Connection data rate optimisation of IEEE 802.15.3 scatternets with multirate carriers

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Abstract: Designed for high rate Wireless Personal Area Networks (WPANs), the IEEE 802.15.3 Medium Access Control (MAC) protocol supports peer-to-peer communications in a piconet. In this paper, we analyse the link rate distribution of an 802.15.3 piconet employing multirate carriers at the physical layer, and show that the Expected Piconet Link Rate (EPLR) decreases gradually with the piconet radius. Furthermore, the Effective Scatternet Connection Rate (ESCR) is defined to minimise the stream connection cost in a scatternet by optimising the piconet coverage. Analytical and simulation results show that direct peer-to-peer communications in 802.15.3 piconets bring a huge gain in the expected data rate of intra-piconet links. While the maximum piconet size minimises the number of piconets in the scatternet and the number of hops for a randomly chosen connection, a medium sized piconet radius optimises the scatternet connection data rate. Thus, configuration of scatternets needs to consider the piconet size and channel reuse given the number of logical channels available.

Keywords: Wireless Personal Area Network; WPAN; 802.15.3; Medium Access Control; MAC; scatternet; peer-to-peer communications.

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1 Introduction

Wireless Personal Area Network (WPAN) technologies, such as Bluetooth, offer a new frontier of ubiquitous wireless networking. However, the data rate in Bluetooth is too low to support high bandwidth multimedia applications. With the rapid proliferation of digital consumer electronic devices, there is an ever increasing demand for WPAN solutions that simultaneously provide high data rates, low cost, low power consumption and Quality of Service (QoS) support. The IEEE 802.15.3 standard (IEEE Std 802.15.3, 2003) has been developed to satisfy these needs. The standard specifies the physical (PHY) and Medium Access Control (MAC) protocols for High Date Rate (HDR) WPANs, which support peer-to-peer communications between devices and QoS provisioning.

Ultra-Wideband (UWB), as a new approach to data transmissions, holds a great promise for short distance communications, especially in WPANs (Stroh, 2003; Park et al., 2003; Porcino and Hirt, 2003). The commercial exploitation of UWB has accelerated after the Federal Communications Commission (FCC) in the USA allocated 7.5 GHz of spectrum in 2002 for unlicensed use by UWB devices with strict emission limits (FCC02-48A1, 2002). The extremely low power consumption and high spatial...
capacity make UWB an ideal candidate for the physical layer of HDR-WPANs. Although the efforts of defining a UWB-based PHY standard for 802.15.3 WPANs failed with the dissolution of the IEEE 802.15.3a task group (IEEE 802.15 TG3a, 2006), the task group consolidated 23 UWB proposals into two merged PHY proposals: Multi-Band Orthogonal Frequency Division Multiplexing (MB-OFDM) (Batra et al., 2004) supported by the WiMedia Alliance and direct sequence UWB (DS-UWB) supported by the UWB Forum (Fisher et al., 2005). Both proposals can be used directly under the 802.15.3 MAC. While existing analyses of MAC protocols often assume a fixed data rate for all transmissions, many contemporary data transmission techniques, including the above UWB PHY, support multiple data rates that adapt to the changing path gains between transmitters and receivers. Therefore, it is important to take rate adaptation into account when investigating the engineering design of 802.15.3 WPANs, which is the objective of this paper.

For a large scale scatternet consisting of multiple piconets, extensive studies on Bluetooth scatternet formation (Petrioli et al., 2003, 2004; Wang et al., 2002; Zaruba et al., 2001) have focused on the topology configuration problem that arises due to the constraint of master-slave communications between no more than eight active devices in a Bluetooth piconet. In contrast, the 802.15.3 MAC allows a large number of active devices (DEVs) to communicate directly in a peer-to-peer manner over a piconet, under the control of the Piconet Coordinator (PNC). Therefore, scatternet formation for 802.15.3 WPANs presents the new problem of how to optimise the coverage area of the piconets to form the scatternet in order to satisfy certain communication performance criteria. For multimedia transmissions in HDR-WPANs, it is desirable to maximise the overall effective connection data rates of the multimedia data streams. With constant link rate, the best result is obviously achieved by minimising the number of piconets in the scatternet by maximising the size of each piconet, thus, minimising the number of hops in a connection. However, in a piconet with multirate carriers, reducing the piconet size could increase the average data rate between DEVs. This trade-off will be explored in this paper. Given such a trade-off, how to choose the right piconet size to minimise overall transmission cost in a scatternet is an important open issue that will be further addressed in this paper.

The rest of this paper is organised as follows. Section 2 briefly reviews the 802.15.3 MAC and describes the piconet and scatternet formation issues. In Section 3, with the full connectivity support by PNC forwarding, the Expected Piconet Link Rate (EPLR) of an 802.15.3 piconet is derived. In Section 4, the expected scatternet connection rate is defined and analysed with respect to the piconet size. Numerical results for 802.15.3 WPANs based on the MB-OFDM UWB PHY are presented in Section 5. Our analyses in Sections 3–5 are based on an idealised channel model for the physical layer. Section 6 discusses how the results of these analyses can be applied when practical channel conditions are taken into account. Section 7 concludes this paper.

2 IEEE 802.15.3 MAC and network formation issues

Designed for HDR-WPANs, the IEEE 802.15.3 MAC employs centralised PNC control of direct peer-to-peer communications among DEVs, as shown in Figure 1. The PNC provides basic timing for network synchronisation, performs admission control and allocates network resources for data transfers based on QoS policies, etc.

