

HARD SCATTERING IN QCD¹

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Abstract

Recent developments in the theory and experiment of QCD hard scattering are described.

1 Preamble: How Perturbative QCD Works

This has been an active year for experimental results and theoretical developments in quantum chromodynamics. In this talk I will cover a selection of topics presented at this conference, tied together by the interplay of soft and hard physics. QCD is certainly the most complex component of the standard model at current energy scales, evolving as it does from an asymptotically-free perturbative theory of quarks and gluons at short distances to a confining theory of hadrons at long distances. As experiment and theory become more precise, it becomes both possible and necessary to explore the relations between these two limits. I begin with a short review of how we employ the simplicity of the theory at short distances even in the presence of nonperturbative low-energy dynamics. Thus, let us recall a few recurring themes in QCD theory. [1]

Infrared Safety. Our first observation is that there is a class of high-energy cross sections and decay rates that may be calculated directly in perturbation theory. These “infrared safe”

¹Presented at the 28th International Conference on High Energy Physics, July 1996, Warsaw, Poland.

quantities are free of long-distance dependence at leading *power* in a large momentum scale Q , such as the total c.m. energy for e^+e^- annihilation (at lowest order in electroweak couplings). Such a quantity, $\hat{\sigma}$ may be written as a power series in the strong coupling evaluated at scale μ ,

$$Q^2 \hat{\sigma}(Q^2, \mu^2, \alpha_s(\mu^2)) = \sum_n c_n(Q^2/\mu^2) \alpha_s^n(\mu^2), \quad (1)$$

where the c_n are infrared finite coefficients, free of dependence on long-distance parameters. Corrections due to dimensional parameters such as quark masses and vacuum condensates are suppressed by powers of Q . Other examples are jet cross sections in e^+e^- annihilation and the total width of the Z boson.

Factorization. The class of infrared-safe cross sections is a small subset of all high-energy cross sections. In particular, hard-scattering cross sections that involve one or more hadron in the initial state cannot be infrared safe. This is because they depend on the distributions of quark and gluon degrees of freedom in the hadrons, which reflect the nonperturbative, long-time processes that bind the hadrons. Nevertheless, it is often possible to factorize such a cross section into a convolution of an infrared-safe short-distance “hard part” $\hat{\sigma}$, calculable in perturbation theory and specific to the process, and a long-distance function ϕ_{NP} , which, although nonperturbative, is universal among different processes. Thus, for example, a deeply inelastic scattering (DIS) cross section may be written as

$$Q^2 \sigma_{\text{phys}}(Q, m) = \hat{\sigma}_{\text{PT}}(Q/\mu, \alpha_s(\mu)) \otimes \phi_{\text{NP}}(\mu, m), \quad (2)$$

where the subscript “phys” on the left emphasizes that this is the physically observed cross section. The parameter μ on the right separates short-distance dynamics, incorporated into $\hat{\sigma}_{\text{PT}}$, and long-distance dynamics, at the scale of parameters m , absorbed into ϕ_{NP} . For DIS, the convolution is in terms of the familiar fractional momentum ξ of the parton that initiates the scattering, and implicitly includes a sum over parton types. Factorizations of this sort are at the basis of much of perturbative QCD (pQCD). The same physical principles recur in other field-theoretic contexts, under different names, such as the identification of effective field theories.

Evolution. Perhaps the most powerful result in pQCD is evolution with momentum transfer. This is not a single theorem, but a wide-ranging method, which can be exploited whenever we succeed in factorizing dynamics at different length scales, as above. The essential observation is that the scale μ in Eq. 2 is arbitrary, subject only to $\alpha_s(\mu) \ll 1$ (so that $\hat{\sigma}$ may be truly perturbative). The *physical* cross section, however, must be independent of the choice of this scale of separation,

$$\mu \frac{d\sigma_{\text{phys}}}{d\mu} = 0. \quad (3)$$

The chain rule then leads to separate equations for the short- and long-distance components of the fields,

$$\begin{aligned} \mu \frac{d\phi_{\text{NP}}}{d\mu} &= P(\alpha_s(\mu)) \otimes \phi_{\text{NP}} + \mathcal{O}(1/Q^2) \\ \mu \frac{d\hat{\sigma}_{\text{PT}}}{d\mu} &= -P(\alpha_s(\mu)) \otimes \hat{\sigma}_{\text{PT}} + \mathcal{O}(1/Q^2), \end{aligned} \quad (4)$$

where the convolution is of the same form as in Eq. 2. Applied to QCD, these are the DGLAP evolution equations. [2] The “evolution kernel” P plays the role of a parameter in the separation

of variables between the factors $\hat{\sigma}$ and ϕ . It can depend on $\alpha_s(\mu)$ and momentum fractions only, because these are the only variables that $\hat{\sigma}$ and ϕ hold in common. Thus, the perturbative calculability of the familiar DGLAP evolution kernels is a direct result of the factorizability of cross sections.

2 From Low x to Reggeons via BFKL

With this perspective on the basic methods of QCD, we are ready to turn to one of the major topics of this conference, the new results on low- x structure functions. [3] This will lead us naturally into a discussion of the theory of “small- x ” and its relation to an old friend, the Regge limit.

2.1 The ZEUS, H1 1994 Data

At the colliding beams of HERA, lepton-proton deeply inelastic scattering has been brought to a new kinematic range, with an unprecedented sensitivity to soft partons. [4, 5] These results are often presented in terms of the dimensionless structure function F_2 , which for electromagnetic scattering (Fig. 1a) is related to the cross section at fixed momentum transfer $q^2 = -Q^2$ and variables $x = Q^2/2p \cdot q$ and $y = p \cdot q/p \cdot k$, with k and p the positron and proton momenta, respectively. To a good approximation we have, with $s = (p + k)^2$,

$$\frac{d\sigma}{dx dy} = \frac{2\pi\alpha^2(1 + (1 - y)^2)s}{Q^4} F_2(x, Q^2). \quad (5)$$

Data from the 1994 run of H1 and ZEUS cover the general range of $10^{-6} < x < 10^{-2}$ for momentum transfers $0.3 < Q^2 < 4$ GeV, over which pQCD “turns on”. The data for $10^{-4} < x$ extend up to much higher Q^2 . We recall that, as in Eq. 2, to lowest order in α_s , x has the interpretation of the fractional momentum of the struck parton. The range in Q^2 is limited at the smallest values of x by the kinematic restriction that the total c.m. energy must be larger than the invariant mass of the hadrons in the final state,

$$(p + k)^2 \geq W^2 = \frac{Q^2}{x}(1 - x). \quad (6)$$

The data [3, 4, 5] shows a distinct rise in F_2 toward $x = 0$, which becomes increasingly striking as Q^2 increases. In the range of $x \sim 10^{-6}$, $xG(x)$ increases to the order of 30. To put this value into perspective, we may recall a typical scaling parton distribution, consistent with early DIS experiments, $xG(x) = 3(1 - x)^5$.

Our current understanding of this rise is based on the evolution equations for nonsinglet (f_{ns}) and singlet ($f_q^{(s)} = \sum_f (q_f(x) + \bar{q}_f(x))$, $f_G^{(s)} = G(x)$) sectors of nucleon parton distributions,

$$\begin{aligned} \frac{dq_{\text{ns}}}{d \ln Q^2} &= P_{qq} \otimes q_{\text{ns}} \\ \frac{df_i^{(s)}}{d \ln Q^2} &= P_{ij} \otimes f_j^{(s)}, \end{aligned} \quad (7)$$

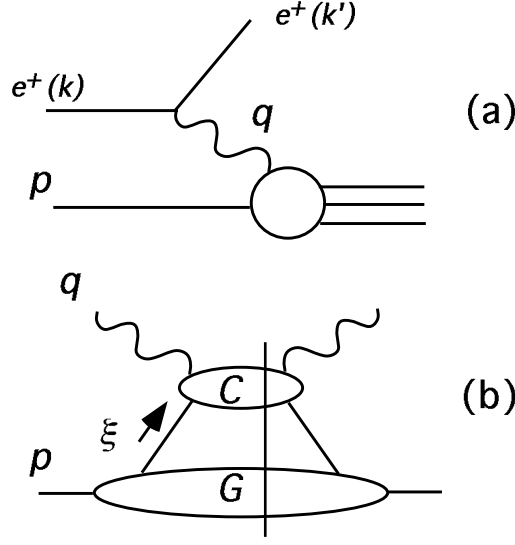


Figure 1: (a) Deeply inelastic scattering. (b) DGLAP factorization of a DIS structure function.

