Fast Indexing of Lattice Vectors for Image Compression

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Summary
Visual communication is becoming increasingly important with applications in several areas such as multimedia, communication, data transmission and storage of remote sensing images, satellite images, education, medical etc….The image data occupies large space. Meeting bandwidth requirements and maintaining acceptable image quality simultaneously is a challenge. Hence image compression is required. There are mainly two types of compression systems- lossy and lossless. When quantization is involved in compression process, compression will be a lossy compression. Lattice Vector Quantization is a simple but powerful tool for vector quantization. After quantization of vectors using lattice structure, indexing of lattice vectors is required. In this work our attention is on the problem of efficient indexing. MSE and PSNR of different images using proposed method are calculated. Perceptual performance of image coding is also shown in the result.

Key words:
Lattice Vector Quantization (LVQ), Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR)

1. Introduction

Vector Quantization has been applied in many wavelet based image coding algorithm due to its superior performance over scalar quantization. But the LBG algorithm [1] which is commonly used to design vector quantizer causes high computational cost and coding complexity especially as the vector dimension and bitrate increase. LVQ is simple but powerful technique for vector quantization and can be viewed as a vector generalization of uniform scalar quantization. Whatever the source distribution is, LVQ will always outperform uniform scalar quantizers. Fast encoding and decoding algorithms making use of simple rounding operations for lattice quantizers have been proposed by Conway and Sloane [2] and [3]. Consequently the encoding and decoding speed does not depend on the number of codewords within the codebook. Its computational simplicity and codebook robustness make it attractive and widely used in the lossy data compression field. Even though the idea behind LVQ being quite simple, the indexing of lattice codevectors is not a trivial operation. To solve the indexing problem, methods have been introduced for indexing Laplacian or Gaussian distributions, as for example in [5], [6]. A large number of important sources of data, including subband image and speech coefficients, especially those obtained from wavelet transformation, can be modeled by the probability density function (pdf) of type Laplacian or Gaussian or Generalized Gaussian. One interesting property of the sources with these distributions is that shells of norm l_1 or l_2 or l_p corresponds to surface of constant probability. It leads to the development of effective product codes. In [7] indexing is based on leader and makes use of the theory of partitions. The use of theory of partition overcomes the complexity and storage requirement for generating and indexing the leaders.
The indexing operation must be as fast as possible at the coding stage, as well as at the decoding one. In this paper we propose a low complexity indexing method for four dimension vectors. This method directly calculates the index of the vector, not depends on the leaders and cardinalityof the vector.

2. Lattice Vector Quantization

The computational complexity and large storage is one of the drawbacks of simple, unstructured VQ in practical implementation. To overcome this problem, structured codebooks are developed with different structure i.e. different lattices corresponding to different types of LVQ. A lattice is the set of all vectors of the form

$$L = \sum_{i=1}^{n} u_i a_i \text{ ------------------------------(1)}$$

where { u_1 , u_2, -------------, u_n } are all integers and {a_1 , a_2 ,------------- ,a_n } is a set of n linearly independent vectors. In lattice vector quantization (LVQ), the input data is mapped to the lattice points of a certain chosen lattice type.

2.1 Optimum Lattice

In [4] properties of different lattices are investigated and the optimal lattices for several dimensions are determined. Optimum lattice in a specific dimension is the lattice by

Manuscript received May 5, 2012
Manuscript revised May 20, 2012

which we can best cover that space. The optimum lattice problem finds a strong analogy with the sphere packing theory, where it is searched to arrange the maximal number of equal non overlapping spheres in a given volume in n-dimensional space. The best packing lattice will be the one providing the densest packing of identical spheres together. In this paper we have used the D_4 lattice. This lattice is having densest packing in four dimension. It consists of all points with integer coordinates (x_1, x_2, x_3, x_4) with $x_1 + x_2 + x_3 + x_4$ even.