Figure 1  Piconet operation in IEEE 802.15.3 WPAN

Data transmissions in an 802.15.3 piconet are based on the superframe, which consists of three parts: the beacon, the optional Contention Access Period (CAP) and the Channel Time Allocation Period (CTAP). The PNC sends beacons to synchronise the piconet, set timing allocations and communicate management information to the DEVs in the piconet. DEVs employ Carrier Sensed Multiple Access with Collision Avoidance (CSMA/CA) for channel access in CAPs to communicate command and/or asynchronous data. DeVs request to the PNC for assignment of Channel Time Allocations (CTAs) in the CTAPs to send isochronous streams and large amounts of asynchronous data by means of TDMA. The guaranteed start time and duration of each CTA enable both good power saving and QoS support for DEVs. Alternatively, the PNC can choose to use Management CTAs (MCTAs) instead of CAP for command exchanges. Slotted Aloha is used for channel access during open and association MCTAs.

The 802.15.3 MAC works with a multirate PHY, in which a mandatory base rate is used for the beacon, command frames as well as the PHY and MAC headers for robustness. The frame payload is sent with a data rate selected from a set of predefined rate-dependent modulation parameters according to the link quality requirement, for example, the Frame Error Rate (FER) threshold, and the perceived link condition, which depends on the distance between the transmitter and the receiver, the propagation environment (e.g. shadowing and fading effects), and interference from cochannel and adjacent channel piconets. While the propagation environment and interference conditions are hard to quantify, in general
closer DEVs may communicate with higher data rates over the shorter links between them. Before a DEV requests a CTA from the PNC, it selects the data rate based on previous estimates of the link condition and calculates the amount of time required in the CTA. The CTA duration is then conveyed to the PNC in the CTA request. The CTA, once assigned, remains unchanged until the DEV sends a new request to the PNC, for example, when the traffic condition or link quality has changed.

Depending on the piconet size and DEV positions, some DEVs may not be able to directly communicate with each other as the distances between them exceed the maximum transmission range. In Yin and Leung (2006c), a third-party handshake protocol is proposed to solve this problem by employing PNC forwarding to provide full MAC layer connectivity between directly unreachable DEV pairs in a piconet.

Unlike pure ad hoc wireless networks, where all mobile devices share a common wireless channel, each 802.15.3 piconet works on a different logical channel except for child and neighbour piconets. Thus, unless there is a bridge DEV associated with the PNCs of adjacent piconets, communication between DEVs in the two piconets is not possible. Due to the very low power emission limit, especially for UWB transmissions, a DEV should always use the maximum allowed power when sending data to minimise the FER. This principle, when applied to the PNC, results in a piconet with the maximum coverage by default.

Two methods are provided by the 802.15.3 standard for controlling transmitter power (IEEE Std 802.15.3, 2003). The first method allows the PNC to set the maximum transmit power for the CAP, beacon, and MCTAs, but excluding association MCTAs. As all DEVs associated with a piconet need to correctly decode the beacons from the PNC, the PNC can reduce the transmission power of beacon frames to limit the distance of DEVs that can be associated with it. Thus, the piconet coverage area and hence the size of the piconet are determined by the transmit power of the beacons. Controlling the power during these times allows the PNC to reduce transmit power without adversely affecting operation of the piconet.

The second method allows a DEV to use a CTA transmission to request the remote DEV to change its transmit power. Thus, if two DEVs have a ‘good’ link in a CTA, they can reduce their transmitter power to decrease the power usage, and to reduce interference to other networks, while still satisfying the channel quality threshold for the application.

As this paper addresses the issue of piconet size, we consider that the PNC can adjust its beacon power to the maximum transmission range, and all DEVs transmit data at maximum power using the highest data rate achievable within the link reliability constraint.

Wireless spectrum is a scarce and valuable resource, especially for HDR-WPANs; for example, the 802.15.3 PHY at 2.4 GHz has no more than 4 channels available (IEEE Std 802.15.3, 2003); the MB-OFDM UWB PHY has only 4 logical channels for Mode 1 devices and a maximum of 16 if all bands are used (Batra et al., 2004); the DS-UWB has 2 bands each with only 6 channels (Fisher et al., 2005). Therefore, the number of 802.15.3 piconets simultaneously operating within radio range is quite limited compared with Bluetooth, which is channelised using pseudorandom hopping sequences with 79 frequencies. As the interference region is much larger than the coverage area of a piconet, the limited number of channels also limits the ability of channel reuse in a large scale network.

With the trade-off between data rate and distance, in a system with a given density of DEVs, a smaller piconet covers a smaller number of DEVs with a higher average data rate within the piconet. This is in contradiction with the fact that a larger piconet covering more DEVs tends to have a higher traffic demand and larger bandwidth requirements. On the other hand, for a scatternet covering a large area, a smaller piconet size leads to more inter-piconet forwarding over a larger number of piconets, which in turn lowers the effective data rate in the overall system. From a system perspective, it is desired to maximise the average stream transmission cost, that is, to maximise the effective mean connection data rate, between randomly chosen DEV pairs in the scatternet. Thus, the link data rate, the piconet traffic load and the piconet coverage area need to be considered systematically with channel reuse considerations.

### 3 Expected piconet link rate

Data rate adaptation in the multirate PHY employed by 802.15.3 WPANs in the presence of propagation impairments and interference is a complex problem that is outside of the scope of the present paper. Assuming all data frames are sent with the maximum allowed power, the data rate generally decreases with increasing distance between communicating DEVs if we assume that the same impairments due to propagation and interference affect all the links. Therefore, in this paper we optimistically employ the following simplified model, which ignores the effects of propagation impairments and interference, to facilitate theoretical analysis at the MAC layer. We model the payload data rate simply as a discrete function of the distance \( d \) between the transmitter and receiver.

\[
\text{Rate}(d) = S_i; R_{i+1} < d \leq R_i, 1 \leq i \leq k \quad (1)
\]

where \( R_{i+1} = 0 \) and \( R_i \) is the transmission range of data rate \( S_i \) that meets a given FER objective. Thus, \( R_i \) corresponds to the maximum transmission range with the base (lowest) rate \( S_i \) at maximum power and \( S_k \) is the maximum data rate. Based on (1), each piconet is abstracted as a circle with the PNC at the centre. In practice, propagation effects and interference result in a reduction of the transmission range, and (1) should be modified by reducing the values of \( R_i \) accordingly so that the effects of fading, shadowing and interference could be taken into account.