which are the practical versions of the general relations (4) above, for quarks and gluons. In the singlet (matrix) equation, the gluon-gluon kernel is singular as x vanishes,

$$P_{GG}(x) = \frac{\alpha_s}{\pi} C_A \frac{1}{x} + \dots, \quad (8)$$

where $C_A = N = 3$ for QCD. This singularity, the only one of its kind in eq. 7, is generally recognized as the origin of the rise in F_2 for small x . The rise is not unexpected, but its *detailed* origin is still debated. [6, 7] Most of us agree that it is the gluon density that is growing for small x , but there are at least two possibilities for the dynamics of its growth. They may be illustrated by a simplified evolution equation, involving only the gluonic distribution, and the singular term in P_{GG} from Eq. 8,

$$\begin{aligned} \frac{d xG(x, Q^2)}{d \ln Q^2} &= x \int_x^1 \frac{d\xi}{\xi} \left(\frac{N\alpha_s(Q^2)}{\pi(x/\xi)} \right) G(\xi, Q^2) \\ &= N \frac{\alpha_s(Q^2)}{\pi} \int_x^1 \frac{d\xi}{\xi} [\xi G(\xi, Q^2)]. \end{aligned} \quad (9)$$

We consider two possibilities for the gluon distribution at the value Q_0^2 at which we begin the evolution: either (i) $\xi G(\xi, Q_0^2) \sim 1$, or (ii) $\xi G(\xi, Q_0^2) \sim \xi^{-\lambda}$. In the first case the ξ integral in Eq. 9 is itself logarithmic in x , $\int_x (d\xi/\xi) \sim \ln x$, and the x -dependence of the distribution changes, and steepens, as Q^2 increases. To leading logarithm in x and Q , case (i) results in a well-known x -dependence that reflects the nature of the kernel,

$$xG(x, Q^2) \sim xG(x, Q_0^2) e^{4\sqrt{(N/b_2) \ln(1/x) \ln(t)}}, \quad (10)$$

where $t \equiv \ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)$, with Λ the scale of the QCD running coupling, and $b_2 = 11 - 2n_f/3$ the one-loop coefficient from the QCD beta function. Notice that this x -dependence,

although more singular than $1/x$, and increasingly steep with Q , remains still less singular than any power like $1/x^{1+\lambda}$.

To this “kernel-driven” x -dependence we may contrast case (ii), for which the ξ integral in the evolution equation leaves the x -dependence stable, since then

$$\frac{d xG(x, Q^2)}{d \ln Q^2} \sim x^{-\lambda}. \quad (11)$$

In summary, if the initial conditions for $\xi G(\xi, Q^2)$ are smooth as $\xi \rightarrow 1$, the perturbative kernel determines the asymptotic x -dependence, while if the initial conditions are singular, the asymptotic behavior is stable under evolution, and in this sense is fundamentally nonperturbative (but see below).

In DIS, which is it? In fact, the question may not be so well posed, since evolution is not fully determined by the $x \rightarrow 0$ behavior of the gluon distribution. In addition, the electromagnetic structure functions are not adequate to determine the large number of parton distributions, including valence and sea quarks. Therefore, more than one parameterization may, and does, fit the data. On the one hand, “global” fitting groups, MRS [8, 9] and CTEQ, [10] with inputs from a wide range of x and Q^2 in a variety of experiments, find that the increasing steepness of F_2 as Q^2 increases is correlated with a steepening gluon distribution, with an effective exponent $\lambda(Q^2)$ that obeys $d\lambda/d \ln Q^2 > 0$. This behavior is built into the GRV parton distributions, [11] starting with a negative value of λ for small Q^2 . The actual evolution of each of these gluon distributions is much more complex than simply case (i) or (ii). On the other hand Lopez *et al.* have described a fit [7] that realizes case (ii) in the sense that the leading power of x in the gluon distribution is Q^2 -independent. In their parameterization, F_2 softens at lower Q^2 because of a Q^2 -dependent prefactor for $x^{-\lambda}$. In any case, the ability of the factorization formalism to describe DIS down to $x \sim 10^{-6}$ – corresponding to a parton that carries one one-millionth of the proton’s momentum – is a testimony to the extraordinary range of the perturbative picture that generalizes the parton model.

In this context we may also observe that there is a perturbative evolution that appears to be kernel-driven. This is the MLLA (“modified leading logarithm approximation”) for partonic multiplicity distributions in jets [12]. Distributions for partons are identified with those for hadrons; a method known as “parton-hadron duality”. Such an identification is particularly natural for a process in which evolution is driven by a perturbative mechanism that may be followed from the shortest distances out to $1/\Lambda_{\text{QCD}}$ self-consistently, although the nature of corrections to parton-hadron duality predictions are not fully understood. MLLA evolution is quite different from DIS, however, because in the former QCD interference actually suppresses the emission of soft gluons. New data that compares well with the predictions of the MLLA has been presented by OPAL [13] at this conference.

By the same token, we may ask whether it is possible to find a singular initial condition like case (ii) above, which is perturbative. The answer is yes, at least at the level of leading logarithmic approximations, in the context of the BFKL treatment of DIS, to which we now turn.

2.2 BFKL Evolution

The BFKL equation for DIS follows from a modified version of the factorized expression, Eq. 4, in which the parton fractional momentum ξ itself plays the role of a factorization scale. To

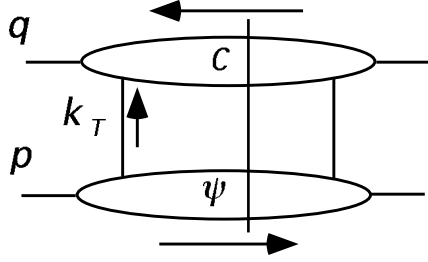


Figure 2: Factorization associated with BFKL resummation of logarithms of x .

emphasize this point, consider the following two factorized expressions for a structure function F (F_2 for instance),

$$\begin{aligned}
 F(x, Q^2) &= \int_x^1 \frac{d\xi}{\xi} C\left(\frac{x}{\xi}, \mu^2/Q^2\right) G(\xi, Q^2) \\
 &\quad + \mathcal{O}\left(\frac{1}{Q^2}\right) \\
 &= \int d^2k_T c\left(\frac{x}{\xi'}, Q, k_T\right) \psi(\xi', k_T) \\
 &\quad + \mathcal{O}\left(\frac{1}{\ln(1/x)}\right). \tag{12}
 \end{aligned}$$

The first equality is the factorization Eq. 2 for F , specialized to gluons only, and represented by Fig. 1b above. (In DIS, we “cut” this diagram: that is, separate it into amplitude and complex conjugate to form a cross section.) The “coefficient function” C plays the role of the hard-scattering cross section $\hat{\sigma}_{\text{PT}}$ in Eq. 2. Corrections to this equality are suppressed by $1/Q^2$ (neglecting for this argument the effects of quarks). We recall that in this expression we have organized momentum scales larger than the factorization scale μ into C , and those smaller into $G(\xi, \mu^2)$.

The second expression in Eq. 12 is a reorganization of the same structure function F , based upon a factorization of partons with fractional momenta smaller than scale ξ' into a new coefficient function c . Partons of larger fractional momenta are absorbed into a new, long-distance wave function ψ . This factorization is illustrated schematically in Fig. 2, in which the process consists of two “jets” of virtual particles, one set nearly parallel to p , the other nearly parallel to q , which exchange a pair of quanta. Note that the picture requires that $Q^2 = -q^2$ be much less than $p \cdot q \sim s$, so that q^μ may be thought of as having a definite direction. This is precisely the small- x region in which we are interested. The factorization is based on the observation that as $p \cdot q \rightarrow \infty$, the assignment of quanta between the two “jets” is somewhat arbitrary, and will change as we sample frames that differ by boosts along the collision axis.

Now, instead of a convolution in fractional momenta, the convolution is in transverse momenta. ψ is related to the standard gluon distribution by

$$G(x, \mu^2) = \int^\mu d^2k_T \psi(x, k_T). \tag{13}$$

Despite the analogous structure of the two convolutions in Eq. 12, corrections to the second form are very different, suppressed only by inverse powers of $\ln(1/x)$. Nevertheless, up to these corrections, both forms are simultaneously valid.

