiliary function
- \( k \) Thermal conductivity, \( \text{W/m} \cdot \text{K} \)
- \( L \) Length of the fin, \( \text{m} \)
- \( L(\theta) \) Linear part
- \( M \) Dimensionless conductive convective parameter
- \( n \) Exponent in Eq. (1)
- \( N(\theta) \) Nonlinear part
- \( P \) Perimeter of the fin, \( \text{m} \)
- \( p \) Embedding parameter
- \( Q_b \) Dimensionless that transfer at fin base
- \( R \) Residual
- \( T \) Temperature, \( \text{K} \)
- \( x \) Non-dimensional space coordinate
- \( y(x) \) Function involved in OHAM

**Greek Symbols**

- \( \theta \) Dimensionless temperature
- \( \zeta \) Dimensionless distance \( \equiv x/L \)
- \( \eta \) Fin efficiency

**Subscripts**

- \( a \) Ambiant
- \( b \) Base
- \( 0 \) Zeroth order
- \( 1 \) First order
- \( 2 \) Second order

**PROBLEM FORMULATION**

Consider a homogeneous and isotropic rectangular fin of arbitrary uniform cross section (Fig. 1) with cross-sectional area \( A_c \), perimeter \( P \), length \( L \) and constant thermal conductivity \( k \). The fin is attached to a prime surface at constant temperature \( T_b \) and extends into fluid at ambient temperature \( T_a \). The tip of the fin is insulated and heat is removed from the surface of the fin via natural convection and local heat transfer coefficient \( h(T) \) depends upon temperature difference between the fin and ambient temperature outside the fin in the power-law type written as follows

\[
h(T) = a(T - T_a)^n
\]  

(2.1)

where ‘\( a \)’ is dimensional constant, \( T \) is the local temperature on the fin surface and \( n \) depends on the heat transfer mode.
Fig. 1: Geometry of the problem

Steady state heat balance equation for the fin of arbitrary uniform cross section can be written as

\[
\frac{d}{dx} \left[ \frac{k}{A_c} \frac{dT}{dx} \right] = -\frac{\rho h(T)k}{A_c} (T - T_a) \tag{2.2}
\]

With boundary conditions

\[
x = L, T = T_a, x = 0, \frac{dT}{dx} = 0 \tag{2.3}
\]

In dimensionless form, Eq. (2.2) and Eq. (2.3) can be written as

\[
\frac{d^2 \theta}{d\zeta^2} = M^2 \theta^{(n+1)} \tag{2.4}
\]

and

\[
\zeta = 1, \theta = 1, \zeta = 0, \frac{d\theta}{d\zeta} = 0 \tag{2.5}
\]

The dimensionless variables are

\[
\theta = \frac{T - T_b}{T_b - T_a}, \zeta = \frac{x}{L},
\]

\[
M = \sqrt{\frac{\rho h - L^2}{k A_c}} = \left[ \frac{\rho P L^2}{k A_c} \left( T_b - T_a \right)^n \right]^{1/2} \tag{2.6}
\]

where \( \zeta \) the axial distance is measured from the tip, \( T_b \) is the temperature at the base of the fin and \( M \) is the dimensionless fin conductive-convective parameter, \( h_b \) is the heat transfer coefficient at the base of fin.

**BASIC IDEA OF OHAM**

Consider a differential equation of the form

\[
L(y(x)) + g(x) + N(y(x)) = 0, B \left( y, \frac{dy}{dx} \right) = 0 \tag{3.1}
\]

where \( L \) is linear operator, \( x \) is independent variable, \( y(x) \) is unknown function, \( g(x) \) is known function, \( N \) is nonlinear operator and \( B \) is boundary operator.

According to OHAM a deformation is constructed as follows:

\[
(1 - p)[L(\phi(x,p)) + g(x)] = H(p)[L(\phi(x,p)) + g(x) + N(\phi(x,p))] \tag{3.2}
\]

\[
B \left( \phi(x,p), \frac{d\phi(x,p)}{dx} \right) = 0
\]

where \( p \in [0,1] \) is an embedding parameter, \( H(p) \) is a nonzero auxiliary function for \( p \neq 0 \) and \( H(0) = 0 \), \( \phi(x,p) \) is an unknown function, obviously, when \( p = 0 \) and \( p = 1 \) it holds \( \phi(x,0) = y_0(x) \) and \( \phi(x,1) = y(x) \).