Without considering any child and/or neighbour piconet, we assume that:

- A piconet consists of \( N \) DEVs in which one DEV acts as the PNC and all other DEVs are uniformly distributed in the coverage area of the piconet.
3.1 Type A: connection between a DEV and the PNC

Since \( r \leq R \), the maximum coverage range, all DEVs can exchange data directly with the PNC. A DEV can communicate with the PNC at rate \( S_i \) if the DEV is within the circle of radius \( R \) centred at the PNC. With uniformly distributed DEVs in the piconet, the probability distribution of the data rates for type A connections is given as follows.

\[
P_A(S_i) = \begin{cases} 
0 & ; r \leq R_{i+1} \\
1 - \left( \frac{R_{i+1}^2}{r^2} \right) & ; R_{i+1} < r \leq R_i \\
\left( \frac{R_i^2 - R_{i+1}^2}{r^2} \right) & ; R_{i+1} < R_i < r 
\end{cases}
\]

(2)

In 802.15.3 MAC, each data stream is sent in a CTA requested from the PNC. Multiple connections utilise separate CTAs in a TDMA manner without interference between them. Suppose there are \( k \) connections, each sending the same amount of data \( L \) in its allocated CTA. Thus, for connection \( j \) with rate \( S_j \), the CTA length is \( L/S_j \), and the average CTA length for these connections is the sum of all these CTAs divided by the number of connections. Therefore, the expected CTA length for a type A connection is given by:

\[
E_A[T] = \sum_{i=1}^{k} \frac{P_A(S_i)}{S_i} \frac{L}{S_i}
\]

(3)

And the expected data rate for a type A connection is:

\[
E_A[S] = \frac{L}{E_A[T]} \frac{1}{\sum_{i=1}^{k} P_A(S_i)(1/S_i)}
\]

(4)

3.2 Type B: intra-piconet connections between DEVs other than the PNC

As peer-to-peer communications are supported in an 802.15.3 piconet, direct connections shall be used for intra-piconet streams whenever possible. However, if a pair of DEVs are out of each other’s radio range, the stream between them has to be forwarded by some other DEV. In the following analysis, the PNC is used to forward frames between DEVs with no direct connection as proposed in Yin and Leung (2006c). We employ the Common Overlap Area (COLA) function \( \text{COLA}(R, r, x) \) defined in Yin and Leung (2006c) to obtain the intersection area between two circles of radii \( R \) and \( r \), where \( x \) is the distance between the centres of the two circles. For a piconet with a coverage radius of \( r \), the probability of each data rate can be derived as follows.

The probability that the source DEV is at distance \( x \) to \( x + \Delta x \) from the PNC is:

\[
P(x) = \frac{2x}{r^2} \Delta x
\]

(5)

Given the source DEV at distance \( x \) from the PNC, the probability of a data stream can be sent at rate \( S_i \) is simply the probability that the destination DEV is in the same piconet and within the circular rings of \( R \) and \( R_{i+1} \) centred around the source DEV. Thus,

\[
P(S_i | x) = \frac{\text{COLA}(r, R, x) - \text{COLA}(r, R_{i+1}, x)}{\pi r^2}; 1 \leq i \leq k
\]

(6)

\[
P(\text{no direct link} | x) = 1 - \sum_{i=1}^{k} P(S_i | x); 1 \leq i \leq k
\]

(7)

Therefore, the probability of \( S_i \) for a type B intra-piconet connections is:

\[
P_b(S_i) = \int_{0}^{\Delta x} P(S_i | x) P(x)dx; 1 \leq i \leq k
\]

(8)

\[
P_b(\text{no direct link}) = 1 - \sum_{i=1}^{k} P(S_i)
\]

(9)

For connections with no direct link, the PNC is used to forward the frames. With the source DEV at distance \( x \) from the PNC, if the source and destination DEVs are out of range, the destination DEV should be in the same piconet but outside the coverage region of the source DEV, within an area given by \( \pi r^2 - \text{COLA}(R, r, x) \). As to the data rates, the source to PNC link rate is given by (1), and the forwarding link rate depends on the destination position. Thus, the conditional probability that the forwarding link has rate \( S_i \) is given by (10).

Combining the results of (1) and (10), the conditional probability that an unreachable DEV pair is linked by the PNC with rate \( S_i \) from the source to the PNC and rate \( S_j \) from the PNC to the destination is given by (11).
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\[ P(i, x) = \begin{cases} 0; & \text{if } r \leq R_{i+1}, 1 \leq i < k \\ \pi R_i^2 - \text{COLA} \left( R_i, R_{i+1}, x \right); & \text{if } R_{i+1} < r \leq R_i, 1 \leq i < k \\ \pi R_k^2 - \text{COLA} \left( R_k, R_{i+1}, x \right); & \text{if } i = k \end{cases} \]

\[ Fw_{\text{Prob}}(i, j) = \begin{cases} \int_{\max(r, x)}^{\min(R, x)} \pi R_i^2 \nu P(n) \text{(no direct link)} \text{P}(x) \text{dx}; & r > R_{i+1} \\ 0; & r \leq R_{i+1} \end{cases} \]

Thus, the combined probability distribution of PNC-forwarded connections can be represented by a lower triangular matrix with elements:

\[ Fw_{\text{Distribution}}_{ij} = \begin{cases} Fw_{\text{Prob}}(i, j) + Fw_{\text{Prob}}(j, i); & i \geq j \\ 0; & i < j \end{cases} \]

where \( Fw_{\text{Distribution}}_{ij} \) denotes the conditional probability that given no direct link, the connection consists of two links with rates \( S_i \) and \( S_j \) respectively. Thus, the equivalent data rate for type \( B \) intra-piconet connections can be estimated by:

\[ E[S] = \left[ \sum_{i=1}^{k} \frac{1}{S_i} + \frac{1}{S_j} \right]^{-1} \]