Just as the first factorization form leads to DGLAP evolution by invoking the independence of F on μ , so the independence of F on ξ' leads to another evolution equation, analogous to Eq. 4, but now with a convolution in transverse momentum rather than parton fraction. This is the BFKL equation, [14]

$$\xi \frac{d\psi(\xi, k_T)}{d\xi} = \int d^2 k'_T \mathcal{K}(k_T - k'_T) \psi(\xi, k'_T). \quad (14)$$

The kernel \mathcal{K} turns out to be independent of ξ in this approximation, although by dimensional analysis it could have depended upon it. Solutions to the BFKL equation may be found by substituting trial solutions in the form of powers of ξ and k_T . These solutions take a continuous range of powers of ξ , of which the most singular is the famous BFKL result,

$$\xi \psi(\xi, k_T) \sim \xi^{-4N \ln 2(\alpha_s/\pi)}. \quad (15)$$

This is precisely of the form of case (ii) above, and lends further plausibility to that approach.

Concerning this result, however, we should make a number of observations. First, the BFKL equation for DIS is not in contradiction to DGLAP evolution in general. Indeed, as we have seen above, it is simply based on a reformulation of the same underlying factorization of long- and short-distances. In this sense, it picks out those terms that produce leading logarithms of $1/x$, to all orders in the strong coupling α_s , from the DGLAP kernel. The question arises, however, which α_s ? If we pick $\alpha_s(Q^2)$, the phenomenology of small x structure functions definitely disfavors Eq. 15, because this distribution would become less rather than more steep as Q^2 increases. Because the BFKL equation results from a purely leading log analysis, however, the argument of the coupling is not determined. Eventually, the two-loop version of the kernel \mathcal{K} in Eq. 14 will help, but there is another, even more important, consideration. In the DGLAP equation, corrections at second and higher order in α_s modify the kernels P_{ij} , but do not change the equation itself. Nonleading logarithms in $1/x$, on the other hand, actually change the form of the BFKL equation, requiring more k_T integrals. [15] (At large Q^2 , however, Eq. 12 may be replaced by a convolution in k_T and ξ' . [16]) In the next section, we will review some of the additional features of BFKL evolution that, despite all this, make it such an attractive subject of study.

2.3 BFKL: the Larger Context

DIS is only one application of the BFKL analysis. Others may be found by noting the relation of the DIS distribution ψ to the total cross section for the scattering of a nucleon by an off-shell photon. In the $x \rightarrow 0$ limit, x is proportional to the ratio of the squared invariant mass of the photon-nucleon system to the “mass” of the photon. Thus, starting with

$$\psi \rightarrow s_{\gamma^*p} \sigma_{\gamma^*p}^{\text{tot}} \sim s_{\gamma^*p} \frac{1}{x^{1+\delta}}, \quad (16)$$

where δ is given by the exponent in Eq. 15, we find

$$s_{\gamma^*p} \sigma_{\gamma^*p}^{\text{tot}} \sim \left(\frac{W^2}{Q^2} \right)^{1+\delta} \sim \left(\frac{s_{\gamma^*p}}{m_{\gamma^*}^2} \right)^{1+\delta}. \quad (17)$$

In this picture, the total γ^*p cross section increases as a power of the relevant c.m. energy squared. This result is at the center of an entire literature. In fact, it was derived first in the context of hadron-hadron scattering, for which analogous reasoning gives [14]

$$\sigma_{AB}^{\text{tot}} \sim s^{\alpha-1}, \quad (18)$$

with $\alpha > 1$ for any two hadrons A and B to leading logarithm in s .

The result Eq. 18 is interesting for (at least) two reasons. First, because it is strongly reminiscent of “Regge” behavior, which was long ago conjectured for the $s \rightarrow \infty$, t fixed limit in field theory,

$$\sigma_{AB}^{\text{tot}}(s, t) \sim \sum_i \beta_{Ai}(t) s^{\alpha_i(t)-1} \beta_{Bi}(t). \quad (19)$$

Here i labels an exchanged “reggeon”, a composite degree of freedom. The function $\alpha_i(t)$, when analytically continued to specific timelike values of t , is identified with the spin of the particle whose exchange describes the reggeon, $\alpha_i(t = m_{iJ}^2) = J$. Although the pomeron’s timelike continuation is a matter of debate, we recognize the tantalizing possibility that here high-energy analysis has found a window to hadronic degrees of freedom.

The second source of interest in Eq. 18 is a problem: such a power behavior extended to arbitrarily large s would eventually violate unitarity bounds on the total cross section. Thus, we are naturally led to study corrections to Eq. 18, and, given the accuracy of the factorization that leads to it, Eq. 12.

Current theoretical work on BFKL resummation and the Regge limit in QCD may be classified roughly into two categories. One might call the first “visionary BFKL”, which attempts to grapple with the two fundamental issues of Regge behavior and unitarity directly as mathematical problems in the presence of strong coupling, going far beyond perturbation theory. This is to be contrasted with “pragmatic BFKL”, which attacks problems where the coupling is small, either in model systems or in special kinematic limits.

Attempts to understand the BFKL equation in a nonperturbative fashion start with the observation that, rewritten as

$$\frac{d}{d \ln x} \psi = \mathcal{K} \otimes \psi, \quad (20)$$

the BFKL equation (Eq. 14 above) is similar to a Schrödinger equation, with $\ln x$ playing the role of time, and \mathcal{K} of the Hamiltonian. [17, 18] For a solution with behavior $x^{-\alpha}$, the Regge exponent α (see Eq. 19) plays the role of an energy. As illustrated in Fig. 2, the wave function is characterized by two external particles, which at lowest order are gluons, while at higher orders have the interpretation of “reggeized” gluons – corresponding to a gluonic Regge trajectory in the general form of Eq. 19 [19]. Evidently confinement prevents such an exchange from contributing to physical hadronic scattering, but its bound states can contribute, and the simplest of these is the two-reggeon bound state, the pomeron, which is the solution to eq. 20. From this point of view, it is possible to generalize the Hamiltonian \mathcal{K} to \mathcal{H}_n , which describes the dynamics of $n > 2$ reggeized gluons, at least when $N \rightarrow \infty$, with N the number of colors. As discussed at this conference, [20, 21, 22] the spectra of the \mathcal{H}_n may be determined in principle by the methods of statistical mechanics. Quite recently, progress has been made in determining the specific “energy” of the three-reggeon bound state, known as the “odderon”. [21] If, as suggested at this conference, the odderon has an exponent above unity, then it might be possible to detect it experimentally. [23] In the long run, if all goes well, and all the α ’s are found as functions of t , one may analytically continue and really solve for the bound states of QCD. This vision is far from realization, however, and we should emphasize that the construction of the multireggeon Hamiltonians requires approximations, such as large numbers of colors, whose validity it is currently difficult to judge.

On the pragmatic side, a laboratory for studying the BFKL pomeron in a perturbative context is in the hypothetical scattering of heavy-quark “onia”, as illustrated in Fig. 3. [24] Because the

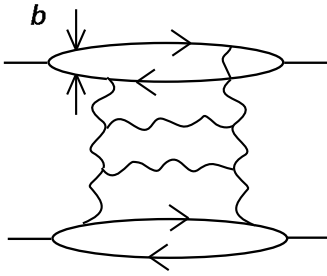


Figure 3: Gluon ladder exchange in onium-onium scattering.

size b of a $Q\bar{Q}$ bound state shrinks with the quark's mass, such a bound state behaves as a “color dipole”, and b as an infrared cutoff on the running of the coupling. In this case, the sum of ladders, as in the figure, may be studied perturbatively, because we may arrange for $\alpha_s(1/b)$ to be small, while $\alpha_s \ln s$ is large. In addition, in the large- N limit, the gluons themselves act as color dipoles, and the process of ladder formation becomes iterative, allowing for an explicit construction of the relevant wave functions. This model serves as an alternate approach to generalize the leading-logarithm, BFKL equation, and to study how unitarity is realized.

There is, in addition, a series of proposals for observing the BFKL pomeron phenomenologically, in hard, high-energy scattering. In each case, a process has been identified in which logarithms of s (analogous to $1/x$, as we have observed above) are generated, with an infrared cutoff set in the perturbative region by a momentum transfer Q , with $\Lambda_{\text{QCD}} \ll Q \ll \sqrt{s}$. Examples are in the rapidity-dependence and angular correlations of jets of transverse momentum Q in very high-energy hadronic collisions. [25, 26, 27] The masses of virtual photons in $\gamma^*\gamma^*$ scattering at very high energy also make it possible to derive an infrared safe, BFKL-resummable cross section. [28] Generally speaking, however, this sort of experimental signal for BFKL has not turned out to be easily accessible, in large part because the leading-logarithm approximation requires a very high energy to provide numerically reliable predictions.

