The effect of Majorana phase in degenerate neutrinos

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Abstract

There are physical Majorana phases in the lepton flavor mixing matrix when neutrinos are Majorana fermions. In the case of two degenerate neutrinos, the physical Majorana phase plays the crucial role for the stability of the maximal flavor mixing between the second and the third generations against quantum corrections. The physical Majorana phase of π guarantees the maximal mixing to be stable against quantum corrections, while the Majorana phase of zero lets the maximal mixing be spoiled by quantum corrections when neutrino masses are of $O(\mathrm{eV})$. The continuous change of the Majorana phase from π to 0 makes the maximal mixing be spoiled by quantum corrections with $O(\mathrm{eV})$ degenerate neutrino masses. On the other hand, when there is the large mass hierarchy between neutrinos, the maximal flavor mixing is not spoiled by quantum corrections independently of the Majorana phase.

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1 Introduction

Recent neutrino oscillation experiments suggest the strong evidences of the tiny neutrino masses and lepton flavor mixings[1]-[3]. Studies of the lepton flavor mixing matrix, which is so-called Maki-Nakagawa-Sakata(MNS) matrix[4], will give us important cues of the physics beyond the standard model. One of the most important studies of the lepton flavor mixing is the analysis of the quantum correction on the MNS matrix[5]-[10].

In this paper we analyze the effect of the Majorana phase for the stability against quantum corrections of the maximal mixing between the second and the third generations, which is suggested by the atmospheric neutrino experiments[2, 3]. There is one physical Majorana phase in the 2×2 MNS matrix when neutrinos are Majorana fermions. This Majorana phase plays the crucial role for the stability against quantum corrections when two neutrino masses are degenerate. When Majorana phase is equal to zero, the maximal mixing is spoiled by quantum corrections when neutrino masses are of $O(\mathrm{eV})$[7]. On the other hand, the maximal mixing is not spoiled by quantum corrections when Majorana phase is equal to π because the neutrino mass matrix has the pseudo-Dirac texture[11]. The continuous change of the Majorana phase from π to 0 makes the maximal mixing be spoiled by quantum corrections when neutrino masses are of $O(\mathrm{eV})$.

When there is the large mass hierarchy between neutrinos, the lepton flavor mixing is stable against quantum corrections independently of the Majorana phase.

2 Stability of the maximal mixing and Majorana phase

The neutrino mass matrix of the second and the third generations

$$\kappa = \begin{pmatrix} \kappa_{22} & \kappa_{23} \\ \kappa_{23} & \kappa_{33} \end{pmatrix} \tag{1}$$

is diagonalized as

$$U^T \kappa \, U = D_\kappa \,, \tag{2}$$

where D_κ is given by

$$D_\kappa = \begin{pmatrix} m_2 & 0 \\ 0 & m_3 \end{pmatrix}, \tag{3}$$

with $m_i \geq 0$ $(i = 2, 3)$. The unitary matrix U is defined as

$$U = \begin{pmatrix} \cos\theta_{23} & \sin\theta_{23} \\ -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi/2} \end{pmatrix}, \tag{4}$$

where θ_{23} is the mixing angle between the second and the third generations, and ϕ denotes the physical Majorana phase of neutrinos. In the diagonal base of charged lepton masses, U is just the MNS matrix.

Since the atmospheric neutrino experiments suggest[2, 3]

$$\sin^2 2\theta_{23} \simeq 1 \,, \tag{5}$$

we analyze the stability of the mixing angle θ_{23} against quantum corrections around $\theta_{23} = \pi/4$. In this case, eq.(2) shows

$$
\begin{aligned}
\kappa &= U^* D_\kappa U^\dagger \\
&= \frac{1}{2} \begin{pmatrix} m_2 + m_3 e^{-i\phi} & -m_2 + m_3 e^{-i\phi} \\ -m_2 + m_3 e^{-i\phi} & m_2 + m_3 e^{-i\phi} \end{pmatrix} .
\end{aligned} \tag{6}
$$

Quantum corrections change the form of κ[8, 9] as

$$
\begin{aligned}
\kappa' &= \begin{pmatrix} 1 & 0 \\ 0 & 1+\epsilon \end{pmatrix} \kappa \begin{pmatrix} 1 & 0 \\ 0 & 1+\epsilon \end{pmatrix} , \\
&= \frac{1}{2} \begin{pmatrix} m_2 + m_3 e^{-i\phi} & (-m_2 + m_3 e^{-i\phi})(1+\epsilon) \\ (-m_2 + m_3 e^{-i\phi})(1+\epsilon) & (m_2 + m_3 e^{-i\phi})(1+2\epsilon) \end{pmatrix} + O(\epsilon^2) .
\end{aligned} \tag{7}
$$