(16)

In a piconet consisting of \( N \) DEVs uniformly distributed over the area of the piconet, the total number of DEV pairs is \( N(N-1)/2 \), of which \( (N-1) \) pairs involve the PNC and have type A connections between them. Therefore, the probability that a connection between a randomly selected pair of DEVs is a type A connection is simply \( 2/N \), and the overall piconet data rate distribution and the EPLR are given by:

\[ P(S_i) = \frac{2P_a(S_i) + (N-2)P_b(S_i)}{N}; \quad 1 \leq i \leq k \]

(14)

\[ P(\text{no direct link}) = 1 - \sum_{i=1}^{k} P(S_i) \]

(15)

Note that (13) and (16) do not account for the low data rate for headers and extra cost of frame forwarding, such as headers, Inter Frame Spaces (IFS) and acknowledgments. Thus in practice the EPLR values will be slightly lower than the results from the derivation. In Yin and Leung (2006a), this issue is further investigated with the effective CTA link rates under various application traffic parameters.

Compared with other wireless networks with centralised control, such as infrastructure mode WLAN and Bluetooth, where all intra-cell or intra-piconet connections have to be relayed by the central node, the combination of direct peer-to-peer communications and centralised control in 802.15.3 WPAN provides great flexibility, efficiency and performance gain in terms of throughput for intra-piconet connections.

4 Scatternet formation considerations

The 802.15.3 MAC allows peer-to-peer communications among DEVs, and up to 236 active DEVs can be associated with a PNC in an 802.15.3 piconet, compared with only 7 active slaves in a Bluetooth piconet (IEEE Std 802.15.1, 2002; IEEE Std 802.15.3, 2003). Thus, the scatternet formation problem for 802.15.3 WPANs is fundamentally different from that for Bluetooth. In a dense 802.15.3 scatternet with a large number of DEVs, the scatternet formation problem has two parts:

1. how to optimise coverage areas of the piconets with respect to certain performance criteria, such as the average number of piconets along the route of a connection or the average cost of a connection.
2. how to connect the piconets with the given size into the scatternet.

In this paper, we aim to solve problem (1); particularly, we consider optimizing the piconet coverage during scatternet formation to minimize the transmission cost. Assume that:

- The scatternet covers a two dimensional region with an area of \( A \), where \( A \) is much larger than the piconet size, with each piconet represented by a circle with radius \( r \).
- The piconet coverage coefficient \( c \) is defined to represent the ratio of effective piconet area considering overlaps between adjacent piconets. Thus, \( c \) is always smaller than 1; for example, with regular hexagonal topology, \( c = 1.5\sqrt{3}/\pi \approx 0.827 \).
- The number of piconets \( n \) in the scatternet is approximately \( n = A/(c\pi r^2) \).
- The scatternet is fully connected; and all links among DEVs within the same piconet has data rate \( \lambda_d \), where \( \lambda_d = E[S] \) given by (16) with piconet radius \( r \).

In wireless transmissions, the interference distance \( (R_i) \) is always larger than the data transmission range \( R \) due to the difference between data reception and interference thresholds. Thus, \( R_i \) can be much larger than the piconet radius \( r \) that is determined by the beacon power. Since each piconet occupies a logical channel, no other piconet operating simultaneously in its interference region should use the same channel except for child and/or neighbour piconets. Because a child and/or neighbour piconet uses a private CTA reserved from the parent piconet, the use of child and/or neighbour piconets does not increase the piconet capacity. Define the cluster size \( M \) as the required number of different logical channels to assign to the piconets in the interference region without channel reuse. Clearly, the minimum value \( M_{\text{min}} \) is obtained when all piconets are operating with the maximum coverage, that is, \( r = R \). Let \( \rho \) be the cluster factor, given by \( M/M_{\text{min}} \). A larger \( \rho \) means more channels are required to provide channel reuse without degrading channel quality. Thus,

\[
M = \frac{1}{c} \left( \frac{R_i}{r} \right)^2; \quad M_{\text{min}} = \frac{1}{c} \left( \frac{R_i}{R} \right)^2; \quad \rho = \left( \frac{R_i}{r} \right)^2.
\]

(17)

In 802.15.3, TDMA is used to assign CTAs for admitted data streams. Therefore, when a stream connection is established, the PNC of each piconet along its route allocates a guaranteed CTA time slot for the stream. Thus, the total cost of the stream transmission is simply the sum of all these CTAs. Obviously, if each hop has a rate of \( \lambda \), the throughput of the connection is still \( \lambda \) regardless of the number of hops. To evaluate the effective data rate at the system level, a packet forwarded along a multihop route is counted only once although it consumes resources over multiple piconets. Define the Effective Scatternet Connection Rate (ESCR) as the expected effective rate of a connection from a source DEV to a randomly chosen destination DEV in the scatternet, passing through an average of \( L \) piconets. Thus, \( \text{ESCR} \approx \lambda/L \). In a two dimensional scatternet consisting of \( n \) piconets laid out in a regular pattern, \( L \sim O(\sqrt{n}) \). Thus, the upper limit of ESCR with piconet radius of \( r \) is given by:

\[
\text{ESCR}_{\text{max}}(r) = \frac{\lambda_d}{L} = O\left( \frac{\lambda_d}{\sqrt{n}} \right) = k_1 \frac{\lambda_d}{\sqrt{n}} = O\left( \frac{\lambda_d r}{\sqrt{A}} \right) = k_2 \frac{\lambda_d r}{\sqrt{A}} \quad (18)
\]

where \( k_1 \) and \( k_2 \) are constants that depend on the pattern of piconets in the scatternet configuration. Once the piconet radius \( r \) is decided, it is desirable to cover the scatternet with a minimum number of piconets, which is achieved, in a given planar area, by a regular hexagonal configuration as shown in Figure 2.