Despite the limitations of the quantitative predictions of BFKL resummation, there remain a number of experimental hints that the physics behind it is relevant. At this conference, we heard, for instance, about the distribution of particles emitted as a function of transverse momentum for various ranges of x . [29] In this connection, we may also mention treatments of DIS final states based on coherence analysis and “angular ordering”, [30] which allow for interpolations at the leading-log level between BFKL and DGLAP equations. The lack of transverse-momentum ordering in BFKL evolution (see below) suggests that it will produce a “harder tail” at high p_T than might be the case for (leading or next-to-leading logarithm) DGLAP evolution. [31] H1 reported some indications of this effect in their 1994 data, although the interpretation is based on comparison to the outputs of event generators. The other experimental signals that we may connect to the program of color singlet exchange are the “rapidity gap” events, which comprise measurable fractions of jet cross-sections at both HERA and the Tevatron. [32, 33] In a rapidity gap event, a pair of jets is produced at very high p_T , with very little radiation between them, a phenomenon that is most naturally attributed to the exchange of some sort of color singlet. [34] A full perturbative treatment has not, so far as I know, yet been given for these processes, and remains as a challenge for the coming years. As the overall momentum transfer vanishes compared to s , rapidity events merge smoothly into “diffractive” events, on which there is now much more data from DIS, and toward which much more theoretical speculation has been

directed. It is to this constellation of signals that we now turn.

3 Evolution and Hard Diffraction

3.1 Interpreting Evolution

One of the most attractive features of DIS is found in the interpretation of its final states as virtual states of the proton “brought to life” by the energy provided by the lepton. By inspecting distributions of hadrons in the final state, we may hope to learn, not only about the distribution of scattered partons, but also about the sequence of virtual processes by which they are produced. Such a linkage of hadrons to partons takes us beyond the strict application of “infrared safety”, in which quantities are computed in perturbation theory and applied directly to inclusive cross sections. Rather, we shall apply the still not fully quantified concept of parton-hadron duality, referred to above.

To discuss distributions of final-state partons, we note that evolution equations arrive at (perturbative) final states by creating many short-lived states from fewer long-lived states. A highly-virtual exchange, such as in DIS, produces a cross section that is dominated by short-lived states, simply because there are so many of them. Nevertheless, the dominant contribution to a short-lived state contains many footprints of the path taken to arrive at that state. The DGLAP and BFKL equations, the primary contenders for describing DIS at low x , each has a characteristic “history”, embedded within its final states. Let us be clear, however, that in this context by “DGLAP evolution”, we mean evolution according to Eq. 4 with a finite-order approximation to the DGLAP kernels P_{ij} . BFKL evolution is consistent with all-orders DGLAP evolution, but has certain qualitative differences from its low-order approximations.

For DGLAP evolution defined in this sense, final states are characterized by partons with ordered k_T , with a moderate decrease in x as k_T increases. Hadrons with the largest k_T are descendants of those partons emitted at the last stage of evolution. BFKL evolution, on the other hand, is characterized by a rapid decrease in x , coupled with slow, perhaps random, variations in k_T . The distinction between these two pictures has an intriguing heuristic relation to the diffractive events, observed with surprising frequency at HERA, and at fixed-target experiments. [35]

3.2 HERA “Stories”

The relation of evolution to distributions in the final state may be illustrated heuristically by two “stories” of evolution, corresponding to DGLAP and BFKL evolution. The first, a DGLAP-like history, is illustrated by Fig. 4a, in which partons are emitted in a ladder strongly-ordered in transverse momenta, $k_{i+1,T} \gg k_{i,T}$. Assuming parton-hadron duality, and ignoring perturbative final-state interactions, suppressed by factors of at least $\alpha_s(k_{i,T})$, we anticipate a distribution of final-state hadrons that reflects the distribution of the partons that emerge from the ladder. Because of the k_T -ordering, the hadrons spread out in the final state, filling a range of directions between the scattered quark (the “current jet”) and the unscattered partons (the “proton remnant”). In the c.m. frame such a final state would appear as in Fig. 4b. In particular, we expect such a process to have no large angular or rapidity gaps in the final state, because there are no large gaps in a leading-log ladder, and because the partons are rapidly separating from each other as they propagate into the final state.

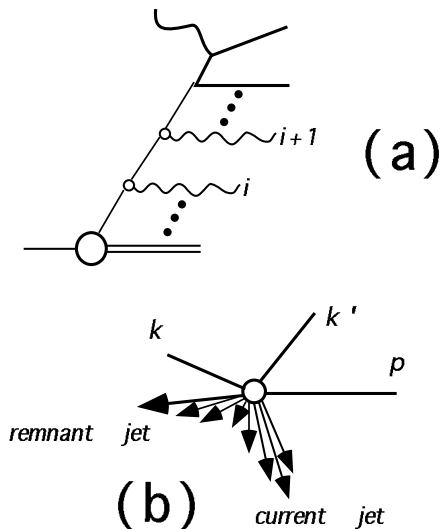


Figure 4: DGLAP story of evolution.

To the above picture we may contrast the “BFKL-like” story of Fig. 5a-c. Here, (5a) the ladder consists of partons emitted with comparable transverse momenta, $k_{i,T} \sim k_{i+1,T}$, but highly-ordered longitudinal momenta, $x_i \gg x_{i+1}$. If, in addition, none of these (randomly distributed) transverse momenta is too large, then there may well be a substantial spatial overlap of the radiated ladder gluons with each other *and with the target proton*. Finally, if the ladder gluons are in a singlet state, [36] which costs them only the emission of one extra gluon, they may have a substantial overlap with the proton remnant to form an outgoing proton, or nucleon resonance (5b). Since the ladder gluons, except perhaps for some at the very top of the ladder, do not appear in the final state, this corresponds to an event of the type shown in 5c, in which the current jet is isolated, with a large “gap” in rapidity between it and the scattered proton p' .

The ladder of virtual gluons plays the role of a “pomeron”. The pomeron in this picture of diffractive scattering is a process, rather than a particle. Nevertheless, it is natural to interpret diffractive hard scattering as being initiated by a parton, not simply from the proton, but from the ladder, or pomeron. In this interpretation, we reduce the diffractive scattering into a two-stage process: the (effective) emission of a pomeron from the proton, with fraction $x_{\mathcal{P}}$ of the proton’s momentum, followed by a hard scattering of a parton with a fraction β of the pomeron’s, and hence fraction $\beta x_{\mathcal{P}}$ of the proton’s, momentum. [37] The variable β may be determined from the invariant mass M_X of the current jet, through the standard relation,

$$M_X^2 = \frac{Q^2}{\beta}(1 - \beta) = (x_{\mathcal{P}}p + q)^2, \quad (21)$$

or, equivalently, $\beta = x/x_{\mathcal{P}}$, with x the usual scaling variable.

If this partonic interpretation is to be valid, it is also plausible to explore the possibility of approximate scaling for hard diffractive scattering, with calculable evolution. This expectation has been formalized [38] in a generalization of DIS factorization to inclusive structure functions for the diffractive cross section (compare Eq. 2),

$$\begin{aligned} F_2^D &= C_{2,i}^D \otimes \phi_{i/p}^D \\ &= \int_{\beta}^1 \frac{d\beta'}{\beta'} C_{2,i}^D(\beta/\beta') \phi_{i/p}^D(\beta'). \end{aligned} \quad (22)$$

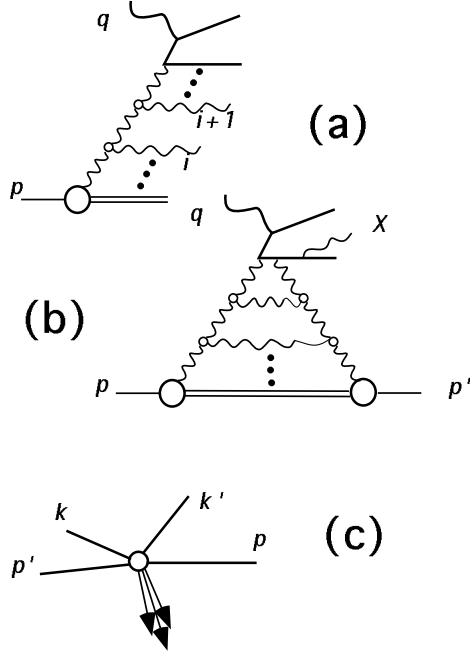


Figure 5: BFKL story of evolution

If this factorization holds, F_2^D evolves according to DGLAP evolution, but with yet another initial condition. The H1 collaboration, by performing an analysis of Q^2 dependence for different values of β , has concluded that the distribution of partons in the pomeron may be concentrated near unity in β , in distinction to normal hadronic parton distributions. [39]