Thus as \( p \) varies from 0 to 1, the solution \( \phi(x,p) \) varies from \( y_0(x) \) to the solution \( y(x) \), where \( y_0(x) \) is obtained from Eq. (3.2) for \( p = 0 \):

\[
L(y_0(x)) + g(x) = 0, B \left( y_0, \frac{dy_0}{dx} = 0 \right) \tag{3.3}
\]

We choose auxiliary function \( H(p) \) in the form

\[
H(p) = pC_1 + p^2C_2 + \ldots \tag{3.4}
\]

Where \( C_1 \) and \( C_2 \) are constants

By expansion the Taylor series of \( \phi(x,p,C_i) \) about \( p \), we obtain

\[
\phi(x,p,\zeta) = y_0(x) + \sum_{k=1}^{\infty} y_k(x,C_1,C_2,\ldots,C_k)p^k \tag{3.5}
\]

Now substituting Eq. (3.5) in Eq. (3.2) and equating the equal powers of \( p \), we obtain the following linear equations:

Zeroth order problem is given by Eq. (3.3), first and second order problems are as follows:

\[
L(y_1(x)) + g(x) = C_1 N_0(y_0(x)), B \left( y_1, \frac{dy_1}{dx} = 0 \right) = 0 \tag{3.6}
\]

\[
L(y_2(x)) - L(y_1(x)) = C_2 N_0(y_0(x)) + C_1 [L(y_1(x)) + N_1(y_0(x),y(x))] \tag{3.7}
\]

\[
B \left( y_2, \frac{dy_2}{dx} \right) = 0
\]

The general governing equations for \( y_k(x) \) are given by
In order to find temperature distribution \( \theta(\zeta) \), we decompose the differential equation Eq. (2.4) in the following form

\[
L(0) = \frac{d^2 \theta}{d \zeta^2} N(\theta) - M^2 \theta^{(n+1)} G(\zeta) = 0 \tag{4.1}
\]

According to the OHAM, we can construct a homotopy \( \theta(\zeta; p) : \Omega \times [-1, 1] \rightarrow \mathbb{R} \) which satisfies

\[
(1 - p) L(\theta(p) + G(\zeta)) - H(p)[L(\theta(p)) + G(\zeta) + N(\theta(p))] = 0 \tag{4.2}
\]

where \( p \in [0, 1] \) is an embedding parameter, \( H(p) = pC_1 + p^2 C_2 \) is non-zero auxiliary function for \( p \neq 0 \). To get the approximate solution we can express \( \theta(\zeta) \) in the following series:

\[
\theta(\zeta, p, C_i) = \sum_{i=1}^{\infty} \theta_i(\zeta) C_i \tag{4.3}
\]

By substituting Eq. (4.3) in Eq. (4.2) along with boundary conditions Eq. (2.5) and equation the coefficients of powers of \( p \), we obtain the following linear equations.

**Zeroth Order of \( p^0 \):**

\[
\frac{d^2 \theta_0}{d \zeta^2} = 0, \quad \theta_0(1) = 1, \quad \frac{d \theta_0(0)}{d \zeta} = 0 \tag{4.3.1}
\]

**First Order of \( p^1 \):**

\[
\frac{d^2 \theta_1}{d \zeta^2} - \frac{d^2 \theta_0}{d \zeta^2} - C_1 \left( -M^2 \theta_0^{(n+1)} \right) + \frac{d^2 \theta_0}{d \zeta^2} = 0 \tag{4.3.2}
\]

\[
\theta_1(1) = 0, \quad \frac{d \theta_1(0)}{d \zeta} = 0
\]