Figure 2 Scatternet with minimum number of piconets arranged in regular hexagonal configuration

Therefore, in a dense network, the regular hexagonal configuration gives the upper limit of ESCR. Assume a scatternet coverage area with length of \( X \) and width of \( Y \). With the hexagonal configuration in Figure 2, the centres of the piconets are separated by \( dx = 1.5r \) and \( dy = \sqrt{3}r \) along the \( x \) and \( y \) axes, respectively. Let \( n_x \) and \( n_y \) be the number of piconets along the \( x \) and \( y \) axis, respectively, thus,

\[
n_x \approx \frac{X}{1.5r}; \quad n_y \approx \frac{Y}{\sqrt{3}r}; \quad n = n_x n_y \approx \frac{XY}{1.5\sqrt{3}r^2} = \frac{1}{1.5\sqrt{3}} A
\]

(19)

Assuming the scatternet is constructed by the regular hexagonal configuration with DEVs uniformly distributed in it, and data streams are generated between random DEV pairs. If we consider only one dimension, let \( X_i \) and \( X_j \) be the position of source and destination DEVs that are uniformly distributed in the dimension with length of \( X \). Thus, \( X_i \) and \( X_j \) are random variables with uniform distribution, and \( (X_i - X_j) \) follows a uniform difference distribution. And the distance between the two DEVs in this dimension, represented by \( \Delta X = |X_i - X_j| \), has a probability density function of

\[
P_{[\cdot, x]} \begin{cases} \frac{2}{X} \left( 1 - \frac{x}{X} \right) & 0 \leq x \leq X \\ 0 & \text{otherwise} \end{cases}
\]

(20)
Thus, the mean of $|X_i - X_j|$ is given by

$$E[\Delta X] = E[|X_i - X_j|] = \int_0^X x p_{x,x} dx = \frac{X}{3} \tag{21}$$

Similarly, the mean of $\Delta Y = |Y_i - Y_j|$ for the other dimension is simply $Y/3$. Let $\Delta n_i$ and $\Delta n_i$ be the number of piconents along the $x$ and $y$ axes, respectively. Thus,

$$\Delta n_k \approx \frac{\Delta X}{1.5r} = \frac{X}{4.5r}; \quad \Delta n_y \approx \frac{\Delta Y}{\sqrt{3}r} = \frac{Y}{3\sqrt{3}r} \tag{22}$$

As shown in Figure 2, inter-picponent hops can occur diagonally in the hexagonal topology. Thus, the expected number of hops for two randomly chosen DEV is given by:

$$L = \begin{cases} \max(\Delta n_x, \Delta n_y), & \text{if } \Delta n_x \leq \Delta n_y / 2 \\ \min(\Delta n_x, \Delta n_y) + \frac{\max(\Delta n_x, \Delta n_y)}{2}, & \text{otherwise} \end{cases} \tag{23}$$

Particularly, for $X = Y = \sqrt{A}$,

$$L = \frac{3 + \sqrt{3} \sqrt{A}}{9\sqrt{3} r} \tag{24}$$

Substituting $L$ into (18) and (19), one can find:

$$k_3' = \frac{9\sqrt{3}}{3 + \sqrt{3}} \approx 3.294; \quad k_1 = \frac{k_3'}{\sqrt{1.5\sqrt{3}}} \approx 2.044 \tag{25}$$

Thus approximately, given the piconet radius $r$, the upper limit of ESCR is given by:

$$ESCR\max(r) \approx 2.044 \frac{\lambda_r}{\sqrt{n}} + 3.294 \frac{\lambda_r}{\sqrt{A}} \tag{26}$$

The result in (18) is the same as that shown in Gupta and Kumar (2000). In fact, the problem is similar to that considered in Gupta and Kumar (2000) if a piconet is abstracted as a node, and the scatternet is modelled as a network with $n$ identical nodes, each capable of transmitting at a rate of $\lambda$. As shown in Gupta and Kumar (2000), (18) is the upper bound for the optimal network configuration; for randomly located nodes, the throughput, that is, the practical ESCR, will be smaller than the $ESCR\max$, and is given by:

$$ESCR(r) = O\left(\frac{\lambda}{\sqrt{n \log n}}\right) = k_1 \frac{\lambda_r}{\sqrt{n \log n}} \tag{27}$$

$$= O\left(\frac{\lambda_r}{\sqrt{A \log n}}\right) = k_1 \frac{\lambda_r}{\sqrt{A \log n}}$$

where $k_1$ and $k_3$ are constants that depends on the scatternet size and the randomness of the configuration pattern. Equations (18) and (27) show that both $ESCR\max$ and ESCR decrease with the scatternet coverage area $A$ according to $\sqrt{A}$ in the denominator. Although $\lambda$ decreases with the piconet radius $r$, $ESCR\max$ and ESCR, as functions of $\lambda / r$, are not monotonous. With a given scatternet area $A$, denote $\lambda_x$ as the EPLR when the piconet operates at the maximum transmission range $R$. To show the effects of piconet size on connection rate performance, using the $ESCR\max$ at maximum piconet radius $R$ as the reference, the ESCR Normalised Index (NI) is defined as:

$$NI\max(r) = \frac{ESCR\max(r)}{ESCR\max(R)} = \frac{\lambda_r}{\lambda_n R} \tag{28}$$

$$NI(r) = \frac{ESCR(r)}{ESCR\max(R)} = k_2 \frac{\lambda_r}{\lambda_n R \log n} = k_2 \frac{NI\max(r)}{\log n} \tag{29}$$

In (29), $k_2$ is also a constant that depends on the scatternet size and topology. However, it is difficult to find actual values for $k_1$ to $k_2$, as they vary with the scatternet configuration. However, the values of $k_1$ to $k_2$ are not critical as the main objective of NI is to show the relative changes of ESCR versus piconet radius $r$. With $ESCR\max(R)$ normalised to 1, a larger NI$(r)$ means a relatively higher ESCR compared to a scatternet with the minimum number of piconents. Thus, $NI\max(r)$ represents the upper limit of ESCR relative to $ESCR\max(R)$. As a function of $\lambda / R$, $NI\max(r)$ is independent of the scatternet area. In practice, $NI(r) < NI\max(r)$ and decreases with the scatternet area $A$ due to a denominator of $\sqrt{\log n}$. As will be seen in the results in the next section, $\lambda_x < \lambda_r$ for $r < R$; therefore, piconents with the maximum coverage will not necessarily give the best ESCR. On the other hand, with a reduced piconet size, the cluster factor $\rho$ is increased, thus more distinct logical channels are required for channel reuse without cochannel interference.