The unitary matrix U' which diagonalizes κ' shows us whether the maximal mixing between the second and the third generations is spoiled by quantum corrections or not. The mixing angle $\hat{\theta}_{23}$ which diagonalizes κ' in eq.(7) is given by

$$\tan 2\hat{\theta}_{23} = \frac{1}{\epsilon} \frac{\delta m^2}{m_2^2 + m_3^2 + 2 m_2 m_3 \cos\phi} + O(\epsilon^0) \,, \tag{8}$$

where

$$\delta m^2 \equiv m_3^2 - m_2^2 \,, \tag{9}$$

which is determined by the atmospheric neutrino experiments[2, 3]. Equation (8) shows that the mixing angle $\hat{\theta}_{23}$ is stable (unstable) against quantum corrections when the value of $\delta m^2/(m_2^2 + m_3^2 + 2 m_2 m_3 \cos\hat{\phi})$ is larger (smaller) than that of ϵ.

When two mass eigenvalues are degenerate as $m_2 \simeq m_3$, we can easily see the following facts at $\phi = 0$, and π:

(i) When $\phi = 0$, eq.(8) derives

$$\tan 2\hat{\theta}_{23} \simeq \frac{1}{\epsilon} \frac{m_3 - m_2}{m_3 + m_2} \,. \tag{10}$$

This means that the mixing angle $\hat{\theta}_{23}$ is stable (unstable) when the value of $(m_3 - m_2)/(m_3 + m_2)$ is larger (smaller) than that of ϵ. When $m_3 = O(1)$ eV, the value of $(m_3 - m_2)/(m_3 + m_2)$ is small enough for the maximal mixing to be spoiled by quantum corrections[7].

(ii) When $\phi = \pi$, eq.(8) derives

$$\tan 2\hat{\theta}_{23} \simeq \frac{1}{\epsilon} \frac{m_3 + m_2}{m_3 - m_2} . \tag{11}$$

This means that the maximal mixing is not spoiled by quantum corrections. It is because the absolute values of off-diagonal elements in κ' are much larger than those of diagonal elements, which is so-called the pseudo-Dirac texture. Equation (11) is induced just from the initial value of $\theta_{23} = \pi/4$. For the general values of θ_{23}, it is satisfied that $\tan 2\hat{\theta}_{23} = \tan 2\theta_{23} + O(\epsilon^2)$[10]. This means the mixing angle of $\hat{\theta}_{23}$ receives little changes from quantum corrections for the general values of θ_{23}.

The behaviors against quantum corrections in cases of (i) and (ii) are completely different from each other as shown above. The physical Majorana phase of π guarantees the maximal flavor mixing to be stable against quantum corrections, while the Majorana phase of zero lets the maximal mixing be spoiled by quantum corrections when $m_3 = O(\text{eV})$. However eq.(4) suggests that they are connected with each other by the continuous change of Majorana phase ϕ. Therefore this Majorana phase ϕ plays the crucial role for the stability of the lepton flavor mixing angle against quantum corrections.

Figure 1 shows the contour plot of $\sin^2 2\hat{\theta}_{23}$ for the continuous changes of ϕ and $m_2{}^2$. We use the value of $\delta m^2 = 3 \times 10^{-3} \text{eV}^2$ [2, 3]. The value of ϵ is determined by two parameters of $\tan \beta = 20$ and the intermediate scale $M_R = 10^{13}$ GeV[10]. We input the low-energy data to θ_{23} and δm^2, and show the value of $\hat{\theta}_{23}$ at $M_R = 10^{13}$ GeV in Fig.1. The continuous change of the Majorana phase from π to 0 makes the maximal mixing between the second and the third generations be spoiled by quantum corrections when $m_2^2 = O(1)\text{eV}^2$. The larger the value of $m_2{}^2$ becomes, the smaller the angle $\hat{\theta}_{23}$ becomes. As we have shown at (ii), the maximal mixing is not spoiled by quantum corrections around $\phi = \pi$ independently of the value of $m_2{}^2$.

Figure 1 also shows that the maximal mixing is stable against quantum corrections around $m_2{}^2 \simeq 0$ independently of the value of ϕ. It is because eq.(8) derives