At the same time, because this effective pomeron appears in both the amplitude for this process and its complex conjugate, the dependence of the diffractive structure function on the *total* hadronic energy $W = \sqrt{(p+q)^2}$ behaves as the *square* of a total cross section,

$$F_2^D(W) \sim (\text{kinematics}) \times \left(W^2\right)^{2(\alpha_{\mathcal{P}}-1)}. \quad (23)$$

Here $\alpha_{\mathcal{P}}$ is the power of s characteristic of single-“pomeron” exchange. If this pomeron were the same at all momentum transfers, it would be directly related by the optical theorem to the exponent in Eq. 18 for the total hadronic cross section. Experimentally, however, the exponent $\alpha_{\mathcal{P}}$ depends upon the momentum transfer, and F_2^D increases faster with W when $Q^2 \gg \Lambda_{\text{QCD}}^2$ than in the photoproduction limit. [39, 40] Thus, it may well be useful to distinguish between two “pomerons”, a “soft” pomeron that generates the total cross section Eq. 19, with a smaller exponent $\alpha_{\mathcal{P}} \sim 1$, and a “hard” pomeron that couples to high-energy scattering processes, with a somewhat larger exponent, perhaps suggestive of the BFKL result, Eq. 15. [41] It is not entirely clear, however, whether the experimental results are consistent with a purely pomeron-like picture. [39]

3.3 Exclusive Electroproduction

Further insight into diffractive processes may be gathered by demanding an *exclusive* final state, as in electroproduction of vector bosons, $\sigma_{\gamma p \rightarrow V p+X}$, with $V = \rho, \psi$, etc., at photon virtuality

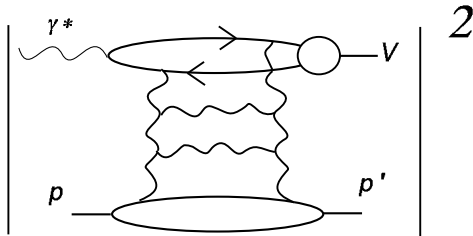


Figure 6: Exclusive electroproduction.

$Q^2 \gg \Lambda_{\text{QCD}}^2$. The price of demanding an exclusive final state is that the cross section is power-suppressed in Q^2 . The leading-logarithm process is illustrated in Fig. 6, in which the gluonic ladder structure is manifest at the amplitude level. Identifying the ladder via unitarity with the gluon distribution, again valid to leading logarithm, one finds, [42, 43]

$$\frac{d\sigma}{dt} \sim \frac{\alpha_s^2}{Q^6} |xG(x, Q^2)|^2, \quad (24)$$

where the constant of proportionality depends on the quark-antiquark wave function of the vector boson. In Eq. 24, $x \sim 1/W^2$ by the usual kinematics. If $xG(x) \sim x^{-\lambda}$, then $d\sigma/dt \sim W^{4\lambda}$, and this cross section, although suppressed in Q , increases dramatically with W . Such an increase has, indeed, been observed. [44] At the same time, we should note that the gluon ladder in Fig. 6 is evaluated at $t = (p - p')^2 < 0$, while the true gluon distribution requires $t = 0$ exactly. It is possible, however, to analyze this approximation systematically. [45]

4 High- p_T , Heavy Quarks and Resummation

4.1 Jet and Heavy Quark Cross Sections

Jet physics is coming of age. Presentations at the parallel sessions on hard processes and perturbative QCD described an increasingly sophisticated analysis of jet cross sections. At HERA, ZEUS and H1, at LEP, the ALEPH, DELPHI OPAL and L3 collaborations, and at SLC, the SLD collaboration, have measured multijet cross sections, analyzed jet-related event shapes and fragmentation functions. The LEP collaborations reported on methods for routinely distinguishing quark and gluon jets. CDF and D0 are also having a close look at jet substructure and multijet events. I have space here for only a few of the many physics issues raised by these important investigations.

High- p_T jet cross sections at hadron colliders are one of the proverbial successes of perturbative QCD. [46, 47] Within the foregoing year, however, jet cross sections at the highest energies received a new burst of attention with the description by CDF [32, 48] of a substantial, and apparently growing, excess of events at the highest transverse momenta, up to approximately one half of a TeV. D0 does not see such a rise, [49] although the uncertainties in both experiments are such that they are not in strong disagreement. This conference has seen a general consensus that this is an inconclusive signal of new physics, and may be more a reflection of remaining uncertainties in parton distributions, [50] particularly the gluon distribution [51, 9] at relatively large values of fractional momentum. [47] With suitably-adjusted parton distributions, the now-classic next-to-leading order (NLO) formalism may well provide a good picture of the data, at least at the current level of statistical and systematic errors.

Another, and closely related, subject of keen interest over the past year has been the computation of the total cross section for the production of top quarks in hadronic collisions, particularly at the Tevatron. [52] Here, the most widely-quoted analyses go beyond NLO, resumming “large”, positive corrections at “partonic threshold” to all orders in perturbation theory. [53, 54, 55] A number of viewpoints have recently been raised on the best way of going about this resummation, and whether or not it results in substantial modifications of NLO results.

The outcome of these discussions is relevant to our confidence in the NLO cross sections for jet and heavy particle production, and hence to signs for new physics. Indeed, as we shall see, the perturbative calculation of hard-scattering cross sections through factorization leads to a nonconvergent series in the coupling. This problem has a resolution, however, that is surprisingly consistent with the successes of NLO approximations, and which contains within it a connection to an as-yet undeveloped formalism for power corrections in general QCD cross sections. To try and clarify these issues, I shall first discuss the origin and resummation of “Sudakov” corrections in factorized cross sections.

The cross sections for jet production, inclusive top production, and any other final states F of mass Q in hadron (A)-hadron (B) scattering take the detailed form, [56]

$$\begin{aligned} \frac{d\sigma_{AB\rightarrow FX}}{dQ^2 dy} &= \sum_{ab} \int_{Q^2/S}^1 dz \int \frac{dx_a}{x_a} \frac{dx_b}{x_b} \\ &\times \phi_{a/A}(x_a, Q^2) \phi_{b/B}(x_b, Q^2) \\ &\times \delta\left(z - \frac{Q^2}{x_a x_b S}\right) \\ &\times \hat{\sigma}_{ab\rightarrow FX}\left(z, y, x_a/x_b, \alpha_s(Q^2)\right), \end{aligned} \tag{25}$$

where the perturbative short-distance function begins with the Born cross section.

$$\hat{\sigma} = \sigma_{\text{Born}} + \frac{\alpha_s}{\pi} \hat{\sigma}^{(1)} + \dots \tag{26}$$

In the following, we shall discuss briefly the nature of corrections in $\hat{\sigma}$ beyond one loop, and how they have been used to improve upon NLO predictions. We shall see once again that factorization plays a central role in this resummation.

4.2 Partonic “Threshold” and Nonconvergence

Partonic threshold in Eq. 25 refers to the limit $z \rightarrow 1$, in which the total partonic c.m. energy $\sqrt{x_a x_b S}$ is just large enough to produce the observed final-state system F , with no energy left over for QCD radiation. (In the production of heavy quarks, partonic threshold should be distinguished from “true” threshold in which the pair is produced at rest, $Q = 2m_q$.) Not surprisingly, partonic threshold is a singular limit, since, even classically, hard scattering demands copious radiation.

The singular nature of partonic threshold is reflected in the presence of potentially large corrections. At n loops, the dominant “large” correction is the “plus distribution”, $-\frac{\alpha_s^n}{n!} \times \left[\frac{\ln^{2n+1}((1-z)^{-1})}{1-z} \right]_+$, which is defined by its integrals with smooth functions $\mathcal{F}(z)$ (such as the

parton distributions in Eq. 25), through,

$$\begin{aligned}
& - \frac{\alpha_s^n}{n!} \int_0^1 dz \frac{\mathcal{F}(z) - \mathcal{F}(1)}{1-z} \ln^{2n+1} \left((1-z)^{-1} \right) \\
& = \frac{\alpha_s^n}{n!} \int_0^1 dz \mathcal{F}'(1) \ln^{2n+1} \left((1-z)^{-1} \right) + \dots \\
& \sim \mathcal{F}'(1) \frac{\alpha_s^n}{n!} (2n+1)! + \dots,
\end{aligned} \tag{27}$$

where $\mathcal{F}'(1)$ is the first derivative to \mathcal{F} at $z = 1$. At n th order, such a correction produces a coefficient of α_s^n that grows faster than $n!$. These corrections have the same sign for every value of n , and for n high enough are large, positive, and nonconvergent.