2.2 Lattice Codebook

LVQ codebook is a set of finite number of lattice points out of infinite lattice points. Lattice points in a codebook are called code vectors or code words. The finite number of lattice points selected from the truncation of the lattice. In our case spherical shape is used for truncation. For selecting the lattice points for spherical truncation l_2 is used .

If $x = x_1, x_2, x_3 ----, x_t$ is a codeword or codevector closest to the point (0, 0, 0, 0), then l_2 norm is given by following equation.

$$l_2 = x_1^2 + x_2^2 + ---- + x_t^2 \text{ ------------------- } (2)$$

The elements of the codevector are quantized values of the input samples. The same codebook must be maintained both at the transmitter and the receiver. The codebook is searched to find the codevector closest to the input vector based on a distortion error measure. The index of the selected codevector is transmitted to the receiver. The receiver requires a simple table lookup, the index received is used to select the reproduction code vector that approximates the input vector.

2.3 Quantization Algorithm

Conway and Sloane [3] developed a fast quantization algorithm which makes searching of the closest lattice point to a given vector extremely fast. For a given vector x, the closest point of D_4 is whichever of f(x) and g(x) has an even sum of components (one will have an even sum, the other an odd sum). This procedure works because f(x) and g(x) differ by one in exactly one coordinate, and so precisely one of $\sum f(x_i)$ and $\sum g(x_i)$ is even and the other is odd. For example find the closest point of D_4 to x= (0.2, 1.8, -0.7, 0.4)

f(x)=(0, 2, -1, 0) and g(x)= (0, 2, -1, 1) since the last component of x is the furthest from an integer, so it is changed from 0 to 1 in g(x). $\sum f(x_i)$=odd, while that of $\sum g(x_i)$=even. Therefore g(x) is the point of D_4 closest to x.

3. Indexing of Lattice Vectors

Once the vectors are quantized into lattice vectors, we may assign a unique and decodable index for each lattice vector.

3.1 Indexing based on product code

When indexing the lattice vectors by a product code, the index is constructed by the concatenation of two other indices: one corresponding to the index of the norm of the vector (prefix) and the other corresponding to the position of the vector on the given shell of constant norm (suffix). Even if the computation of the prefix is trivial, the suffix needs the enumeration and indexing of the lattice vectors lying on given hypersurfaces. Furthermore, increasing the dimension of the space can make the indexing operation prohibitive since the number of vectors lying on a shell grows dramatically with the norm. The indexing of the suffix is usually done according to two different techniques. The most common attributes an index taking into account the total number of vectors lying on a given hyper-surface (cardinality) [1], [2] ,[9]. Another approach, proposed in [5], exploits the leaders of a lattice.

3.2 Indexing based on enumeration

A number of enumeration solutions have been proposed for the case of laplacian and gaussian distributions and for different lattices. A recursive formula to compute the total number of lattice vectors lying on a l_1 norm hyper pyramid has been intoduced in [1] . This enumeration formula has been extended in [6] for generalized gaussian source distributions with shape factor p between 0 and 2. However the work of [6] proposes a solution to count the number of vectors lying inside a given l_p norm, but it does not propose an algorithm to assign an effective index to the vectors of the Z^n lattice. Furthermore, it does not count the number of vectors lying on a given hyper-surface, which makes difficult the use of product codes. In addition, because the cardinality of an hyper-surfaces can rapidly achieve non tractable values for practical implementations.

3.3 Indexing based on leaders

The method proposed in [6] presents the advantage that the vectors have an effective indexing algorithm on the shells of constant norm and does not attribute the index based on the total number of vectors of the lattice, but based on a small amount of vectors called leaders. On the contrary, the approach used in [12] allows one to design good product codes for indexing the vectors. In [6], the authors have shown that indexing vectors using leaders can save much more memory than LBG-based algorithms. However, this method in its original conception is not so attractive. Indeed, in order to avoid a heavy computational complexity to index the leaders and allow direct addressing of the leaders' coordinates, one must construct a look-up-table in the dimension of the hyper-space containing all the leaders. This means that for each norm and each lattice dimension, one should store a table which actually is a tensor of high order n (i.e. which is as high as the dimension of the vectors). Furthermore, in order to attribute an index to a leader it is necessary to generate all leaders, which remains a highly complex operation, especially for high dimensions and norms. For applications where one needs high-dimensional vectors with large dynamic ranges (implying many shells), as for example in the multiresolution data compression context, to generate and store a large amount of huge tensors is quite prohibitive for practical purposes.