## 5 Simulation and numerical results

### 5.1 Simulation method

The PHY used in the simulation is based on the MB-OFDM UWB specification (Batra et al., 2004), which has now been adopted as an ECMA standard (ECMA-368 Std, 2005). It is also considered in wireless USB and high-speed Bluetooth. Basically, MB-OFDM divides the 3.1 to 10.6 GHz spectrum into 14 528 MHz bands grouped into 5 band groups. Within each band group, each Time Frequency Code (TFC) corresponds to a logical channel, which can be used to form a piconet. Thus it supports up to 4 piconents within interference range if working in Mode 1, where only the first band group is used; and up to 16 piconents within interference range if all band groups are used.

MB-OFDM supports multiple data rates from 53.3 to 480 Mbps. In the simulations, only a subset of the supported data rates is used. The range at which the MB-OFDM system, operating in Mode 1, can achieve a Packet Error Rate (PER) of 8% with a link success probability of 90% for Channel Model 1 (CM1) together with some rate dependent parameters are listed in Table 1. The ranges for 110–480 Mbps are from the proposed specification (Batra et al., 2004) and the range for 53.3 Mbps is estimated by multiply $\sqrt{2}$ over the range for 110 Mbps, which is approximately 17 m.
Table 1 Data rates and range parameters for model1 DEVs

<table>
<thead>
<tr>
<th>Data rate (Mbps)</th>
<th>Code rate (R)</th>
<th>Time spreading gain (T SF)</th>
<th>OFDM Coded OFDM bits/symbol (N_{code})</th>
<th>Range to achieve 8% PER with 90% link success rate in CMI (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>53.3</td>
<td>1/3</td>
<td>2</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>110</td>
<td>11/32</td>
<td>2</td>
<td>200</td>
<td>12.0</td>
</tr>
<tr>
<td>200</td>
<td>5/8</td>
<td>2</td>
<td>200</td>
<td>7.4</td>
</tr>
<tr>
<td>480</td>
<td>3/4</td>
<td>1</td>
<td>200</td>
<td>3.2</td>
</tr>
</tbody>
</table>

For simplicity, in the simulations, all links within the transmission range of each specific data rate are considered error free, while all out-of-range transmissions fail, and all data transmissions employ Immediate Acknowledgments (Imm-ACKs), that is, an Imm-ACK is sent for each successful data or command frame transmission. Since the transmission range is obtained to satisfy a FER threshold, the error free assumption is valid as long as the FER threshold presents an acceptable QoS to the application. The corresponding MB-OFDM and MAC parameters used in the simulation are summarised in Table 2.

Table 2 MB-OFDM and 802.15.3 MAC simulation parameters

<table>
<thead>
<tr>
<th>DEV mode and channel model</th>
<th>OFDM symbol ( T_{sym} )</th>
<th>T_HDR(PLCP preamble + header)</th>
<th>SIFS ( T_{SIFS} )</th>
<th>No. of DEVs in the piconet (N)</th>
<th>Piconet coverage</th>
<th>Payload size (LENGTH + FCS)</th>
<th>Payload transmission time ( T_{payload} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1, CM1</td>
<td>0.3125 ( \mu s )</td>
<td>13.125 ( \mu s )</td>
<td>10 ( \mu s )</td>
<td>10</td>
<td>1–17 m</td>
<td>1024 and 4096 bytes</td>
<td>( T_{sym} \times \text{Ceiling} (\text{Ceiling}(1/R) \times (8 \times \text{payload} + 6) \times \text{TSF})/N_{bps} )</td>
</tr>
</tbody>
</table>

The simulator to investigate EPLR in a piconet is written in c++. In each simulation, a PNC DEV is generated first, and the piconet size is set with radius \( r \). We assume the piconet size is decided upon scatternet formation, with no dynamic piconet size adjustment later on. Then, \( N-1 \) DEVs are generated and placed in the piconet coverage area with a uniform distribution. Connections are generated between randomly chosen DEV pairs. Traffic streams over all connections have fixed data frame payloads. The simulator first computes the distance between the two chosen DEVs and set the link rate accordingly. If the link distance is greater than the range of 53.3 Mbps, the connection employs PNC forwarding. With the given data stream parameters, the CTA length for each link is calculated in consideration of the applicable IFSs and ACKs as specified in IEEE Std 802.15.3 (2003). The simulations find the data rate distributions and then compute the equivalent data rates taking into account of PNC forwarding. The presented results are averaged from one million randomly generated piconets.

A separate simulator is written in c++ to investigate the ESCR. As the main focus of the paper is to find the optimal piconet size, we assume once a connection is established, each piconet along the path allocates a CTA for it. In the simulations, the DEV density is set as 0.075 DEVs/m², as a higher density makes the scatternet more likely to be fully connected. The scatternet is formed by a simple but most practical stochastic algorithm studied in Yin and Leung (2006b). It randomly chooses an unassociated DEV to act as a PNC. After the new PNC is generated, all DEVs within its coverage area are associated with it. This ensures no PNCs are directly reachable to each other. The process continues until no unassociated DEV is left, that is, all DEVs in the scatternet area belong to at least one piconet. A bridge DEV between adjacent piconets is chosen from the DEVs that are reachable to both PNCs.