**Second Order of \( p^2 \):**

\[
\frac{d^2 \theta_2}{d \zeta^2} - \frac{d^2 \theta_1}{d \zeta^2} - C_2 \left( -M^2 \theta_0^{(n+1)} \right) + \frac{d^2 \theta_0}{d \zeta^2} = 0 \tag{4.3.3}
\]

\[
\theta_2(1) = 0, \quad \frac{d \theta_2(0)}{d \zeta} = 0
\]
The zeroth order solution is obtained from Eq. (4.3.1) by applying boundary conditions:

\[ \theta_0(\xi) = 1 \]  
\[ \text{(4.4)} \]

The first order solution is obtained by substituting Eq. (4.4) in Eq. (4.3.2)

\[ \theta_1(\xi) = \frac{1}{2} M^2 C_1 \left(1 - \xi^2\right) \]  
\[ \text{(4.5)} \]

The second order solution is obtained by substituting Eq. (4.5) and Eq. (4.4) in Eq (4.3.3)

\[ \theta_2(\xi) = \frac{1}{24} \left(12M^2 \xi(1 - \xi^2)(C_1^2 + C_1 + C_2) + M^2 C_1^2(1 + n)(5 - 6\xi^2 + \xi^4)\right) \]  
\[ \text{(4.6)} \]

We obtain \( \theta(\xi, 1; C_1, C_2) \) from \( \lim_{\beta \to \infty} \theta(\xi, \beta; C_1, C_2) \) as follows

\[ \theta(\xi, 1; C_1, C_2) = B_1 \xi^4 + B_2 \xi^2 + B_3 \]  
\[ \text{(4.7)} \]

Where

\[ B_1 = \frac{(1 + n)}{24} M^2 C_1^2 \]

\[ B_2 = \left(\frac{1}{2} M^2 C_1^2 - \frac{1}{2} M^2 \left(C_1^2 + C_1 + C_2\right) - \frac{(1 + n)}{4} M^2 C_1^2\right) \]

\[ B_3 = \left(1 + \frac{1}{2} M^2 C_1 + \frac{1}{2} M^2 \left(C_1^2 + C_1 + C_2\right) + \frac{5(1 + n)}{24} M^2 C_1^2\right) \]

Substituting Eq. (4.7) in Eq. (2.2), we obtain the residual

\[ R(\xi, C_1, C_2) = 12B_1 \xi^2 + 2B_2 - M^2 (B_1 \xi^4 + B_2 \xi^2 + B_3)^{(1+n)} \]  
\[ \text{(4.8)} \]

For the calculation of \( C_1 \) and \( C_2 \) we will minimize the functional as follows

\[ J(C_1, C_2) = \int_{0}^{1} \left(12B_1 \xi^2 + 2B_2 - M^2 (B_1 \xi^4 + B_2 \xi^2 + B_3)^{(1+n)}\right)^2 \, d\xi \]  
\[ \text{(4.9)} \]

The unknown constants \( C_1 \) and \( C_2 \) can be identified by using the following conditions

\[ \frac{\partial J(C_1, C_2)}{\partial C_1} = 0 \]  
\[ \frac{\partial J(C_1, C_2)}{\partial C_2} = 0 \]  
\[ \text{(4.10)} \]

We obtain \( C_1 = -1.0396620259043718 \) and \( C_2 = -0.001573076298392932 \) for \( M = 1 \) and \( n = -1 \)

Thus the temperature distribution will be

\[ \theta(\xi) = 1 + \frac{1}{2} \left(-1.03966 + 1.03966\xi^2\right) + \frac{1}{24} (0.475944 - 0.475944 \xi^2) \]  
\[ \text{(4.11)} \]

We obtain \( C_1 = -0.8337205004453097 \) and \( C_2 = -0.02092667410572506 \) for \( M = 1 \) and \( n = 0 \)

\[ \theta(\xi) = 1 + \frac{1}{2} \left(-1.083372 + 1.083372\xi^2\right) + \frac{1}{24} (1.5618 - 2.25585\xi^2 + 0.695094 \xi^4) \]  
\[ \text{(4.12)} \]

We obtain \( C_1 = -0.6841567541500388 \) and \( C_2 = -0.6408872233670536 \) for \( M = 1 \) and \( n = 1 \)

\[ \theta(\xi) = 1 + \frac{1}{2} \left(-0.684157 + 0.684157\xi^2\right) + \frac{1}{24} (1.3186 - 2.25475\xi^2 + 0.936141 \xi^4) \]  
\[ \text{(4.13)} \]