$$\tan 2\hat{\theta}_{23} \simeq \frac{1}{\epsilon} \tag{12}$$

when $m_2{}^2 = 0$. The physical Majorana phase can be rotated out by the field redefinition in this case. Equation (12) suggests that the maximal mixing is not spoiled by quantum
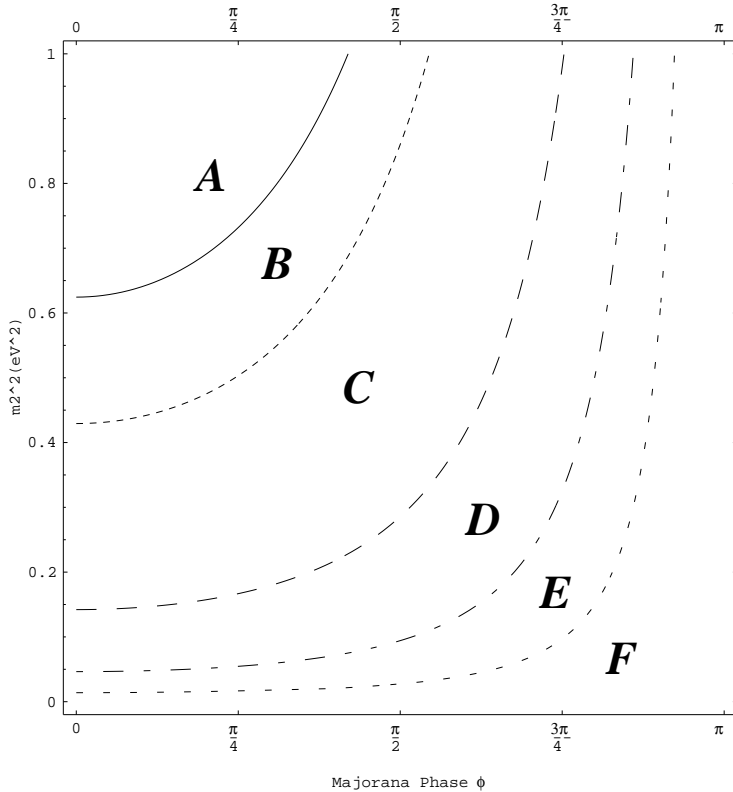
Fig. 1: The contour plot of $\sin^2 2\hat{\theta}_{23}$ for the continuous changes of the Majorana phase ϕ and $m_2{}^2$ (A: $\sin^2 2\hat{\theta}_{23} < 0.05$, B: $0.05 \leq \sin^2 2\hat{\theta}_{23} < 0.1$, C: $0.1 \leq \sin^2 2\hat{\theta}_{23} < 0.5$, D: $0.5 \leq \sin^2 2\hat{\theta}_{23} < 0.9$, E: $0.9 \leq \sin^2 2\hat{\theta}_{23} < 0.99$, F: $0.99 \leq \sin^2 2\hat{\theta}_{23}$). We use the experimental value of $\delta m^2 = 3 \times 10^{-3}$ eV^2[2, 3]. The value of ϵ is determined by two parameters of $\tan\beta = 20$ and $M_R = 10^{13}$ GeV [10]. $\hat{\theta}_{23}$ is the mixing at $M_R = 10^{13}$ GeV.

corrections independently of the Majorana phase ϕ when there is the large mass hierarchy between m_2 and m_3. Equation (12) is induced just from the initial value of $\theta_{23} = \pi/4$, and for the general values of θ_{23}, it is satisfied that $\tan 2\hat{\theta}_{23} = \tan 2\theta_{23}(1 - \epsilon \sec 2\theta_{23}) + O(\epsilon^2)$[10]. This means the mixing angle of $\hat{\theta}_{23}$ receives little changes from quantum corrections for the general values of θ_{23}.

This result implies that the flavor mixing matrix with the large mass hierarchy of $|m_3| \gg |m_2| \gg |m_1|$ is also stable against quantum corrections in the three generation neutrinos. It has been shown by the numerical analyses in Ref.[10].

3   Summary

There are physical Majorana phases in the MNS matrix when neutrinos are Majorana fermions. The Majorana phase in the MNS matrix plays the crucial role for the stability of the maximal mixing between the second and the third generations against quantum

corrections when two neutrino masses are degenerate. When the Majorana phase is equal to zero and neutrino masses are of $O(\text{eV})$, the maximal mixing is spoiled by quantum corrections. On the other hand, when Majorana phase is equal to π, the maximal mixing is not spoiled by the quantum corrections because of the pseudo-Dirac texture of the neutrino mass matrix. The continuous change of the Majorana phase from π to 0 makes the maximal mixing be spoiled by quantum corrections when neutrino masses are of $O(\text{eV})$.

When there is the large mass hierarchy between neutrinos, the maximal mixing is not spoiled by quantum corrections independently of the Majorana phase. This result can explain that the mixing angles are stable against quantum corrections in the three generation neutrinos with the large mass hierarchies of $|m_3| \gg |m_2| \gg |m_1|$.

We can also analyze the effects of neutrino Majorana phases in the cases of three generation neutrinos with mass hierarchies of $|m_2| \sim |m_1| \gg |m_3|$ and $|m_3| \sim |m_2| \sim |m_1|$ [12].

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