After the scatternet is formed, a separate graph is created, in which a piconet is abstracted as a node, and two nodes are connected if there is a bridge DEV. Then, if the generated graph is connected, shortest paths are computed between randomly selected node pairs to yield the minimum number of piconets between any two randomly chosen DEVs not within the same piconet. The average of the results is used as the average number of piconet hops \( L \) for randomly chosen DEV pairs. The simulation result of \( L \) is calculated by the average values from 1000 randomly generated fully connected scatternet configurations. The ESCR is then calculated as \( \lambda/L \). To compare with the theoretical results and as a reference for NI, an abstract regular polygonal topology is created to get \( ESCR_{\text{avg}} \) for the same scatternet area. We evaluate the optimal piconet coverage based on this simple scatternet formation method and MB-OFDM; however, the insight obtained can be applied to any other scatternet formation methods and PHY layers.

5.2 Simulation results

Figure 3 presents the rate distribution versus piconet radius with 10 randomly located DEVs in the piconet. The theoretical and simulation results are in close agreement. It shows that the fraction of lower data rate connections increases with the piconet size. Although, the PNC can always communicate with any DEV within the piconet, some DEV pairs are out of range when the piconet radius is greater than half of the maximum transmission range; for example, if a piconet with 10 DEVs operates with the maximum coverage, more than 41% of type B intra-piconet links (Yin and Leung, 2006c), and overall 33% of all intra-piconet links, are unreachable with direct peer-to-peer connections, and less than 16% can connect with 200 Mbps or more (Figure 3). The use of PNC forwarding ensures that a DEV can communicate with any other DEV in the piconet. However, since the forwarded frame does not increase the overall rate but uses one additional CTA, connections assisted by PNC forwarding normally have lower effective rates than that of direct peer-to-peer connections.
When the piconet radius is increased, the fraction of lower rate links increases, and the EPLR declines gradually, as shown in Figures 4 and 5.

Figure 4 provides the equivalent data rates for different types of connections in a piconet. Type A connections achieve higher data rates than type B connections since the latter has some unreachable pairs and more long links. It also shows that the equivalent rate for type B connections is 35.8% to 100% higher than that of a network without peer-to-peer connection support, where all packets are forwarded by the central controller, such as a Bluetooth piconet or an infrastructure WLAN. Therefore, the support of peer-to-peer communications in 802.15.3 piconets provides a much better performance in terms of system throughput. The overall piconet equivalent data rate, that is, EPLR, becomes very close to the (type B) intra-piconet data rate when the number of DEVs is more than a few, due to the small fraction of type A connections.

The frame overheads and IFSs are taken into consideration in the simulations, and the resulting net throughput values for payload transmissions are converted back to the equivalent PHY rate by calculations to facilitate comparisons with the theoretical calculations. For example, suppose for a PNC forwarded connection,

- $S_i$ and $S_j$ are the link rates of the two forwarding links.
- $P$ is the total payload bits sent in a CTA.
- $TO$ as the total frame transmission overhead in a CTA, which includes the PHY/MAC headers of all transmitted frames and the applicable ACKs and IFSs.

Thus, respectively, the CTA lengths of the two links in the forwarding connection are:

$$CTA_i = TO + \frac{P}{S_i}; \quad CTA_j = TO + \frac{P}{S_j}$$

The equivalent PHY rate ($S_E$) for the forwarded connection in the simulation is then computed as follows:

$$S_E = \frac{P}{(CTA_i + CTA_j) - TO} = \frac{P}{P/S_i + P/S_j + TO}$$

Figure 5 compares the analytical and the simulation results for EPLR with 10 uniformly distributed DEVs in the piconet. The EPLR decreases gradually at a decreasing rate as the piconet becomes bigger. A very high data rate, up to 480 Mbps, can be achieved for a very small piconet. The EPLR is 55.5 Mbps at the maximum possible piconet radius of 17 m, 108.9 Mbps at a 10 m radius, and 131.6 Mbps if the piconet operates at half of the maximum radius such that all DEVs can communicate in a peer-to-peer manner. It also shows that the theoretical results from (16) are slightly higher than the simulation results at a large piconet radius. The differences come from the lower data rate for headers. With a larger payload of 4 kilobytes, the simulation results become very close to the analytical results.

With the trade-off between the piconet size and throughput, a smaller piconet has a higher EPLR, but more piconets are required to form a scatternet that covers a given area, thus requiring more inter-piconet forwarding for randomly chosen DEV pairs in an extended scatternet.
chosen $k_5$ value here does not give an accurate estimation of NI values, but it clearly shows the trends of NI against the scatternet area and the piconet size.

While $\text{NI}_{\text{max}}$ is independent of the scatternet area, Figure 6 shows that NI decreases with increased scatternet area and is much lower than $\text{NI}_{\text{max}}$. The analytical results for $\text{NI}_{\text{max}}$ with simulated EPLR are slightly lower than the theoretical analysis results due to lower $r$ values. In all cases, the curves have two vertexes at piconet radii of approximately 6.5 m and 12 m. Since the PHY only provides a discrete set of data rates, a distance slightly longer than the range threshold of one rate results in a significant drop of the data rate to the next lower level. Therefore, these vertexes are located approximately at transmission ranges of the intermediate rates. The corresponding cluster factors $\rho$, the numbers of distinct logical channels required to avoid inter-piconet interference, at the two vertexes are 6.84 and 2, respectively. As 802.15.3 has only a limited number of logical channels available for channel reuse, the first vertex may result in excessive interference between DEVs in different piconets. The second vertex corresponding to a 12 m piconet radius is much better able to avoid cochannel interference due to the smaller cluster size constraint, and should therefore be chosen for best ESCR performance.