The origin of such terms is in the process of factorization itself, as it is normally implemented. The issue may be illustrated schematically as in Fig. 7 for $\hat{\sigma}$ in the Drell-Yan (DY) cross section. The hard-scattering function $\hat{\sigma}$ is computed from parton-parton cross sections by factorizing, that is, dividing out, distributions for each incoming parton. But these distributions, the denominator of the figure, are *each* the square of an amplitude, in which there are both incoming *and* outgoing partons, both of which have a tendency to radiate. In the numerator, however, we need only worry about the radiation of the incoming partons. This difference is also illustrated by comparing the

$$\hat{\sigma}(N) = \frac{\left| \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \text{---} \end{array} \right. + \left. \begin{array}{c} \text{---} \\ \diagdown \quad \diagup \\ \text{---} \end{array} \right|^2}{\left| \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \text{---} \end{array} \right. + \left. \begin{array}{c} \text{---} \\ \diagdown \quad \diagup \\ \text{---} \end{array} \right|^4}$$

Figure 7: Schematic representation of moments of the Drell-Yan partonic hard-scattering function.

invariant masses of the hadronic final states in the DY and DIS processes,

$$\begin{aligned}
\text{DY} & : \quad W_{\text{had}}^2 = Q^2(1-z)^2 \\
\text{DIS} & : \quad W_{\text{had}}^2 = \frac{Q^2(1-x)}{x}.
\end{aligned} \tag{28}$$

In the former, there is a quark-antiquark pair in the initial state only, while in the latter, which defines a distribution in “DIS scheme”, a quark or antiquark is present with large energy in the final state as well. As we shall see, both the numerator and denominator – the DY and DIS cross sections – vanish at threshold. But the denominator, with *two* factors of the DIS cross section, vanishes faster, and this results in an enhancement in $\hat{\sigma}$. To resolve this problem in a general case, we need to develop a method for organizing large corrections in the elastic limit. This is known as Sudakov resummation, and like DGLAP evolution and BFKL resummation, it is based on a factorization property of QCD cross sections. [57]

4.3 Sudakov Resummation

Sudakov resummation applies to a large class of hard-scattering cross sections in “exclusive” limits, for instance, $w \rightarrow 0$, when [58, 59, 60]

$$\begin{aligned}
 w &= 1 - T \quad e^+e^- \\
 &= 1 - x \quad \text{DIS} \\
 &= 1 - Q^2/s \quad \text{DY total} \\
 &= Q_T/\sqrt{s} \quad \text{DY or 2 Jet},
 \end{aligned}
 \tag{29}$$

where T is the thrust variable. In the limit that any of these variables vanishes, all the final-state and/or initial-state particles in these processes are either part of one or two massless *jets*, or have vanishingly soft momenta. In terms of momentum flow, these configurations are identical to exclusive processes. For the case of thrust, the cross section is illustrated in Fig. 8 in cut diagram notation, as a sum over possible final states C of an amplitude (to the left of C) times its complex conjugate (to the right).

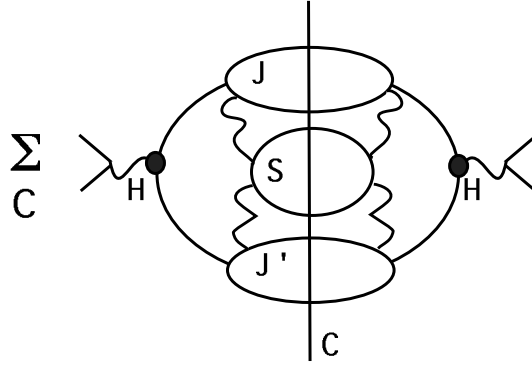


Figure 8: e^+e^- annihilation in the $T \rightarrow 1$ limit. All lines in the J 's are mutually collinear, those in S carry vanishing momenta.

In this case, $w = 1 - T$ is the kinematic variable that vanishes in the two-jet limit for total c.m. energy Q . The singular part of $d\sigma/dw$ as $w \rightarrow 0$ enjoys a factorized form that generalizes Eq. 2, and in which the cross section is a convolution of functions, one for each jet J and J' , and one for the soft radiation S , linked by their contributions to the weight w , [57]

$$\begin{aligned}
 \frac{d\sigma}{dw} &= H(p_i/\mu, \zeta_i, \alpha_s(\mu^2)) \\
 &\times \int \frac{dw_1}{w_1} \frac{dw_2}{w_2} \frac{dw_s}{w_s} \delta(w - w_1 - w_2 - w_s) \\
 &\times S(w_s Q/\mu, v_i, \zeta_i) \\
 &\times J(p_1 \cdot \zeta_1 / w_1 Q^2, \mu) J'(p_2 \cdot \zeta_2 / w_2 Q^2, \mu) \\
 &+ \mathcal{O}(w^0).
 \end{aligned}
 \tag{30}$$

Multiplying the convolution is an overall factor H , which summarizes the effects of quanta that are off-shell by order Q^2 . Here, in addition to a factorization scale μ that separates hard and soft quanta, we must introduce two new vectors ζ_1 and ζ_2 , to distinguish quanta moving in the jet

directions. These may be thought of as gauge-fixing vectors. Finally, the v_i^μ are lightlike vectors in the jet directions. Once again, our freedom in choosing μ , ζ_1 and ζ_2 enables us to derive a useful resummation.

As is often the case, resummation is most easily carried out in a space of moments, under which the convolution, Eq. 30, reduces to a product,

$$\begin{aligned} & \int dw e^{-Nw} \frac{d\sigma}{dw} \\ &= H(p_i/\mu, \zeta_i, \alpha_s(\mu^2)) \tilde{S}(Q/N\mu, v_i, \zeta_i) \\ & \times \tilde{J}_1(p_1 \cdot \zeta_1, Q^2/N, \mu) \tilde{J}_2(p_2 \cdot \zeta_2, Q^2/N, \mu). \end{aligned} \tag{31}$$

The independence of $d\sigma/dw$ from ζ_1 , for instance, requires that

$$\begin{aligned} & p_1 \cdot \zeta_1 \frac{\partial}{\partial p_1 \cdot \zeta_1} \ln \tilde{J}_1(p_1 \cdot \zeta_1, Q^2/N, \mu) \\ &= -p_1 \cdot \zeta_1 \frac{\partial}{\partial p_1 \cdot \zeta_1} H(p_i/\mu, \zeta_i, \alpha_s(\mu^2)) \\ & \quad - p_1 \cdot \zeta_1 \frac{\partial}{\partial p_1 \cdot \zeta_1} \tilde{S}(Q/N\mu, v_i, \zeta_i), \end{aligned} \tag{32}$$

which is readily turned into an evolution equation for N -dependence. The process is closely analogous to the procedure that leads from the μ -independence of Eq. 2 to the DGLAP evolution equations, 4.

The result of this procedure is an exponential of a double integral over the running coupling at variable momentum scale, [57]

$$\begin{aligned} \ln \left(\frac{d\tilde{\sigma}(N)}{dw} \right) &= D_\sigma(\alpha_s(Q), \alpha_s(Q/N)) \\ &+ 2 \int_{Q/N}^Q \frac{d\lambda}{\lambda} \left[\int_{Q^2/\lambda N}^\lambda \frac{d\xi}{\xi} A_\sigma(\alpha_s(\xi^2)) \right. \\ & \quad \left. + B_\sigma(\alpha_s(\lambda)) \right], \end{aligned} \tag{33}$$

with D , A and B power series with infrared finite coefficients, which can be determined at low order by direct comparison with explicit calculations. When the couplings are treated as fixed, Eq. 33 produces an exponential of the form, $\exp[-\text{const} \times \alpha_s \ln^2 N]$. The running of the coupling produces “subleading” logarithms, $\alpha_s^n \ln^m N$, with $m \leq n$, in the exponent. This exponentiated moment must be inverted to apply the result to Eq. 25 in a practical case. At very large $N \geq Q/\Lambda_{\text{QCD}}$, however, Eq. 33 becomes ill-defined, as the perturbative coupling $\alpha_s(\xi^2)$ diverges. We shall return to this problem below. Analogous reasoning applies to both the DIS and DY cross sections in their exclusive limits, and leads to the Sudakov *enhancement* in the factorized hard-scattering function at partonic threshold noted above.