We obtain \( C_1 = -0.5940953439782699 \) and \( C_2 = -0.10055929055285127 \) for \( M = 1 \) and \( n = 2 \)

\[ \theta(\xi) = 1 + \frac{1}{2} \left(-0.594095 + 0.594095\xi^2\right) + \frac{1}{24} (1.19377 - 2.25562\xi^2 + 1.05885 \xi^4) \]  
\[ \text{(4.14)} \]

We obtain \( C_1 = -0.5315977118081278 \) and \( C_2 = -0.1291465295328504 \) for \( M = 1 \) and \( n = 3 \)

\[ \theta(\xi) = 1 + \frac{1}{2} \left(-0.531598 + 0.531598\xi^2\right) + \frac{1}{24} (1.11415 - 2.224453\xi^2 + 1.13038 \xi^4) \]  
\[ \text{(4.15)} \]

We obtain \( C_1 = -0.48474796410187865 \) and \( C_2 = -0.15168226737683604 \) for \( M = 1 \) and \( n = 4 \)

\[ \theta(\xi) = 1 + \frac{1}{2} \left(-0.484748 + 0.484748\xi^2\right) + \frac{1}{24} (1.05712 - 2.23202\xi^2 + 1.1749 \xi^4) \]  
\[ \text{(4.16)} \]
Table 1: Efficiency of fin and Heat transfer rate by OHAM, for \( n = 1 \)

<table>
<thead>
<tr>
<th>M</th>
<th>Optimal constants (C1)</th>
<th>Optimal constants (C2)</th>
<th>Efficiency (( \eta ))</th>
<th>Heat transfer (( Q_b ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.9999425414714733</td>
<td>-2.30646873263E--09</td>
<td>0.999933</td>
<td>9.99933E-05</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.9198371546249328</td>
<td>-0.0044650717292166</td>
<td>0.907789</td>
<td>0.145246</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.7613990651440697</td>
<td>-0.0379672490078687</td>
<td>0.733472</td>
<td>0.469386</td>
</tr>
<tr>
<td>1.2</td>
<td>-0.6144861928519469</td>
<td>-0.092416537795910</td>
<td>0.581733</td>
<td>0.837081</td>
</tr>
<tr>
<td>1.6</td>
<td>-0.49965653605303134</td>
<td>-0.1452293105184308</td>
<td>0.470137</td>
<td>1.20014</td>
</tr>
<tr>
<td>2</td>
<td>-0.41267823838855433</td>
<td>-0.18620275061307986</td>
<td>0.389817</td>
<td>1.54845</td>
</tr>
</tbody>
</table>

Table 2: Efficiency of fin and Heat transfer rate by OHAM, for \( n = 5 \)

<table>
<thead>
<tr>
<th>M</th>
<th>Optimal constants (C1)</th>
<th>Optimal constants (C2)</th>
<th>Efficiency (( \eta ))</th>
<th>Heat transfer (( Q_b ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.0000158594469984</td>
<td>-0.9998260047086994</td>
<td>0.9998</td>
<td>9.99794E-05</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.7845406337298615</td>
<td>-0.0306108337687974</td>
<td>0.787265</td>
<td>0.125956</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.5340928273806823</td>
<td>-0.1280366873500497</td>
<td>0.546761</td>
<td>0.349338</td>
</tr>
<tr>
<td>1.2</td>
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<td>0.576306</td>
</tr>
<tr>
<td>1.6</td>
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<td>0.794419</td>
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<tr>
<td>2</td>
<td>-0.22510984234307682</td>
<td>-0.2566862678526507</td>
<td>0.257729</td>
<td>1.00334</td>
</tr>
</tbody>
</table>

FIN EFFICIENCY AND HEAT TRANSFER RATE BY OHAM

The efficiency of the fin \( \eta \) is defined as the ratio of the total heat transfer rate to the fin at the base temperature

\[
\eta = \int_0^1 \frac{P_b(T - T_b)dz}{P L h_b(T_b - T)} = \int_0^1 \theta(z) (\varepsilon + 1) dz
\]  

(5.1)

The dimensionless heat transfer is defined as

\[
Q_b = \frac{d\theta}{dz}_{z=1}
\]

(5.2)

The Table 1 and 2 represents the values of Efficiency and Heat transfer rate for \( n = 1 \) and \( n = 5 \) at \( 0 < M \leq 2 \), by utilizing the optimal values of \( C_1 \) and \( C_2 \).