In [8] and [11] a partition function q(r,n) is used which not only gives the total number of leaders lying on a given hyper-pyramid but can also be used to provide unique indices for these leaders. This function gives the number of partitions of r with atmost n parts(in partition number theory it is equivalent to say number of partitions of r with no element greater than n with any number of parts). It is more realistic for LVQ where the vector dimension is fixed. By this method indexing a leader can be done even for large vector dimension which is impossible when indexing is done directly on all the vectors of a hyper-surface. In [10] author describes a new alternative for indexing Z^n lattice vectors lying on generalized gaussian distribution shells.

4. Proposed Method for indexing of lattice vectors

In this method there is no need to find out the index of leader, norm of the vector and finally the rank of the vector in a group of similar norm. Following steps are followed to get the encoded data.

Step1- Apply wavelet decomposition

Step 2-Reshape last level Subbands into Rx4 shape, where value of R depends on the DWT composition level and the size of input image. For example if the size of subband is 8x8 then it is reshaped into 16x4 size.

Step3- Apply quantization algorithm of D_4 lattice to quantize vectors (result will be a matrix of Rx4)

Step4- Indexes are assigned to the quantized vectors, at the output of encoder Rx1 matrix is generated. Following procedure is followed to get index of the vector [A B C D]

[A B C D] = quantized vector (QV) + │minimum Value of Rx4 quantized vector (QV) │

base = maximum Value of Rx4 quantized vector (QV) + │minimum Value of Rx4 quantized vector (QV) │ + 1

index = (A * base^3) + (B * base^2) + (C * base) + D

5. Simulation Results

The following test images of size 512x512 are decomposed into 2 DWT levels. 2x2 block size is used for truncating the last level subbands which determine the vector size. In this case 2x2 block size results in four dimensional vectors. Quantization algorithm is used to find the closest point of lattice D_4. Index to the closest lattice points are assigned using proposed indexing method. Indices of the vectors works as input to the decoder, at the decoder reverse logic is applied to get the reconstructed image.

Table 1:MSE and PSNR of different test images using proposed method of indexing for lattice vectors

Test Image	MSE	PSNR
Peppers	72.0748	29.5530
Barbara	192.6546	25.2830
Scenary	113.7380	27.5717
Goldhill	65.1658	29.9906
Scenary	113.7380	27.5717

Table 2: Values of MSE and PSNR for different quantization levels for Barbara image

Wavelet Decomposition level is 2						
No. of quantization levels	105	2x105	3x105	5x105	10x10 6	10x108
MSE	1.2514 e+003	339.97 26	256.1 905	216.1 534	192.7 291	192.65 46
PSNR	17.157 0	22.816 4	24.04 52	24.78 32	25.28 13	25.283 0
Wavelet Decomposition level is 3						
MSE	1.6500e +003	599.9 965	432.9 028	376.0 863	337.4 696	337.36 06
PSNR	15.9560	20.34 93	21.76 69	22.37 79	22.84 85	22.849 9

Original Image

Reconstructed image

Original Image

Reconstructed image

Fig. 1: Perceptual Performance of Pepper and Barbara image coding using proposed indexing method for Lattice Vector Quantization

6. Conclusion

In this paper we have proposed a solution for indexing lattice vectors. In our work, indexing of vectors is not based on leaders, direct index is assigned to the vectors. There is no need to store the codebook thus reducing the memory usage. We have applied this method with D4 lattice and spherical truncation. As future works we will extend the use of this method for lattices Dn (n>4)and E8.

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