The simple exponentiation of Eq. 33 applies, unfortunately, only to QCD corrections to electroweak hard-scattering. For QCD hard-scattering, such as jet and heavy-quark production, which involve color exchange, the exponentiation becomes a matrix problem, which, however, is interesting in its own right. [61, 62, 63] The methods required to treat this problem turn out

to be related, in particular, to the popular “effective theory” approach to heavy quark physics. Consider, for example, heavy quark production. In this case, the dynamics of soft gluons (S in Fig. 8), may be represented as in Fig. 9, in which the hard-scattering function $\hat{\sigma}$ from Eq. 25 is factored into two short-distance functions h_I and h_J , labelled by the color exchange that takes place at short distances, and a soft function S_{IJ} , which is the cross section for eikonized light quarks to annihilate to form eikonized heavy quarks. [64] Here “eikonized” means that these quarks emit gluons without recoil, and that correspondingly their propagators are of the form $1/v \cdot k$, with v^μ the velocity of the quark, familiar from heavy quark theory. The relevant evolution equations may be interpreted as renormalization group equations for the composite vertices that link the eikonized quarks. At *next-to-leading* logarithm, the cross section in moment space is a sum over color exchanges, of different exponentials, [64]

$$\hat{\sigma}_{q\bar{q} \rightarrow Q\bar{Q}}(N) = \sum_{IJ} S_{IJ} h_I(Q) h_J^*(Q) \times e^{E_{IJ}(N, \alpha_s)}. \quad (34)$$

At leading logarithm, E_{IJ} is the same as in Eq. 33, but beyond leading logarithm depends on the color exchanges in the hard scattering. Similar composite operators characterizing color exchange also occur in treatments of hadron-hadron elastic scattering [62, 63] and the decay and production of quarkonia. [65]

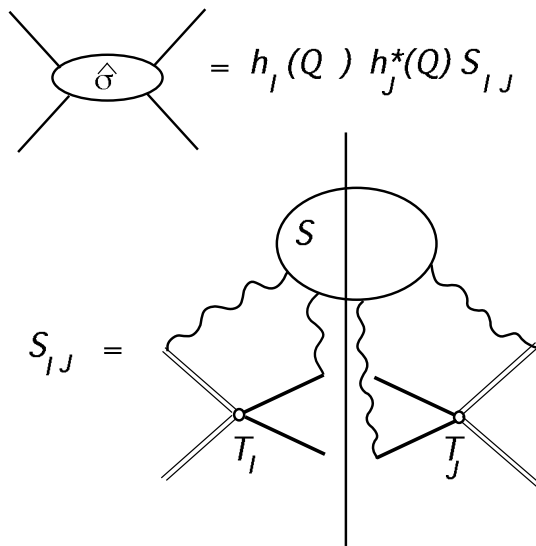


Figure 9: Representation of the factorization of the hard scattering function $\hat{\sigma}$ near the elastic limit.

Up until now, top production is the only inclusive cross section to have been studied extensively with threshold resummation taken into account, and then only at the level of leading logarithmic accuracy. [53, 54, 55] (Very recently, however, a preliminary investigation of non-leading logarithms has appeared. [66]) The various groups agree on the basic exponentiation in N space; differences in their results primarily concern how best to invert the transform. Especially interesting is the question of what to do with the running coupling in Eq. 33 when $\xi_{\min}^2 = Q^2/N^2 \rightarrow \Lambda_{\text{QCD}}^2$. We shall touch on this issue in the next section.

At the current experimental accuracy, there is actually little to choose between the different numerical predictions. In future Tevatron runs, however, we may hope for more accurate measurements of the total top and jet cross sections, and by then some of the issues of numerical accuracy should be resolved. In the meantime, it is worth noting that the only available (and conservative) estimate of jet cross sections [55] shows the resummed jet cross section rising, relative to NLO, at the very highest p_T 's yet measured, by the order of five percent. The effect may be too small for current statistical errors, but it is an intriguing contribution, which requires an understanding of the theory beyond the lowest orders. Before passing to the issue of how to deal with the running coupling, we should note that a fully consistent use of resummed hard scattering functions requires a set of parton distributions derived from experiment by employing the same resummed hard parts. It may take a little while for someone to summon the courage to undertake such a project.

5 Power Corrections and IR Renormalons

Even at the highest energies, precision for unaided NLO calculations is still relatively rare, and this, as in the examples above, has stimulated the development of Sudakov resummation techniques. We have already observed, however, that in Sudakov resummation, going to all orders opens the door to singularities associated with the divergence in the running coupling, as in Eq. 33 when the moment variable N reaches Q/Λ_{QCD} . We may rely on event generators to build in a model of high order corrections, but it's hard to know to what the nonperturbative inputs in these models correspond in the true theory. Indeed, the inclusion of any fixed dimensional scale Λ' in an event generator leads to power-suppressed corrections of the form $(\Lambda'/Q)^a$, $a > 0$, in hard-scattering cross sections at scale Q . [67] All this has led a group of investigators to ask whether there might be a theory of power corrections in QCD cross sections. [68, 69, 70] These and related ideas were discussed extensively at the conference. [71, 72]

Of course, there are famous cases in which a theory of power corrections already exists, based on the operator product expansion (OPE). For instance, the total e^+e^- annihilation cross section is given by

$$\sigma_{\text{tot}}^{e^+e^-} = \frac{(4\pi\alpha)^2}{Q^2} \text{Im } \pi(Q^2) \quad (35)$$

where the function π is found from the products of electromagnetic currents,

$$\pi(Q^2) = \left(\frac{-i}{3Q^2} \right) \int d^4x e^{-iq \cdot x} \langle 0 | T j^\mu(0) j_\mu(x) | 0 \rangle, \quad (36)$$

which at short distances (large q^2) may be expanded in terms of local operators,

$$\begin{aligned} \langle 0 | j^\mu(0) j_\mu(x) | 0 \rangle &= \frac{1}{x^6} C_0(x^2 \mu^2) \\ &+ \frac{1}{x^2} C_{F^2}(x^2 \mu^2) \\ &\times \alpha_s(\mu^2) \langle 0 | F_{\mu\nu} F^{\mu\nu}(0) | 0 \rangle + \dots \end{aligned} \quad (37)$$

Here we have shown only two terms, the leading power, found from perturbation theory with vanishing quark masses (σ_{tot} , of course, is infrared safe), and the ‘‘gluon condensate’’ term, suppressed by $(x^2)^2$ compared to perturbation theory at short distances, but still singular. Suppose

we had never heard of the OPE; could we have discovered the F^2 term in Eq. 37 on the basis of perturbative arguments alone?

In fact, we could have done so, by looking at the large-order behavior of coefficients $C_0^{(n)}$ of $\alpha_s(Q)^n$. [73] Of course, we are not able to compute the full n th order coefficients, but it is not too difficult to identify subintegrals that take the form

$$\begin{aligned} C_0^{(0)} &\sim (1/Q^4) \int_0^{Q^2} dk^2 k^2 \ln^n(k^2/\mu^2) \\ &\sim (1/2) (1/2)^n n!. \end{aligned} \quad (38)$$

Diagrams that contribute to this result are illustrated in Fig. 10, where H absorbs lines that are off-shell by order Q^2 , and T lines that have soft momenta. Each power of the logarithm in Eq. 38 is associated with a self-energy (or other one-loop subdiagram that requires renormalization). A little renormalization group footwork shows that the sum of all diagrams of this sort takes the remarkably simple form

$$\begin{aligned} &\int_0^{Q^2} dk^2 k^2 \alpha_s(k^2) \\ &= \int_0^{Q^2} dk^2 k^2 \frac{\alpha_s(Q^2)}{1 + \left(\frac{\alpha_s(Q^2)}{4\pi}\right) b_2 \ln(k^2/Q^2)}, \end{aligned} \quad (39)$$

in which expanding the denominator of the one-loop perturbative running coupling in powers of $\alpha_s(Q^2)$ generates all the $n!$ coefficients. Behavior of this sort, which is associated with the (infrared) singularity of the running coupling (at $k^2/Q^2 = \exp[-4\pi/b_2\alpha_s(Q^2)]$), is known as an “infrared renormalon”. [74] If we still want to go to all orders, we would be tempted to regularize

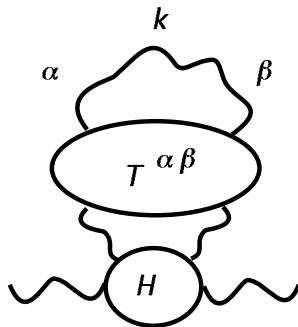


Figure 10: Subintegral that gives rise to nonconvergent behavior in $\sigma_{\text{tot}}^{e^+e^-}$.

perturbation theory by removing contributions from $k^2 \sim \Lambda_{\text{QCD}}^2$ in all integrals like those in Eq. 39, and replacing them by new, nonperturbative parameters. This may be done in a surprisingly simple fashion. [73] We can use gauge invariance arguments to show that there is only one such parameter, which may be identified with the gluon condensate,

$$C_{\text{PT}} \rightarrow C_{\text{PT}}^{\text{reg}} + \frac{\alpha_s \langle 0|F^2(0)|0\rangle}{Q^4}. \quad (40)$$

Order-by-order in perturbation theory, the vacuum expectation value of the gluon condensate generates all of the integrals of Eq. 39 above, for which the upper limit appears as a renormalization scale. Hence, to order $1/Q^4$, it is natural to replace the factorial growth from soft gluons by

a nonperturbative value for the gluon condensate. In summary, if there were no OPE, it would be necessary to invent it, to derive useful information from infinite-order perturbation theory.