NUMERICAL METHOD

The solution obtained by OHAM is validated by solving numerically Eq. (2.4) with boundary conditions (2.5) by using Mathematica. The command Dsolve with the options numeric which uses by default a Fehlberg fourth-fifth order Runge-Kutta method with degree four interpolations was used. The default values for absolute and relative errors are 1e-7. The step size was fixed as

Fig. 2: Temperature distribution for different values of \( n \) at \( M = 1 \) (OHAM Results)

Fig. 3: Temperature distribution for different values of \( n \) at \( M = 1 \) (Numeric Results)
Table 3: Comparison of OHAM and numeric results for temperature distribution

<table>
<thead>
<tr>
<th>n = -1</th>
<th>n = 0</th>
<th>n = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>ζ</td>
<td>OHAM</td>
<td>Numeric</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>0.1</td>
<td>0.505</td>
<td>0.505</td>
</tr>
<tr>
<td>0.2</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>0.3</td>
<td>0.545</td>
<td>0.545</td>
</tr>
<tr>
<td>0.4</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>0.5</td>
<td>0.625</td>
<td>0.625</td>
</tr>
<tr>
<td>0.6</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>0.7</td>
<td>0.745</td>
<td>0.745</td>
</tr>
<tr>
<td>0.8</td>
<td>0.82</td>
<td>0.82</td>
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<tr>
<td>0.9</td>
<td>0.905</td>
<td>0.905</td>
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<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Δξ = 0.001. From numerical experiments we can say that to get the smooth graph it is enough to use 1000 points. The numerical results agreed with the approximate results and confirmed the accuracy of our OHAM.

RESULTS AND DISCUSSIONS

Figure 2 and 3 show the comparison between Numerical and OHAM results Temperature distribution at different values of index n of power Law model by keeping conductive-convective parameter (M) constant. Both results show that at ζ=1, temperature profile is constant for all values of n, but it deceases for 0=ζ<1. It is also recorded that temperature rises with the increase in the value of n. It has been observed from Table 3 that results taken by OHAM have good agreement with numerical results with very small absolute error.

The efficiency of the fin can be calculated by using the expression (5.1). Table 1 and 2 are constructed to show the efficiency of the fin for different values of conductive-convective parameter (M) at n=1 and n=5 by using the optimal values of the constants C1,C2. It can be observed from the Table 1 and 2 that for minimum value of conductive parameter (M) the efficiency (η) is Maximum. But the efficiency (η) decreases as M decreases.

The rate of Heat Transfer is obtained from the expression (5.2). At n = -1, we obtained uniform local heat flux through-out the surface of the fin for all values of conduction parameter (M). Table 1 and 2 represents the heat transfer rate for M<2 at n=1 and 5 with optimum values of constants C1,C2. It is also observed that rate of heat flux at ζ = 1 is maximum at M=2 and minimum at small value of conductive parameter (M). From Table 1 and 2, it can be observed that efficiency of the fin and rate of heat transfer are inversely proportional to each other. Graphical representation of efficiency and base temperature for 0=M= 2, at n=1,3,5 can be observed.

SUMMARY AND CONCLUSIONS

We applied analytic method, Optimal Homotopy asymptotic method (OHAM) to analyze temperature distribution, efficiency and base temperature variation.
as Power Law function. The results obtained from OHAM are highly accurate to numerical results. The method is very useful for highly non-linear boundary problems having large domain. The method does not require the small parameter like perturbation approximation method or discretization like numerical methods. It is easy and fast to reach the convergence point and obtained solutions in explicit form. OHAM can benefits in real sense to researchers in solving & analyzing the highly nonlinear systems.

For symbolic derivations of some of the equations we use the software Wolfram Mathematica.

REFERENCES