To validate the analysis, simulations are performed to determine $L$ only for piconet radii of 7 to 17 m since very small radii are not likely to be used in practice for scatternet formation. All results in Figure 7 are based on ESCR computed from values of $L$ obtained by simulations. Simulation results based on the regular hexagonal scatternet configuration verify that $\text{NI}_{\text{max}}$ shows little variations between different scatternet sizes. The variations are caused by rounding the number of piconets in each dimension to an integer for the regular hexagonal configuration. Figure 7 also shows that, NI for stochastic scatternet formation shows similar trends as $\text{NI}_{\text{max}}$ for regular scatternet configuration, when the piconet radius is higher than 12 m. Although $\text{NI} < \text{NI}_{\text{max}}$, the drop in NI due to the simple stochastic formation algorithm is less than 20%, compared to the $\text{NI}_{\text{max}}$ for regular scatternet configuration. Compared to the analysis results for NI in Figure 6, the corresponding simulation results for stochastic scatternet formation in Figure 7 shows a similar trend but with a more obvious vertex at a 12 m piconet radius and a faster rate of decrease as piconets get smaller. The latter is caused by a larger amount of overlaps between randomly placed piconets and hence a smaller value of piconet coverage coefficient $c$. The variation in $c$ in turn contributes to the deviations in the values of $k_5$ from simulations: 2.0–2.2 and 1.8–2.0 for stochastic scatternet formation over areas of 200 × 200 m$^2$ and 100 × 100 m$^2$, respectively.

6 Practical considerations

The above analyses and simulations are based on a simplified abstract model with circular piconet coverage areas. This approach is commonly used in the literature (Li et al., 2001; Santi et al., 2001). In practice, however, radio links are subject to changing path loss, shadowing, fading, interference, etc., due to obstacles and non-ideal antenna patterns; therefore in reality a piconet is not perfectly circular, and the data rate may not be a direct function of the link distance. Thus, a more accurate modelling of physical layer is important in network-level research on wireless networks (Bettstetter and Hartmann, 2005; Zorzi and Pupolin, 1995). When propagation effects are taken into consideration, a DEV’s transmission range at each data rate while satisfying the FER threshold, which is assumed to be a fixed parameter in the simplified model, becomes a stochastic parameter (Bettstetter and Hartmann, 2005), and the link distance is no longer sufficient to determine the maximum achievable data rate.

As TDMA is used for CTAs, there is only one active connection in a piconet at any time. However, with multiple simultaneously operating piconets in a scatternet, adjacent channel interference between piconets could further degrade link performance. For example, the logical channels in DS-UWB and MB-OFDM are based on code division and TFC, respectively, and neither of them is perfectly orthogonal. Therefore, adjacent piconets should
employ some cooperative mechanisms to prevent simultaneous transmissions by DEVs in close proximity, similar to the mechanisms used in distributed MAC of ECMA-368 (ECMA-368 Std, 2005). Development of such a mechanism is beyond the scope of this paper and left for future research.

Although distances are used in our simplified model for performance evaluations purposes, in practice the scatternet formation process does not rely on distance information. DEVs estimate the link condition and select the data rate by assessing the received signal strength. Given the FER requirement, the maximum transmission range as well as the transmission distance $R_i$ for data rate $S_i$ are reduced when channel impairments and interference are taken into account. However, the trade-off between piconet size and connection data rate revealed in this paper still exists, and our analyses are still applicable if the transmission ranges $R_i$ are reduced accordingly. Our observation that an intermediate piconet size gives the best scatternet connection rate performance remains valid. Therefore, when a DEV is elected to be the PNC, it should decrease the transmission power to the appropriate level when sending beacons, so that only the DEVs within an intermediate distance from it can correctly receive its beacons and associate with the piconet. In our performance evaluations, we have determined that this distance is given by the transmission range of the second lowest data rate, that is, 110 Mbps.

7 Conclusion and future work

The IEEE 802.15.3 WPAN is designed to support multimedia communications with high data rates and peer-to-peer connections. Thus, the EPLR and ESCR are very important performance criteria. In this paper, we have analysed the data rate distribution of 802.15.3 piconets, and estimated the EPLR in a piconet. The ESCR has been defined to represent the connection cost between a randomly chosen pair of DEVs within the scatternet. The ESCR performance has been analysed with a NI to show the impact of piconet size, and the upper limit of ESCR in a scatternet has been derived using a regular hexagonal configuration. We have presented simulation results based on the MB-OFDM UWB PHY, which show that although the EPLR decreases as the piconet radius increases, ESCR can be optimised with respect to the piconet size. Results show that an intermediate piconet radius of 12 m given by the transmission range of the second lowest data rate achieves the best ESCR in a scatternet using the MB-OFDM UWB PHY. Thus, scatternet formation requires joint consideration of piconet size, channel reuse and connection rate.

With the expected adoption of UWB in the Bluetooth PHY, some features of 802.15.3 may be incorporated in the next Bluetooth specifications as well, for example, a higher number of active nodes and peer-to-peer communications within a piconet. Although the analyses in this paper are based on the IEEE 802.15.3 standards, the results give useful insights to the design of HDR-WPANs employing multirate carriers, including WiMedia networks supported by ECMA-368 (ECMA-368 Std, 2005). With a distributed MAC approach in ECMA-368, each DEV is virtually a PNC with all neighbour DEVs as members of its piconet, and the optimal connection rate becomes a critical consideration for efficient service delivery. Furthermore, the observations in this paper are particularly applicable to route selection in mobile ad hoc network and mesh networks. In many of these networks, including WiMedia and next generation Bluetooth, CSMA/CA is used for channel access and data transmission instead of TDMA in 802.15.3. Thus, the analyses presented in this paper can be applied by changing the corresponding link cost to include the expected backoff period and the collision probability. Work is in progress to extend our analyses to these networks. Furthermore, while this paper employs a simple model for link rate adaptation as a function of distance, in our future work we will incorporate realistic propagation models for HDR-WPANs in our analyses when such models become available.

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