Could something similar be going on for resummed cross sections, leading not to the OPE but to something else? There are reasons to suggest that this might be so, although to my knowledge no general formalism yet exists. The simplest, but still very subtle, phenomenological implications for the nature of power corrections associated with infrared renormalons have been widely discussed, especially in the context of $1/Q$ and/or $1/Q^2$ corrections in e^+e^- event shapes [68] and Drell-Yan cross sections. [69, 70] For the former, the DELPHI collaboration has employed the recently-increased LEP energy to compare theoretical expectations to data over a very wide range of c.m. energies. [13]

An example where some progress toward an operator formalism has been made is in Sudakov resummation for the transverse momentum distribution of a Drell-Yan pair, noted above in Eq. 29. [60] Here, as was shown years ago, the natural transformation (the moment of Sec. 4 above) is to impact parameter space, in terms of which the transformed cross section includes a factor $\tilde{P}(b)$ that resums all logarithms of b , according to

$$\begin{aligned} \ln \tilde{P}(b) = & \int_0^{Q^2} \frac{dk_T^2}{k_T^2} A(\alpha_s(k_T^2)) \ln(Q^2/k_T^2) \\ & \times \left(e^{-ik_T \cdot b} - 1 \right), \end{aligned} \quad (41)$$

where Q is the mass of the produced pair, and where at leading logarithm k_T may be interpreted as the transverse momentum of a gluon. In Eq. 41, we have suppressed various nonleading terms for simplicity. Up to such corrections, this is of the form of Eq. 33 order-by-order in logarithms of b , once b is identified as iN/Q . In this form, however, as in Eq. 33, the perturbative running coupling passes through a region where it diverges. Following our reasoning for e^+e^- annihilation above, however, we may replace that region by one or more nonperturbative parameter. This leads to the classic form suggested by Collins and Soper, [60]

$$\begin{aligned} \ln \tilde{P}(b) = & \int_{(1/b^*)^2}^{Q^2} \frac{dk_T^2}{k_T^2} A(\alpha_s(k_T^2)) \ln(Q^2/k_T^2) \\ & \times \left(e^{-ik_T \cdot b} - 1 \right) \\ & + g_1 b^2 \ln(Q^2 b^2) + \dots \end{aligned} \quad (42)$$

Here g_1 is our nonperturbative parameter, which replaces the integral $k_T < 1/b^*$,

$$\int_0^{(1/b^*)^2} \frac{dk_T^2}{k_T^2} A(\alpha_s(k_T^2)) \ln(Q^2/k_T^2) (b^2 k_T^2) + \dots, \quad (43)$$

with b^* an adjustable parameter (upon which g_1 depends).

We now ask whether there is an operator, whose perturbative vacuum expectation value is identical to the infrared perturbative contribution 43, which we have removed? In fact, there is, at least for g_1 , and it turns out to be a rather direct, if nonlocal, generalization of the gluon condensate encountered in the OPE above. [70] It is illustrated in Fig. 11, and consists of a pair of field strengths integrated over the paths of two light-like eikonized quarks, like those encountered above in the resummation of threshold corrections for heavy quark production (Fig.

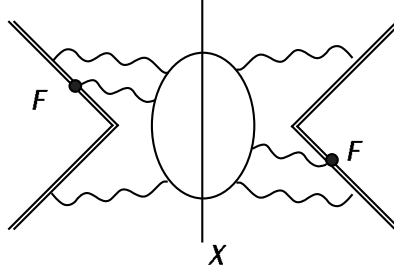


Figure 11: Representation of nonlocal vacuum expectation value.

9). The eikonal lines, which may be written as path-ordered exponentials of the gauge field,

$$\begin{aligned} & \Phi_p[S p + x, S_0 p + x] \\ &= \text{P exp} \left(-ig \int_{S_0}^S ds p^\mu A_\mu(p s + x) \right), \end{aligned} \tag{44}$$

enforce gauge invariance in the nonlocal condensate, defined as

$$\begin{aligned} & \langle 0 | \Phi_{p_2}^\dagger(0, -\infty) (\vec{\mathcal{F}}_{p_1}(0) - \vec{\mathcal{F}}_{p_2}^\dagger(0)) \\ & \quad \times \Phi_{p_1}(0, -\infty) | 0 \rangle, \end{aligned} \tag{45}$$

where \mathcal{F} is constructed from $F^{\mu\nu}$ by

$$\begin{aligned} \mathcal{F}_p^\alpha(x) &= -ig \int_{-\infty}^0 ds \Phi_p(x, x + sp) \\ & \quad \times p_\mu F^{\mu\alpha}(sp + x) \Phi_{-p}(x + ps, x). \end{aligned} \tag{46}$$

Compared to the local gluon condensate of the OPE, the field strengths are now free to migrate along a one-dimensional manifold of lightlike paths, whose directions have been determined by the incoming particles. In this sense, it is a natural first generalization of the OPE. Correlations in even higher dimensions are almost certainly necessary, however, when dealing with quantities like event shapes, which are sensitive to interactions at very large times. It is my hope that the clarification of these issues will be one of the interesting developments of the coming years.

6 Optimistic Summary

I would like to suggest that the qualitative and quantitative successes of perturbative QCD this year are a prologue.

Each of the major currents of research touched on in this talk point to further challenges. The low- x and diffractive behavior of DIS structure functions as seen at HERA is suggesting new avenues to connect hadronic and partonic degrees of freedom. The BFKL program continues to inspire inventive theoretical and experimental advances. At the same time, Sudakov resummation

methods are making it possible to understand the successes and limitations of NLO calculations of high- p_T and related cross sections at past and future collider runs. There is also a growing awareness of the need for a theory of power corrections beyond the OPE, hints for which may be found in infrared renormalon analysis of power corrections. The extended energy reach of LEP affords new opportunities to check and develop these ideas. There were other related and important topics discussed at the conference which space does not allow me to discuss here beyond the merest mention, including the treatment of spin and nuclear effects in hard scattering, photoproduction and direct photon production, and elastic scattering. Many of the same methods, opportunities and challenges sketched above apply to these topics as well.

Theorists will have to work hard to keep up with data like those of this year. Now we are really ready to study quantum chromodynamics.

Acknowledgements

I would like to thank the organizers of ICHEP96 for their invitation, and Wu-Ki Tung for helpful conversations. This work was supported in part by the National Science Foundation under grant PHY930988.

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Questions

G. Farrar, Rutgers Univeristy:

Will theory be able to constrain power-law corrections, e.g. those you mentioned for LEP event shape variables, or will they have to be fit as for higher-twist corrections in DIS?

G. Sterman:

Parameters describing power-law corrections are certainly nonperturbative, and must be taken from experiment, like parton distributions. It is unlikely that a few numbers will describe these corrections. It is my expectation, however, that a well-defined set of functions will turn out to control power corrections to many event shapes. Because these corrections are relatively large, they may well be more accessible experimentally than are higher-twist corrections in DIS.

G. Wolf, DESY:

You have alluded to diffraction scattering in BFKL and DGLAP. You seem to suggest that diffraction scattering should be stronger in BFKL as compared to DGLAP. Did I understand you correctly?

G. Sterman:

Yes, although I would put it slightly differently. I would suggest that the diffractive component of DIS structure functions evolves by a process that is of the BFKL type, lacking strong ordering in transverse momenta.